3rd Financial Risks International Forum

Risk Dependencies. Paris, March 25 & 26, 2010

Pricing CDOs with state dependent stochastic recovery rates

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Pricing CDOs with state dependent stochastic recovery rates

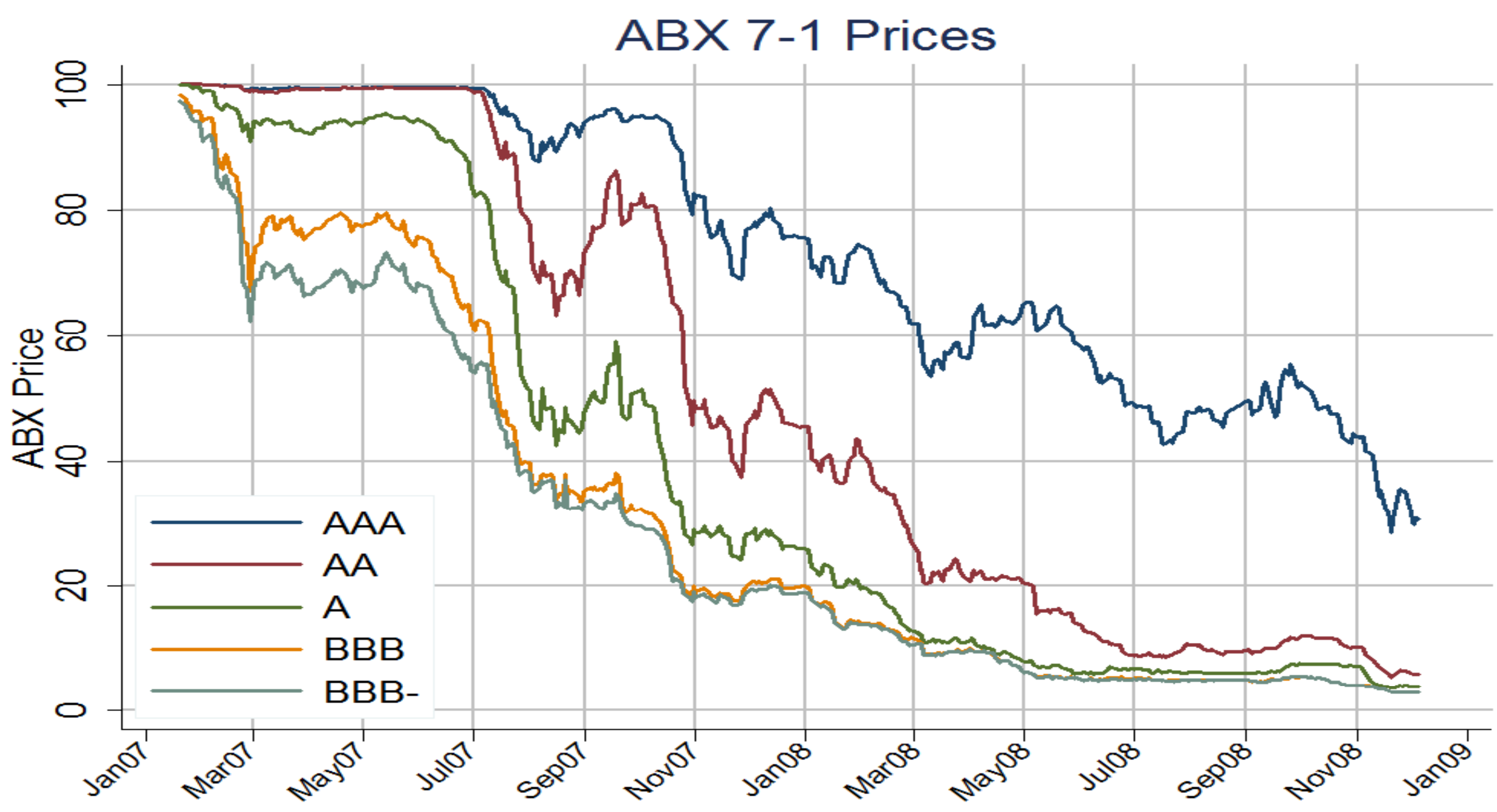
- Outlook
- Practical context : surge in super senior tranche spreads
- Increase of risk for individual losses leads to increase of risk in aggregate losses
  - *For proper positive dependence*
- Consequences of previous analysis
- Comparing risks for granular portfolios sharing the same large portfolio limit
  - *Stochastic recovery rate versus recovery markdown*
- Numerical issues

State dependent recovery rates

- Practical context
  - *Calibration of super senior tranches during the liquidity and credit crisis*
    - Insurance against very large credit losses
    - [30-100] tranche on CDX starts to pay when (approximately) 50% of the 125 major companies in North America are in default
      - Contributed to the collapse of AIG
    - AIG reinsurer of major banks
      - Sold protection through AIG Financial Products (London) and Banque AIG (Paris)
      - Between 440 and 500 billion "CDS" outstanding
      - Issues with accounting, counterparty risk, collateral management and liquidity.
        - Large MTM losses
        - Though no insurance payments were to be made

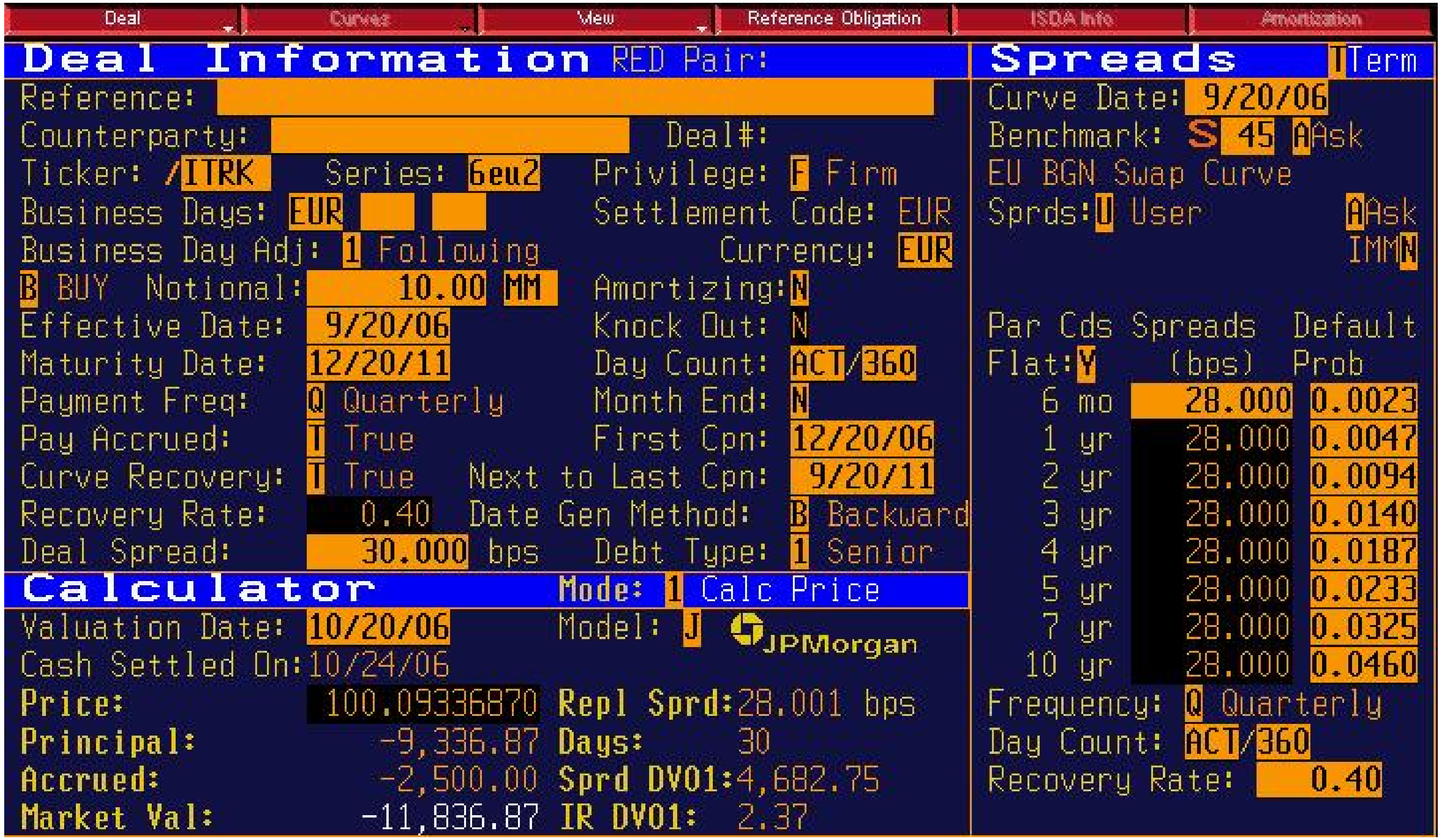
State dependent recovery rates

- Market tsunami on AAA & AA Asset Backed Securities
  - *Increase in spreads induced more damage than actual defaults*
  - *Prices patterns are quite informative for financial modelling*



State dependent recovery rates

- High spreads on super senior tranches
  - *Fixed 40% recovery rate assumption used to be market standard*



State dependent recovery rates

- High spreads on super senior tranches
  - *Could not be calibrated with the standard 40% recovery rate*
  - *[60-100] tranches traded at positive premiums …*

Table 1: Tranche Quotes and Base Correlations for CDX.NA.IG Series 9 5Y

Tranche	Spread (bps)	Base Correlation (%)
0-3%	500, Upfront 68.51 points	39.45
3-7%	773.99	67.12
7-10%	435.52	72.58
10-15%	240.05	85.18
15-30%	126.50	-
30-100%	69.57	-

Source: Data from March 12, supplied by ***Markit***

State dependent recovery rates

- Practical context
  - *Steep "base correlations"*
  - *Implied dependence as measured by implied Gaussian copula correlation*
  - *Increases strongly with respect to attachment point*
    - Reflecting "fat tails" in aggregate loss distributions
    - A bunch of issues of trading desks
      - Negative or increasing tranchelet prices
      - Delta scattering and weird idiosyncratic gamma
  - *These issues are (partly) solved in a stochastic recovery rate approach*
  - *Main issue since 2008 for investment banks*

State dependent recovery rates

- Theoretical context
  - *Aggregate loss = sum of individual losses*
  - *Individual loss = default indicator times loss given default*
  - *Recovery rate = 1 – loss given default / credit notional*
  - *Recovery rates are stochastic*
- Cross dependencies
  - *Amongst default events (copula models, etc.)*
  - *Between default events and recovery rates*
  - *Amongst recovery rates*
- Dependence through common latent factors
  - *For convenience*

State dependent recovery rates

- When does an increase in individual risk leads to an increase in the risk on the aggregate portfolio (sum of individual risks) ?
  - *(Multivariate) Gaussian risks*
    - Individual risks with same expectation
    - Increase in risk = increase in variance
    - Increase in aggregate portfolio risk occurs if and only if pairwise correlations are non negative
  - *What about the general case ?*
    - Stochastic orders
      - Univariate case : convex order (close to second order stochastic dominance)
    - Positive dependence between individual risks

State dependent recovery rates

- Positive dependence
  - *MTP2: Multivariate Total Positivity of Order 2 (Karlin & Rinott (1980))*
    - Log-density is supermodular
  - *Conditionally Increasing*
    - $X = (X_1, \ldots, X_n)$ is CI if and only if $E\left[\phi(X_i) \middle| (X_j)_{j \in J}\right]$ is increasing in $(X_j)_{j \in J}$ for increasing ϕ
  - *Positive association (Esary, Proschan &Walkup (1967))*
  - *PSMD: positive supermodular dependent*
- Gaussian copula
  - *Positive association = PSMD = positive pairwise correlations*
  - *MTP2 = CI (Müller & Scarsini (2001))*

State dependent recovery rates

- Theoretical context
  - *Non Gaussian framework*
    - Individual risks have a probability mass at 0
  - *Increase of risk of individual risks: convex order*
  - *Theorem (Müller & Scarsini (2001))*
    - X and Y random vectors with common conditionally increasing copula
    - X_i smaller than Y_i for all i
    - Then X smaller than Y with respect to dcx (directionally convex) order
      - Then X smaller than Y with respect to stop-loss order
  - *Gaussian copula dependence*
    - Conditionally increasing if and only if the inverse of covariance matrix is a M-matrix
    - Σ non singular, entrywise non negative, Σ^{-1} has positive non diagonal entries

State dependent recovery rates

JOURNAL OF ECONOMIC THEORY 13, 341–360 (1976)

The Arbitrage Theory of Capital Asset Pricing

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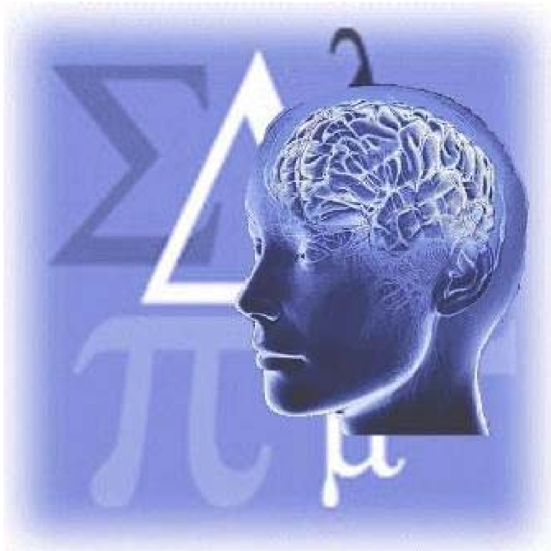
Received March 19, 1973; revised May 19, 1976

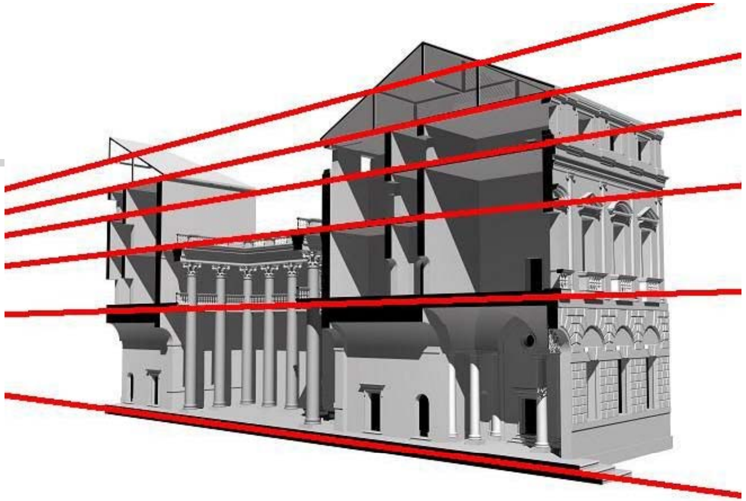
- *Dependence in large dimension*
- *Well known to finance people*
- *Factor models*
  - Arbitrage pricing theory, asymptotic portfolios
    - Chamberlain & Rothschild (1983)
  - Large portfolio approximations (infinite granular portfolios)
    - Conditional law of large numbers
  - Qualitative data with spatial dependence
    - CreditRisk + (Binomial mixtures), CreditMetrics, Basel II (Gaussian copula)
    - Gordy (2000, 2003) Crouhy et al. (2000)
  - Factor models may not be related to a causal view upon dependence
    - De Finetti, exchangeable sequences of Bernoulli variables are Binomial mixtures
    - Mixing random variable latent factor

State dependent recovery rates

- Spatial dependence with qualitative data
  - *Factor models have been used for long in other fields*
    - IQ tests (differential psychology), Bock & Lieberman (1970), Holland (1981)
    - Item Response Models
    - Latent Monotone Univariate Models, Holland (1981), Holland & Rosenbaum (1986)
- Stochastic recovery rates
  - *Modeling of cross dependencies*



State dependent recovery rates



- Stochastic recovery rates
  - *Modeling of cross dependencies*
    - Individual loss = default indicator times loss given default
    - What is important for the computation of tranche premiums (or risk measures) is the joint distribution of individual losses
    - Direct approach: (discretized) individual loss seen as a polychotomous (or multinomial) variable
      - Multivariate Probit model (Krekel (2008))
      - Dual view of CreditMetrics (default side versus ratings)
    - Sequential models
      - Probit or logit models for default events (dichotomous model)
      - Modeling of loss given default : Amraoui & Hitier (2008)

State dependent recovery rates

- Gaussian copula
  - *When is it conditionally increasing?*
  - *One factor case (positive betas)*
    - Gaussian copula is Conditionally Increasing (proof based on Holland & Rosenbaum (1986))
  - *Multifactor case : more intricate, even if all betas are positive, Gaussian copula may not be Conditionally Increasing*
    - Counterexamples
      - Gaussian copula with positive pairwise correlation
      - Increase of marginal risk (convex order)
      - May lead to a decrease of convex risk measures on aggregate portfolio
      - Constraints on conditional covariance matrix
  - *Hierarchical Gaussian copulas*
      - Intra and intersector correlations, Gregory & Laurent (2004)
      - Conditionally Increasing copula (proof based upon Karlin & Rinott (1980))

State dependent recovery rates

- Consequences of previous analysis
  - *Other examples of* ***Conditionally Increasing*** *copulas*
  - *Archimedean copulas, Müller & Scarsini (2005)*
  - *Dichotomous models with monotone unidimensional representation*
    - Default indicators conditionally independent upon scalar V
    - Conditional default probabilities are non decreasing in V
    - Most known and used models
    - Includes additive factor copula models (Cousin & Laurent (2008)), such as generic one factor Lévy model of Albrecher et al. (2007).
- Most portfolio credit risk models associated with CI

State dependent recovery rates

- Consequences of previous analysis
  - *Non stochastic recovery rates*
  - *Analysis of a “recovery markdown”*
  - *Change recovery rate assumption from 40% to 30% (say)*
  - *Change marginal default probability so that expected loss unit is unchanged*
  - *Lemma : increase of marginal risk with respect to convex order*
- Then, given a CI copula, increase of risk of the aggregate portfolio with respect to convex order
  - *Increase in senior tranche premiums*
  - *Or CDO senior tranche spreads*

State dependent recovery rates

- Consequences of previous analysis
  - *Stochastic recovery rate of Amraoui and Hitier (2008)*
  - *Depends only upon latent factor*
    - As in Altman *et al.* (JoB 2005)
  - *Specification of recovery rate is such that conditional upon latent factor is the same as in a recovery mark-down case*
  - *Same conditional expected losses*
    - Same large portfolio approximations
    - Same "infinitely granular" portfolios
    - When number of names tends to infinity, strong convergence of aggregate losses to large portfolio limits
- Stochastic recovery rate (AH) versus recovery markdown
  - *Same infinitely granular portfolios*
  - *But finitely granular portfolios behave (slightly) differently*

State dependent recovery rates

- Stochastic recovery rate (AH) vs recovery markdown
  - *Main comparison result*
  - *Aggregate losses are ordered with respect to convex order*
  - *Smaller risks in stochastic recovery rate specification*
  - *Smaller spreads on senior tranches*
  - *Small numerical discrepancies*
- Ongoing risk management and theoretical issues
  - *Spot recovery versus time to recovery*
    - Bennani &Maetz (2009), Li (2000)
  - *Risk management for distressed names in a stochastic recovery rate framework*
    - Off the run series, bespoke portfolios

State dependent recovery rates

- Numerical issues
  - *Computational efficiency*
    - Especially important when computing Greeks and risk managing CDOs
  - *Needs to be reassessed in case of stochastic recovery models*
  - *Analytical computations of conditional moments*
    - Gram Charlier expansions
    - Same low order approximation than Stein’s method
    - Much quicker than recursions and Monte Carlo