DOUBLE IMPACT

Credit Risk Assessment for Secured Loans

Ali Chabaane
BNP Paribas

Jean-Paul Laurent
ISFA Actuarial School
University of Lyon & BNP Paribas

Julien Salomon
BNP Paribas
julien.salomon@bnpparibas.com

Abstract : The quantitative IRB approach evaluating regulatory capital provides a benchmark framework for credit risk assessment. Nevertheless, the postulated independence between default events and recovery rates seems inappropriate for secured loans such as mortgage loans. The model we introduce is an extension of the regulatory one and takes into consideration correlation effects between default events and collateral market values.

As a result, we show that this is likely to augment capital requirements in comparison with Basel II recommendations.

***Keywords*: Basel II Agreement, Mortgage Loans, Collateral Value, Recovery Rate, Factor Models, Risk Measure, Value at Risk**

Summary

I. Collateral protection

- **Default mechanism**
- Modelling **Default** and **Collateral Value**
- **Dependence** between **Defaults & Collateral Values**

II. Aggregating mortgage portfolios

- **Aggregated loss :** methodology & computation
- **Loss distribution :** Monte-Carlo results

III. Risk measure

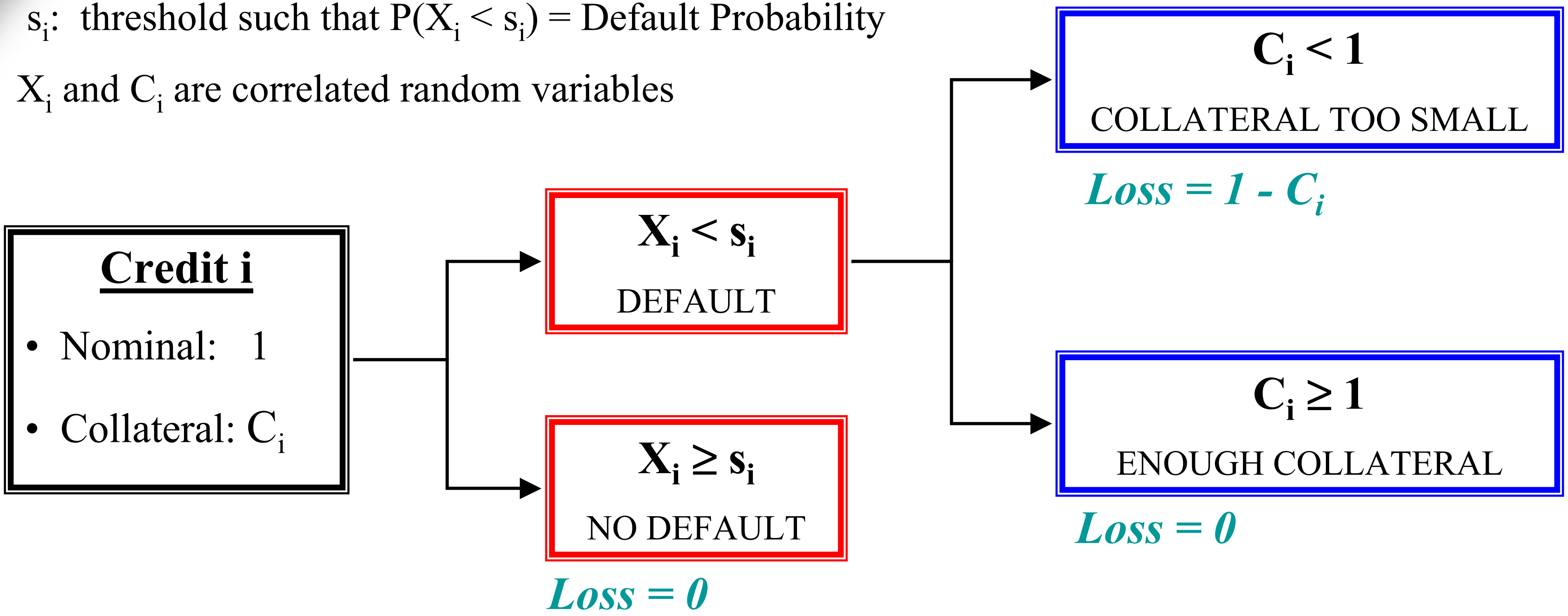
- **Risk measures :** Value at Risk & Expected Shortfall
- **Capital requirements**
- **Comparison** with **Basel II benchmark**

1. Default Mechanism

X_i: **Latent Variable** for the i[th] obligor

s_i: threshold such that $P(X_i < s_i)$ = Default Probability

X_i and C_i are correlated random variables



$$\text{Loss}_i = 1_{\{X_i < s_i\}} \times (1 - C_i)^+$$

2. Modelling Default Latent Variable and Collateral Value

■ Modelling latent variable X_i:

One factor structure : $X_i = \sqrt{\rho}\,\Psi + \sqrt{1-\rho}\,\Psi_i$

- **Ψ systematic risk factor,** gaussian
- Ψ_i specific risk, gaussian i.i.d.
- ρ correlation parameter

■ Modelling Collateral Value C_i

- 1st case: C_i are deterministic $\Rightarrow$ Basel II framework
- 2nd case: C_i are positively correlated variables. Given a **systematic recovery factor** ξ, C_i are independent:
  - J. Frye (Risk, 2000a), E. Canabarro et al. (Risk 2003) : C_i are gaussian
  - M. Pykhtin (Risk 2003), Chabaane, Laurent, Salomon (2003) : C_i are lognormal

3. Modelling Default Latent Variable and Collateral Value

Modelling dependence between X_i and C_i

- **Low recovery rates** associated with **high default rates (**Altman, 2003**)**.
- **<u>Dependence structure</u>** between **<u>Default</u>** & **<u>Collateral Value</u>:**
  - Basel II framework, Canabarro et al (2003): no correlation
  - Frye (2003), Pykhtin (2003): driven by the same risk factor
  - Chabaane, Laurent, Salomon (2003): driven by two correlated risk factors

<u>Remark:</u> assuming the same risk factors is likely to induce harsh collapse of collateral value when default occurs. This strong dependence seems inappropriate for retail banking, especially mortgage portfolio.

4. Credit portfolio Aggregated Loss

- The **aggregated loss** is the **sum of individual** losses.

Credit 1 | **Credit 2** | … | **Credit n** → **Aggregated loss L** for the credit portfolio

$$L = \sum_{i=1}^{n} 1_{\{X_i < s_i\}} \times \left(1 - C_i\right)^+$$

(n obligors)

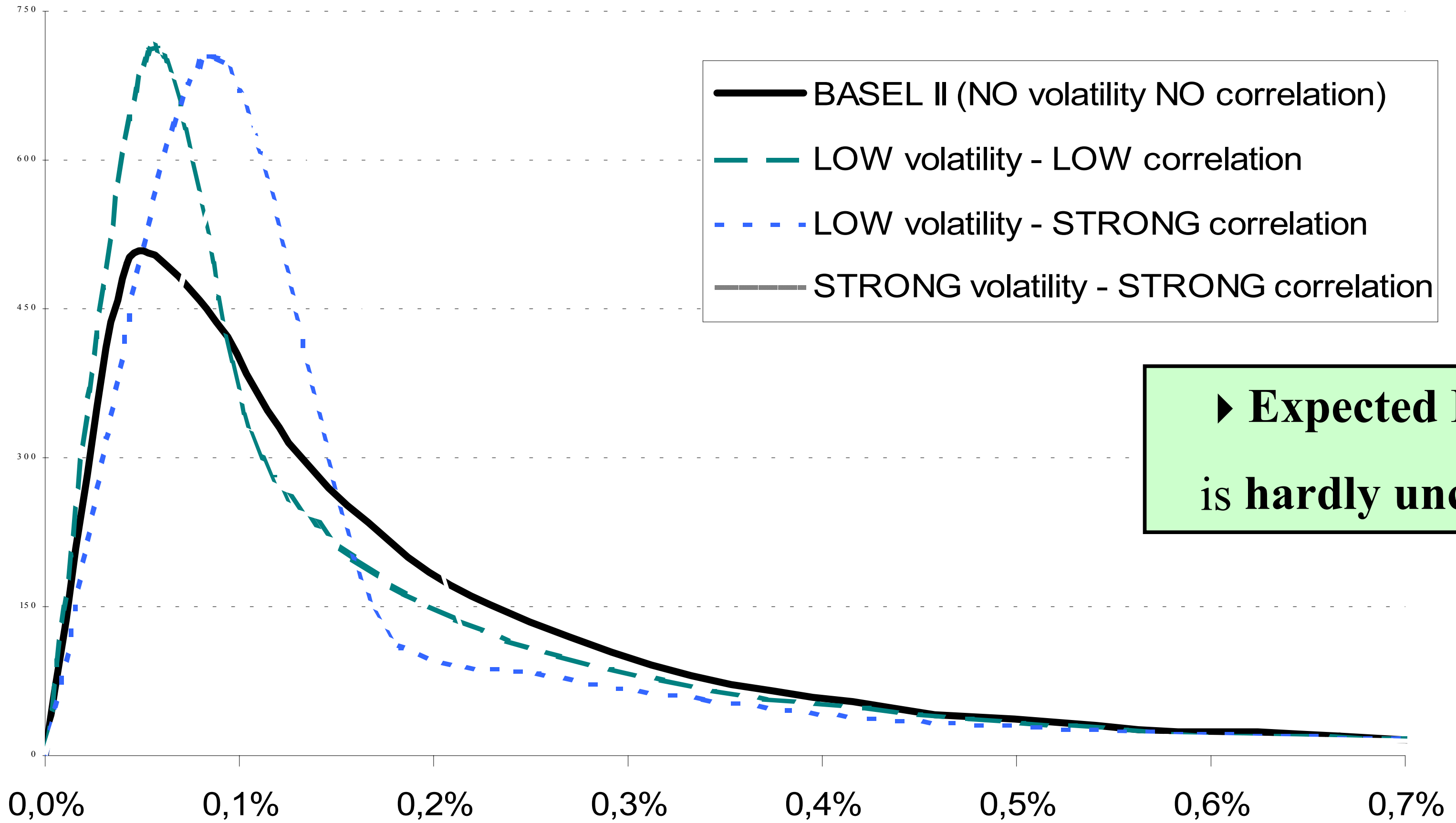
- **Many approaches** may be used to derive the **loss distribution:**
  - Asymptotic expansion (Gordy, Wilde)
  - Monte-Carlo Simulation (individual loss, aggregated loss, …)
  - Fourier inversion techniques

5. Comparison with Basel II benchmark

- Collateral **volatility** leads to **fat tail** distribution

- Default/recovery **correlation increases losses** severity

Portfolio loss distribution (EL = 0,2%)



- **Expected Loss** (EL) is **hardly unchanged**

6. Risk Measures : VaR vs ES

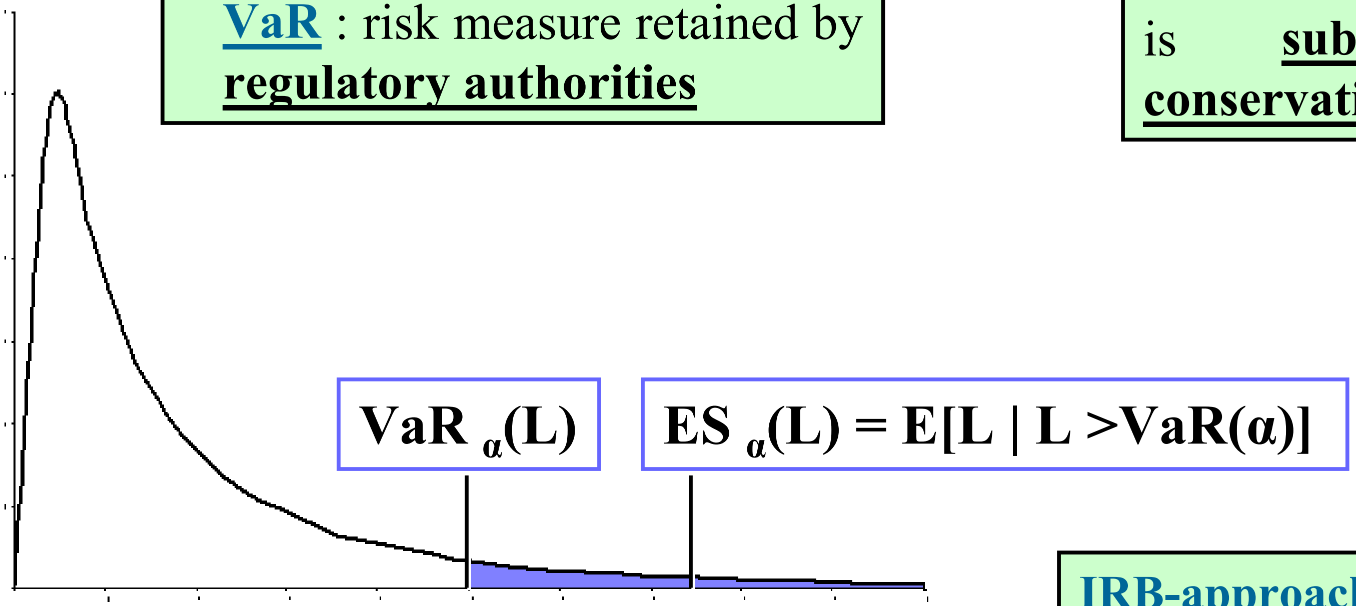
The **Value at Risk** and the **Expected Shortfall** for a confidence level $\alpha \in [0,1]$ are:

$$\mathrm{VaR}_{\alpha}(L) = \inf\left(t,\ P[L \le t] \ge \alpha\right)$$

$$\mathrm{ES}_{\alpha}(L) = E^{P}\left[L \mid L > \mathrm{VaR}_{\alpha}(L)\right]$$

ES : considered a reliable alternative coherent risk measure to VaR, since it is **sub-additive** and more **conservative**.

VaR : risk measure retained by **regulatory authorities**



Loss Distribution

IRB-approach : **bank capital charges match the credit risk** magnitude (L for retail & corporate, L-E[L] for mortgage)

7. VaR computation

- **Basel II Model : VaR given by:**

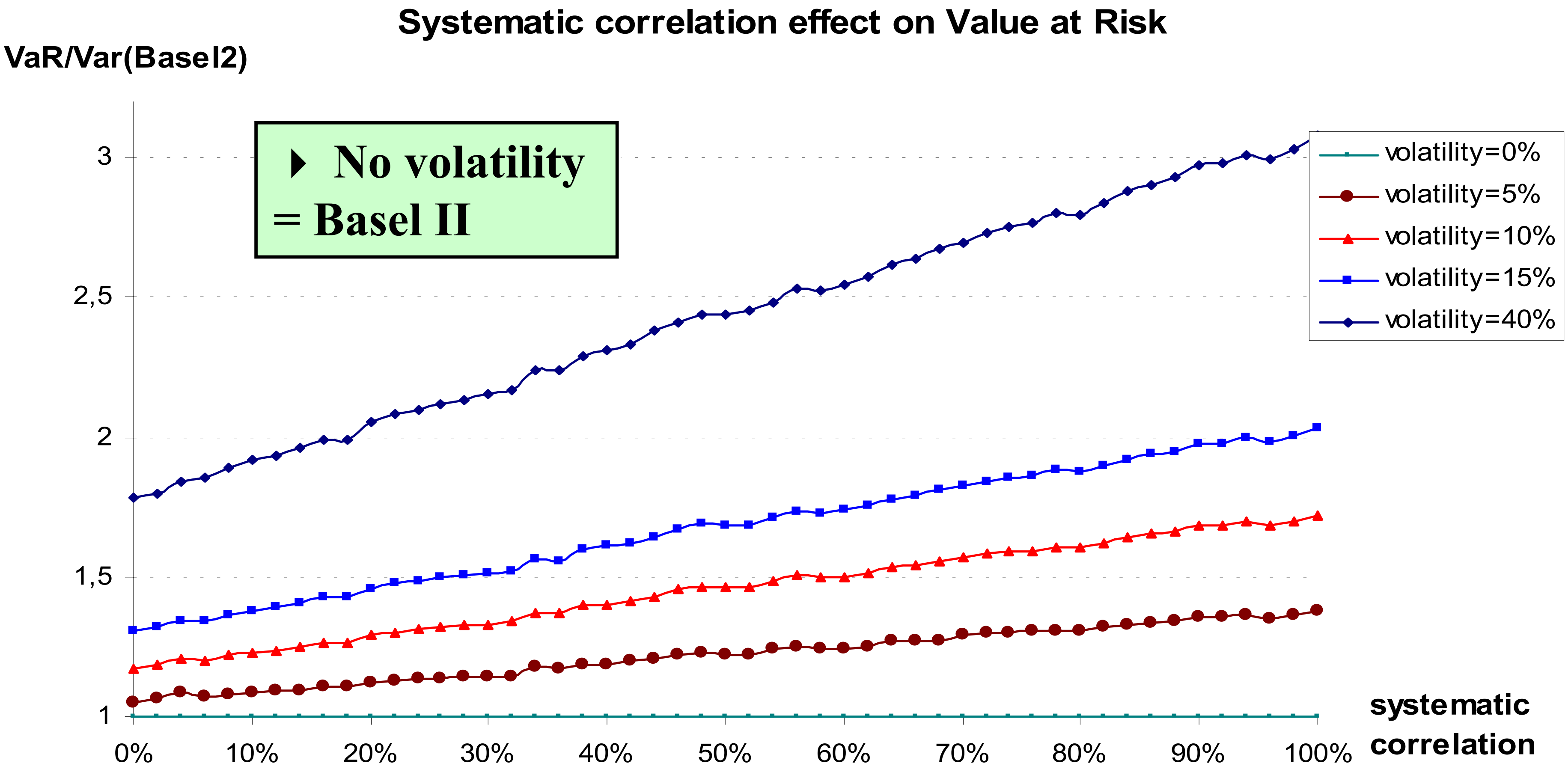
$$\mathrm{VaR}_{\mathrm{Basel2}}(\alpha) = (1 - \mathrm{recovery}) \times \Phi\left[\frac{\Phi^{-1}(PD) + \sqrt{\rho}\,\Phi^{-1}(1-\alpha)}{\sqrt{1-\rho}}\right]$$

- **Default/Collateral Model:**
  - If **default/collateral correlation** is **unspecified** $\Rightarrow$ **Monte-Carlo** simulation.
  - Particular case : **correlation = 100%,** VaR given by the cabalistic expression :

$$\frac{\mathrm{VaR}(\alpha)}{\mathrm{VaR}_{\mathrm{Basel2}}} = PD \times \frac{\Phi\left[\frac{-\mu/\sigma + \sqrt{\beta}\,\Phi^{-1}(\alpha)}{\sqrt{1-\beta}}\right] - e^{\mu+\sigma^2/2} \times e^{-\sigma\sqrt{\beta}\Phi^{-1}(\alpha) - \sigma^2\beta/2} \times \Phi\left[\frac{-\mu/\sigma + \sqrt{\beta}\,\Phi^{-1}(\alpha)}{\sqrt{1-\beta}} - \sigma\sqrt{1-\beta}\right]}{\Phi_2\left[\Phi^{-1}(PD); -\frac{\mu}{\sigma}; \eta\sqrt{\beta\rho}\right] - e^{\mu+\sigma^2/2} \times \Phi_2\left[\Phi^{-1}(PD) - \sigma\eta\sqrt{\beta\rho}; -\frac{\mu}{\sigma} - \sigma; \eta\sqrt{\beta\rho}\right]}$$

- **Monte-Carlo Simulation Results**: **VaR always greater than Basel II VaR**
  - the **higher** the **volatility,** the **higher** the **VaR**
  - the **higher** the **default/collateral correlation,** the **higher** the **VaR**

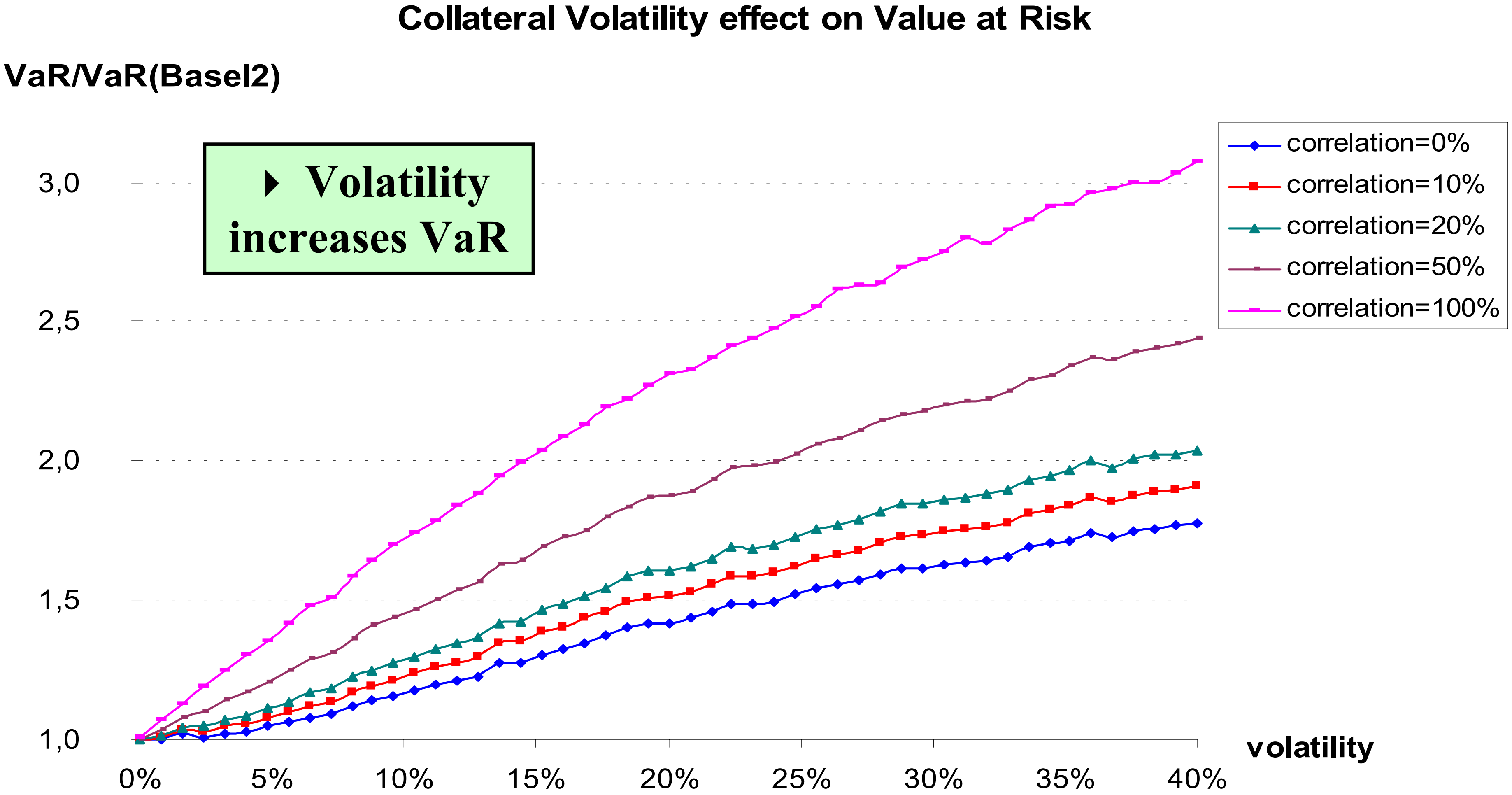
8. VaR result : factors correlation effect



- No volatility = Basel II

- Quasi-**linear dependence** between **VaR** and **correlation**

9. VaR results : volatility effect



- **Volatility increases VaR**

- **Strong** default/recovery **correlations** imply **stronger VaR**

Conclusion

- **Keeping coherence with Basel II**
  - **Factor model** for Latent Default Variable
  - **Factor model** for Collateral Value
  - **Dependence** between **Default** & **Recovery**

- **Some results**
  - **Collateral volatility** clearly **increases VaR**
  - **Murphy's law**: in addition to default, collateral value depreciated
  - **Expected Shortfall** behaves the **same way as VaR**
  - Ability to **split risk** charge into **credit risk** & **market risk**

References & Acknowledgements

The authors wish to thank Antoine Chouillou, Christian Gouriéroux, the Financial Models Team at BNP PARIBAS for helpful discussions.

[1] **E. Altman, B. Brady, A. Resti, A. Sironi,** *The link between default and recovery rates: theory, empirical evidence and implications*, Working Paper, March 2003

[2] **E. Canabarro, E. Picoult, T. Wilde,** *Analytic Methods for Counterparty Risk,* Risk, Sept. 2003.

[3] **A. Chabaane, A. Chouillou, J.-P. Laurent,** *Aggregation and Credit Risk Measurement in Retail Banking*, Forthcoming in EIR Conference.

[4] **R. Frey, A. J. McNeil**, *Dependent Defaults in Models of Portfolio Credit Risk*, To appear in the Journal of Risk 2003.

[5] **J. Frye,** *Collateral Damage*, Risk, April 2000.

[6] **J. Frye,** *Depressing Recoveries,* Risk, November 2000.

[7] **M. Gordy,** *A risk-factor Model foundation for Ratings-based Bank Capital rules*, Journal of Financial Intermediation, July 2003.

[8] **M. Pikhtin,** *Unexpected Recovery Risk*, Risk, August 2003.

[9] **O. Vasicek,** *Loan Portfolio Value*, Risk, December 2002.