Abstract: The quantitative IRB approach evaluating regulatory capital provides a benchmark framework for credit risk assessment. Nevertheless, the postulated independence between default events and recovery rates seems inappropriate for secured loans such as mortgage loans. The model we introduce is an extension of the regulatory one and takes into consideration correlation effects between default events and collateral market values. As a result, we show that this is likely to augment capital requirements in comparison with Basel II recommendations.

Keywords: Basel II Agreement, Mortgage Loans, Collateral Value, Recovery Rate, Factor Models, Risk Measure, Value at Risk
Summary

I. Collateral protection
   - Default mechanism
   - Modelling Default and Collateral Value
   - Dependence between Defaults & Collateral Values

II. Aggregating mortgage portfolios
   - Aggregated loss : methodology & computation
   - Loss distribution : Monte-Carlo results

III. Risk measure
   - Risk measures : Value at Risk & Expected Shortfall
   - Capital requirements
   - Comparison with Basel II benchmark
1. Default Mechanism

\[
\text{Credit } i \\
\begin{aligned}
&\text{Nominal: } 1 \\
&\text{Collateral: } C_i
\end{aligned}
\]

\[
X_i: \text{ Latent Variable for the } i\text{th obligor}
\]

\[
s_i: \text{ threshold such that } P(X_i < s_i) = \text{Default Probability}
\]

\[
X_i \text{ and } C_i \text{ are correlated random variables}
\]

\[
\begin{align*}
X_i < s_i &\quad \text{DEFAULT} \\
X_i \geq s_i &\quad \text{NO DEFAULT}
\end{align*}
\]

\[
C_i < 1 \quad \text{COLLATERAL TOO SMALL} \\
C_i \geq 1 \quad \text{ENOUGH COLLATERAL}
\]

\[
\text{Loss } i = \left\{ X_i < s_i \right\} \times \left( 1 - C_i \right)^+
\]

Dependence Modelling for Credit Portfolios
2. Modelling Default Latent Variable and Collateral Value

■ Modelling latent variable $X_i$:

One factor structure: $X_i = \sqrt{\rho} \Psi + \sqrt{1-\rho} \Psi_i$

- $\Psi$ systematic risk factor, gaussian
- $\Psi_i$ specific risk, gaussian i.i.d.
- $\rho$ correlation parameter

■ Modelling Collateral Value $C_i$

- **1st case**: $C_i$ are deterministic $\Rightarrow$ Basel II framework
- **2nd case**: $C_i$ are positively correlated variables. Given a systematic recovery factor $\xi$, $C_i$ are independent:
  - J. Frye (Risk, 2000a), E. Canabarro et al. (Risk 2003) : $C_i$ are gaussian
  - M. Pykhtin (Risk 2003), Chabaane, Laurent, Salomon (2003) : $C_i$ are lognormal
3. Modelling Default Latent Variable and Collateral Value

- Modelling dependence between $X_i$ and $C_i$
  - Low recovery rates associated with high default rates (Altman, 2003).
  - Dependence structure between Default & Collateral Value:
    - Frye (2003), Pykhtin (2003): driven by the same risk factor
    - Chabaane, Laurent, Salomon (2003): driven by two correlated risk factors

Remark: assuming the same risk factors is likely to induce harsh collapse of collateral value when default occurs. This strong dependence seems inappropriate for retail banking, especially mortgage portfolio.
4. Credit portfolio Aggregated Loss

- The **aggregated loss** is the sum of **individual** losses.

\[
L = \sum_{i=1}^{n} 1_{\{x_i < s_i\}} \times (1 - C_i)^+
\]

- Many approaches may be used to derive the **loss distribution**:
  - Asymptotic expansion (Gordy, Wilde)
  - Monte-Carlo Simulation (individual loss, aggregated loss, …)
  - Fourier inversion techniques
5. Comparison with Basel II benchmark

- Collateral volatility leads to fat tail distribution
- Default/recovery correlation increases losses severity

Portfolio loss distribution (EL = 0.2%)

- Expected Loss (EL) is hardly unchanged
6. Risk Measures : VaR vs ES

The Value at Risk and the Expected Shortfall for a confidence level \( \alpha \in [0, 1] \) are:

\[
\text{VaR}_\alpha(L) = \inf \left( t \mid P[L \leq t] \geq \alpha \right)
\]

\[
\text{ES}_\alpha(L) = E[p \mid L > \text{VaR}_\alpha(L)]
\]

**VaR** : risk measure retained by regulatory authorities

**ES** : considered a reliable alternative coherent risk measure to VaR, since it is sub-additive and more conservative.

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**IRB-approach** : bank capital charges match the credit risk magnitude (L for retail & corporate, L-E[L] for mortgage)
7. VaR computation

**Basel II Model**: VaR given by:

\[
\text{VaR}_{\text{Basel}2}(\alpha) = (1 - \text{recovery}) \times \Phi \left[ \frac{\Phi^{-1}(PD) + \sqrt{\rho} \Phi^{-1}(1 - \alpha)}{\sqrt{1 - \rho}} \right]
\]

**Default/Collateral Model**:

- If default/collateral correlation is unspecified \(\Rightarrow\) Monte-Carlo simulation.
- Particular case: correlation = 100\%, VaR given by the cabalistic expression:

\[
\text{VaR}(\alpha) = \text{PD} \times \Phi \left[ \frac{-\mu/\sigma + \sqrt{\beta} \Phi^{-1}(\alpha)}{\sqrt{1-\beta}} \right] - e^{\mu + \sigma^2/2} \times e^{-\sigma \sqrt{\beta} \Phi^{-1}(\alpha) - \sigma^2/2} \times \Phi \left[ \frac{-\mu/\sigma + \sqrt{\beta} \Phi^{-1}(\alpha) - \sigma \sqrt{1-\beta}}{\sqrt{1-\beta}} \right]
\]

\[
\frac{\text{VaR}}{\text{VaR}_{\text{Basel}2}} = \frac{\Phi_2 \left[ \Phi^{-1}(PD); -\frac{\mu}{\sigma}; \eta \sqrt{\beta \rho} \right] - e^{\mu + \sigma^2/2} \times \Phi_2 \left[ \Phi^{-1}(PD) - \sigma \eta \sqrt{\beta \rho}; -\frac{\mu}{\sigma} - \sigma; \eta \sqrt{\beta \rho} \right]}{\Phi_2 \left[ \Phi^{-1}(PD); -\frac{\mu}{\sigma}; \eta \sqrt{\beta \rho} \right] - e^{\mu + \sigma^2/2} \times \Phi_2 \left[ \Phi^{-1}(PD) - \sigma \eta \sqrt{\beta \rho}; -\frac{\mu}{\sigma} - \sigma; \eta \sqrt{\beta \rho} \right]}
\]

**Monte-Carlo Simulation Results**: VaR always greater than Basel II VaR

- the higher the volatility, the higher the VaR
- the higher the default/collateral correlation, the higher the VaR
8. VaR result: factors correlation effect

Systematic correlation effect on Value at Risk

- No volatility = Basel II

Quasi-linear dependence between VaR and correlation

<table>
<thead>
<tr>
<th>Volatility</th>
<th>Line Color</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>Green</td>
</tr>
<tr>
<td>5%</td>
<td>Red</td>
</tr>
<tr>
<td>10%</td>
<td>Orange</td>
</tr>
<tr>
<td>15%</td>
<td>Blue</td>
</tr>
<tr>
<td>40%</td>
<td>Purple</td>
</tr>
</tbody>
</table>
9. VaR results: volatility effect

Collateral Volatility effect on Value at Risk

- Volatility increases VaR
- Strong default/recovery correlations imply stronger VaR

\[ \text{VaR/VaR(Basel2)} \]

volatility

\[ 0\% \quad 5\% \quad 10\% \quad 15\% \quad 20\% \quad 25\% \quad 30\% \quad 35\% \quad 40\% \]

\[ 1,0 \quad 1,5 \quad 2,0 \quad 2,5 \quad 3,0 \]

correlation=0%
correlation=10%
correlation=20%
correlation=50%
correlation=100%
Conclusion

- Keeping coherence with Basel II
  - Factor model for Latent Default Variable
  - Factor model for Collateral Value
  - Dependence between Default & Recovery

- Some results
  - Collateral volatility clearly increases VaR
  - Murphy’s law: in addition to default, collateral value depreciated
  - Expected Shortfall behaves the same way as VaR
  - Ability to split risk charge into credit risk & market risk
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