Comparing Copula Models for the Pricing of Basket Credit Derivatives and CDO's

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Comparing Copula Models for the Pricing of Basket Credit Derivatives and CDO's

- Semi analytical pricing and hedging of basket default swaps and CDO tranches
- Homogeneous and non homogeneous cases
- Computation of sensitivity with respect to credit curves
- Correlation parameters
- Choice of copula

Pricing of Basket default swaps and CDO tranches

- $i = 1, \ldots, n$ names.
- $\tau_1, \ldots, \tau_n$ default times.
- N_i nominal of credit *i*,
- δ_i recovery rate (between 0 and 1)
  - $N_i(1-\delta_i)$ loss given default (of name i)
  - if $N_i(1-\delta_i)$ does not depend on *i*: <u>homogeneous</u> case
  - otherwise, <u>heterogeneous</u> case.

Pricing of Basket default swaps and CDO tranches

- Credit default swap (CDS) on name *i*:
- Default leg:
  - *payment of* $N_i(1-\delta_i)$ *at* τ_i *if* $\tau_i \leqslant T$
  - *Where T is the maturity of the CDS*
- Premium leg:
- constant periodic premium paid until $\min(\tau_i, T)$
  - *CDS premiums depend on maturity T*
  - *Liquid markets: CDS premiums, inputs of pricing models*

Pricing of Basket default swaps and CDO tranches

- First to default swap:
- Default leg: payment of $N_i(1-\delta_i)$ at:

$$\tau^1 = \min(\tau_1, \dots, \tau_n)$$

  - Where *i* is the name in default
  - If $\tau^1 \leqslant T$ maturity of First to default swap
- Homogeneous case:
  - *payment does not depend on name i in default*
- Premium leg:
  - *constant periodic premium until* $\min(\tau^1, T)$
  - *Remark: payment in case of simultaneous defaults ?*

Pricing of Basket default swaps and CDO tranches

- General Basket k^{th} to default swaps
- $\tau^1, \ldots, \tau^n$ ordered default times
- k^{th} to default swap default leg:
  - Payment of $N_i(1-\delta_i)$ at τ^k
  - where i is the name in default,
  - If $\tau^k \leqslant T$ maturity of k^{th} to default swap
- Premium leg:
  - *constant periodic premium until* $\min(\tau^k, T)$

Pricing of Basket default swaps and CDO tranches

- *Homogeneous case:*
  - payoff does not depend on name *i* in default
  - *simpler payoff*
- Number of names in defaults at *t*: $N(t)$
- Remark that: $\tau^k > t \Longleftrightarrow N(t) < k$
- Default payment when $N(t)$ jumps from k to $k+1$
  - *Default payment* $N_i(1-\delta_i)$ *does not depend on name i.*

Pricing of Basket default swaps and CDO tranches

- Synthetic CDO tranches
- Payments are based on the accumulated losses on the pool of credits
- Accumulated loss at *t*:

$$L(t) = \sum_{1 \le i \le n} N_i(1-\delta_i)N_i(t)$$

  - where $N_i(t) = 1_{\tau_i \le t}$, $N_i(1-\delta_i)$ loss given default.
  - $L(t)$ pure jump process

Pricing of Basket default swaps and CDO tranches

- Tranches with thresholds $0 \leq A \leq B \leq \sum N_j$
  - *Mezzanine: losses are between A and B*
- Cumulated payments at time *t on mezzanine tranche*

$$M(t) = (L(t) - A))\, 1_{[A,B]}(L(t)) + (B - A)1_{]B,\infty[}(L(t))$$

- *Payments on default leg:*

$\Delta M(t) = M(t) - M(t^-)$ at time $t \leq T$

- *Payments on premium leg:*
  - periodic premium,
  - proportional to outstanding nominal: $B - A - M(t)$

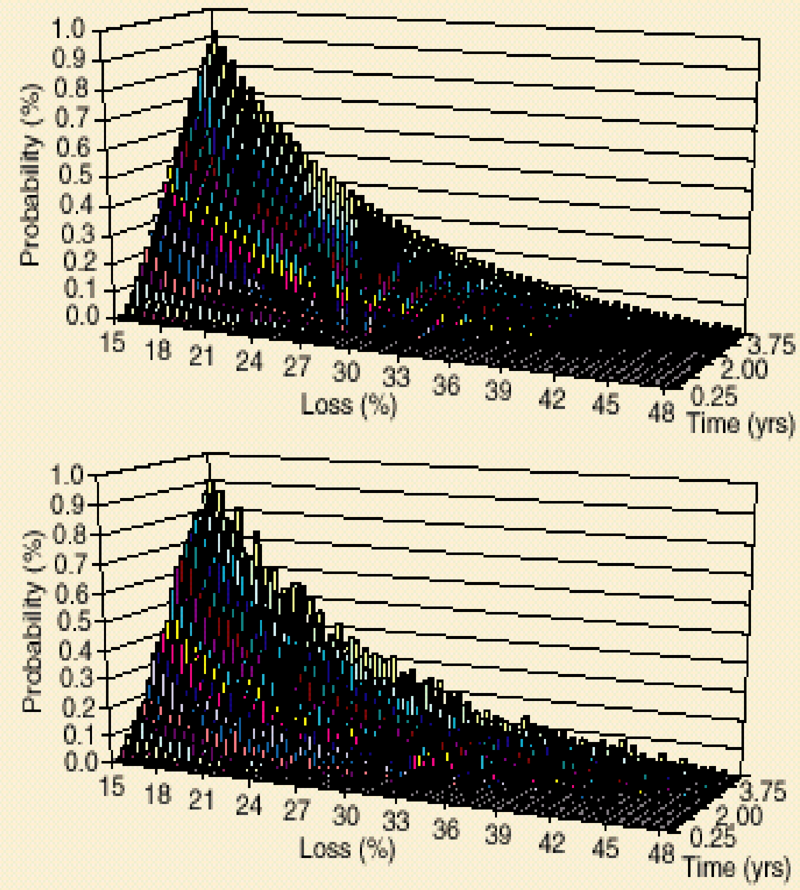
Pricing of Basket default swaps and CDO tranches

- Upfront premium: $E\left[\int_0^T B(t)dM(t)\right]$
    - *B(t) discount factor, T maturity of CDO*
- Integration by parts $B(T)E[M(T)] + \int_0^T E[M(t)]dB(t)$
    - *Where* $E[M(t)] = (B-A)Q(L(t) > B) + \int_A^B (x-A)dF_{L(t)}(x)$
- Premium only involves loss distributions

Pricing of Basket default swaps and CDO tranches

- CDO premiums only involve loss distributions
- One hundred names, same nominal.
- Recovery rates: 40%
- Credit spreads uniformly distributed between 60 and 250 bp.
- Gaussian copula, correlation: 50%
- 10^5 Monte Carlo simulations

3. Loss distribution



Loss distribution over time for the table B example with 50% correlation for the semi-explicit approach (top) and Monte Carlo simulation (bottom)

Pricing of Basket default swaps and CDO tranches

- Contribution of names to the PV of the default leg
  - *See « Basket defaults swaps, CDO's and Factor Copulas » available on www.defaultrisk.com*
- Same methodology applies for homogeneous basket default swaps
  - *i.e. payment in case of default does not depend on name in default*
  - *Since the payoff only involves the number of defaults*
  - *For non homogeneous basket default swaps, pricing formulae also exist, but are more tricky*

Pricing of Basket default swaps and CDO tranches

- One factor Gaussian copula:
  - $V, \bar{V}_i, \ i = 1, \ldots, n$ *independent Gaussian,*

    $$V_i = \rho_i V + \sqrt{1-\rho_i^2}\bar{V}_i$$

  - *Default times:* $\tau_i = F_i^{-1}(\Phi(V_i))$
  - *Conditional default probabilities:* $p_t^{i|V} = \Phi\left(\frac{-\rho_i V + \Phi^{-1}(F_i(t))}{\sqrt{1-\rho_i^2}}\right)$
- Example: non homogeneous first to default swap
  - *Default leg* $\int_0^T \sum_{i=1}^n M_i B(t) E\left[\prod_{j \neq i}\left(1 - p_t^{j|V}\right)\frac{dp_t^{i|V}}{dt}\right] dt$

Sensitivity with respect to credit curves

- Computation of Greeks
  - *Changes in credit curves of individual names*
  - *Changes in correlation parameters*
- Greeks can be computed up to an integration over factor distribution
  - *Lengthy but easy to compute formulas*
  - *The technique is applicable to Gaussian and non Gaussian copulas*
  - *See « I will survive », RISK magazine, June 2003, for more about the derivation.*

Sensitivity with respect to credit curves

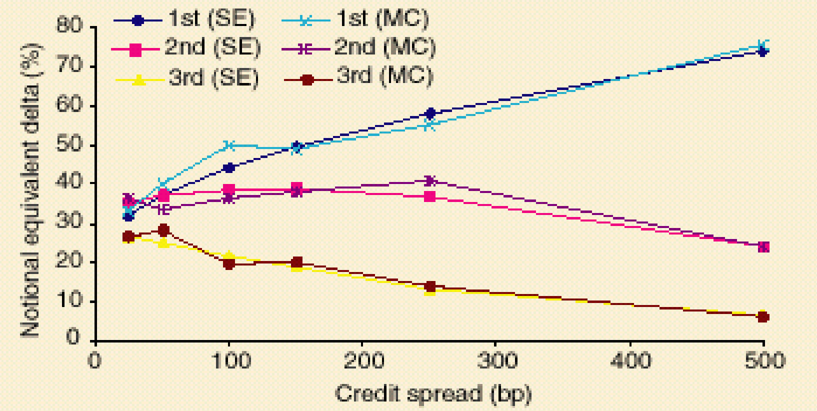
- Example: six names portfolio
- Changes in credit curves of individual names
- Amount of individual CDS to hedge the basket
- Semi-analytical more accurate than 10^5 Monte Carlo simulations.
- Much quicker: about 25 Monte Carlo simulations.

A. Comparison of the semi-explicit formulas with Monte Carlo simulations

	First to default SE	First to default MC	Second to default SE	Second to default MC	Third to default SE	Third to default MC
0%	1,075.1	1,075.9	214.8	214.7	28.2	27.7
20%	927.0	925.9	247.2	247.5	61.4	61.8
30%	859.9	857.9	256.8	257.6	77.6	78.0
40%	796.6	795.2	263.3	264.2	92.7	93.0
60%	679.6	678.0	268.8	268.9	119.5	119.8
80%	573.1	571.7	266.2	266.1	141.0	140.9
100%	500.0	500.0	250.0	250.0	150.0	150.0

Premiums in basis points per annum as a function of correlation for a five-year maturity basket with credit spreads of 25, 50, 100, 150, 250 and 500bp and equal recovery rates of 40%

1. Deltas calculated using semi-explicit formulas and Monte Carlo approaches

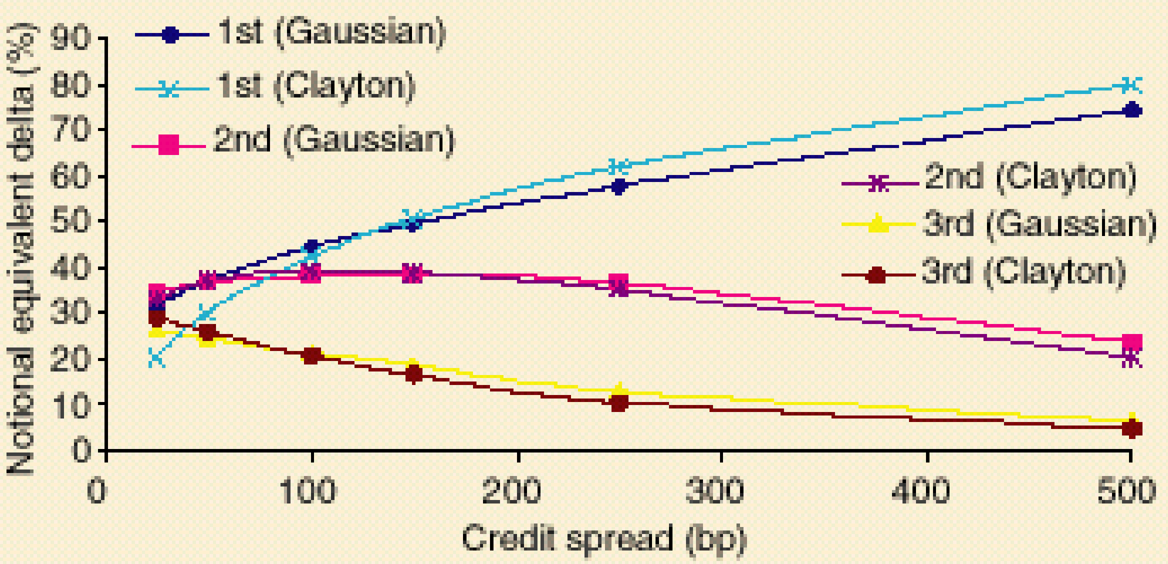


Comparison of deltas calculated using the analytical formulas and 105 Monte Carlo simulations for the example given in table A. The Monte Carlo deltas are calculated by applying a 10bp parallel shift to each curve

Sensitivity with respect to credit curves

- Changes in credit curves of individual names
  - *Dependence upon the choice of copula for defaults*

2. Deltas using Gaussian and Clayton copula

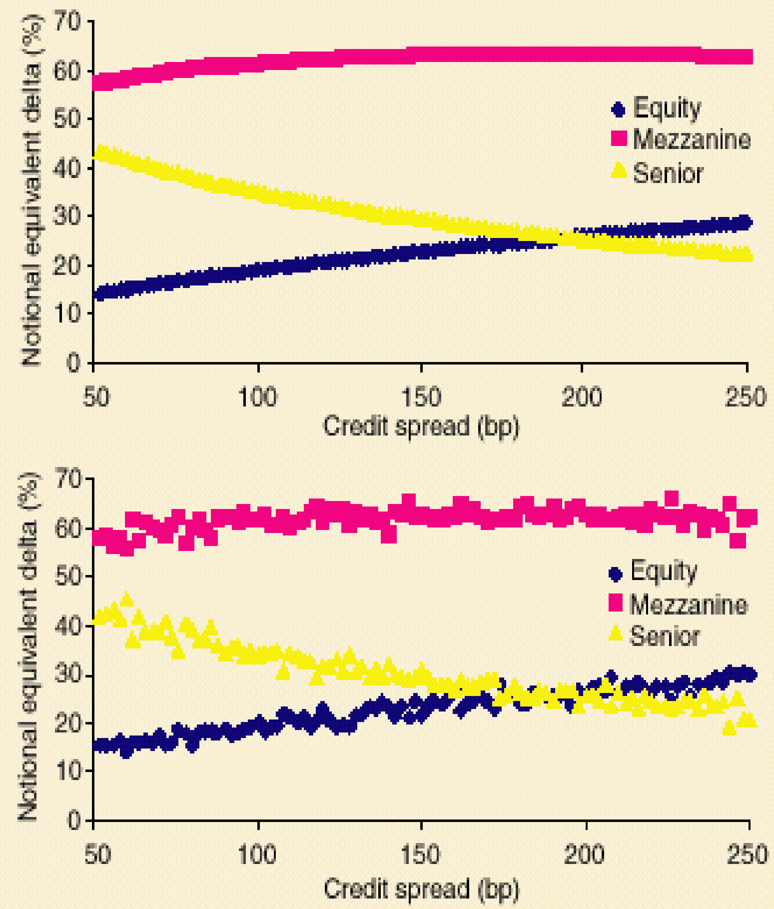


Comparison of deltas calculated using Gaussian (30% correlation) and Clayton copulas ($\theta = 0.27$)

Sensitivity with respect to credit curves

- Hedging of CDO tranches with respect to credit curves of individual names
- Amount of individual CDS to hedge the CDO tranche
- Semi-analytic : some seconds
- Monte Carlo more than one hour and still shaky

4. CDO tranche deltas



CDO tranche deltas using the analytical method (top) and Monte Carlo (bottom) for a correlation of 50%

Correlation Parameters

- CDO premiums (bp pa)
  - *with respect to correlation*
  - *Gaussian copula*
  - *Attachment points: 3%, 10%*
  - *100 names, unit nominal*
  - *5 years maturity, recovery rate 40%*
  - *Credit spreads uniformly distributed between 60 and 150 bp*
- Equity tranche premiums decrease with correlation
- Senior tranche premiums increase with correlation
- Small correlation sensitivity of mezzanine tranche

ρ	equity	mezzanine	senior
0 %	6176	694	0.05
10 %	4046	758	5.8
30 %	2303	698	23
50 %	1489	583	40
70 %	933	470	56

Correlation parameters

- Gaussian copula with sector correlations

$$\begin{pmatrix}
1 & \beta_1 & \beta_1 & & & & & & & & \\
\beta_1 & 1 & \beta_1 & & & & & & \gamma & & \\
\beta_1 & \beta_1 & 1 & & & & & & & & \\
 & & & 1 & & & & & & & \\
 & & & & \cdot & & & & & & \\
 & & & & & \cdot & & & & & \\
 & & & & & & 1 & & & & \\
 & & & & & & & 1 & \beta_m & \beta_m \\
 & \gamma & & & & & & \beta_m & 1 & \beta_m \\
 & & & & & & & \beta_m & \beta_m & 1
\end{pmatrix}$$

  - *Analytical approach still applicable*
  - *"In the Core of Correlation", to appear in Risk Magazine*

Correlation Parameters

- TRAC-X Europe
  - *Names grouped in 5 sectors*
  - *Intersector correlation: 20%*
  - *Intrasector correlation varying from 20% to 80%*
  - *Tranche premiums (bp pa)*
- Increase in intrasector correlation
  - *Less diversification*
  - *Increase in senior tranche premiums*
  - *Decrease in equity tranche premiums*

$$\begin{pmatrix} 1 & 60\% & 60\% & & & & & & & \\ 60\% & 1 & 60\% & & & & & & 20\% & \\ 60\% & 60\% & 1 & & & & & & & \\ & & & 1 & & & & & & \\ & & & & \ddots & & & & & \\ & & & & & \ddots & & & & \\ & & & & & & 1 & & & \\ & & & & & & & 1 & 60\% & 60\% \\ & 20\% & & & & & & 60\% & 1 & 60\% \\ & & & & & & & 60\% & 60\% & 1 \end{pmatrix}$$

	0-3%	3-6%	6-9%	9-12%	12-22%
20%	1273.9	287.5	93.4	33.3	6.0
30%	1226.6	294.4	102.7	39.9	7.9
40%	1168.9	303.5	114.0	47.3	10.3
50%	1100.5	314.2	127.6	56.3	13.3
60%	**1020.9**	**325.8**	**143.8**	**67.2**	**17.0**
70%	929.1	337.5	163.6	80.8	21.6
80%	821.9	349.3	188.0	98.8	27.2

Correlation Parameters

- Implied flat correlation
  - *With respect to intrasector correlation*
- * premium cannot be matched with flat correlation
  - *Due to small correlation sensitivities of mezzanine tranches*
- Negative correlation smile

$$
\begin{pmatrix}
1 & 60\% & 60\% & & & & & & & \\
60\% & 1 & 60\% & & & & & & 20\% & \\
60\% & 60\% & 1 & & & & & & & \\
 & & & 1 & & & & & & \\
 & & & & \cdot & & & & & \\
 & & & & & \cdot & & & & \\
 & & & & & & 1 & & & \\
 & & & & & & & 1 & 60\% & 60\% \\
 & 20\% & & & & & & 60\% & 1 & 60\% \\
 & & & & & & & 60\% & 60\% & 1
\end{pmatrix}
$$

	0-3%	3-6%	6-9%	9-12%	12-22%
20%	20.0%	20.0%	20.0%	20.0%	20.0%
30%	22.2%	22.6%	22.1%	22.2%	22.0%
40%	25.0%	27.6%	25.2%	24.6%	24.2%
50%	28.5%	*	29.7%	27.3%	26.8%
60%	32.8%	*	40.5%	30.6%	29.8%
70%	44.9%	*	*	34.8%	33.1%
80%	44.8%	*	*	41.3%	37.1%

Correlation parameters

- Pairwise correlation sensitivities
  - *not to be confused with sensitivities to factor loadings*

$$V_i = \rho_i V + \sqrt{1-\rho_i^2}\bar{V}_i$$

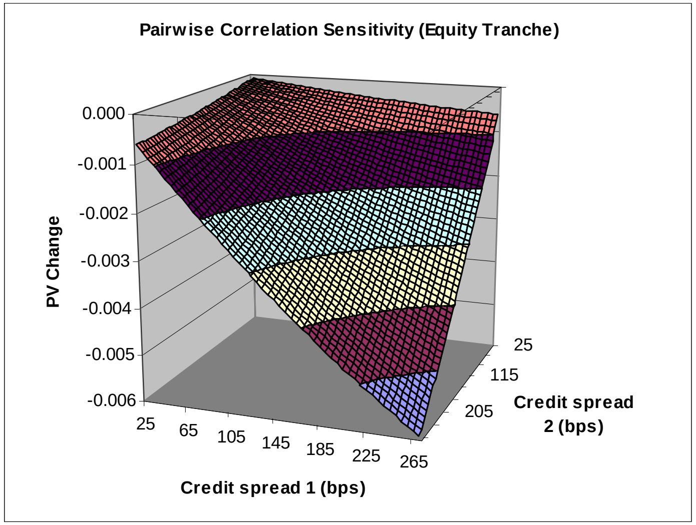
  - *Correlation between names i and j:* $\rho_i \rho_j$
  - *Sensitivity wrt factor loading: shift in* ρ_i
  - *All correlations involving name i are shifted*

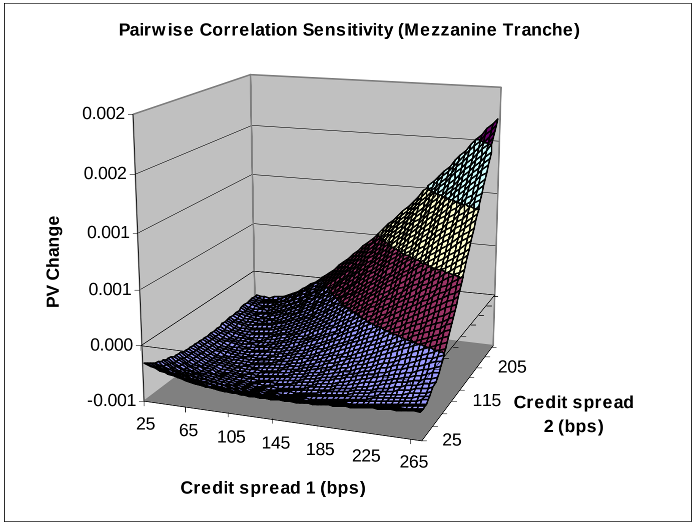
- Pairwise correlation sensitivities
  - *Local effect*

$$\begin{pmatrix} 1 & \rho_{12} & & & & & & \\ \rho_{21} & 1 & & & & & & \\ & & 1 & & \rho_{ij}+\delta & & & \\ & & & . & & & & \\ & & \rho_{ij}+\delta & & . & & & \\ & & & & & 1 & & \\ & & & & & & 1 & . \\ & & & & & & . & 1 \end{pmatrix}$$

Correlation Parameters

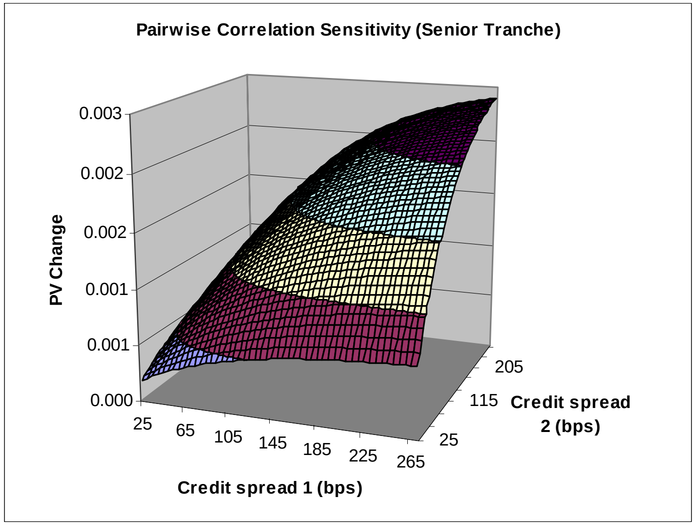
- Pairwise Correlation sensitivities
  - *Protection buyer*
- 50 names
  - *spreads 25, 30,…, 270 bp*
- Three tranches:
  - *attachment points: 4%, 15%*
- Base correlation: 25%
- Shift of pair-wise correlation to 35%
- Correlation sensitivities wrt the names being perturbed
- equity (top), mezzanine (bottom)
  - *Negative equity tranche correlation sensitivities*
  - *Bigger effect for names with high spreads*





Correlation Parameters

- Senior tranche correlation sensitivities
  - *Positive sensitivities*
  - *Protection buyer is long a call on the aggregated loss*
    - Positive vega
  - *Increasing correlation*
    - Implies less diversification
    - Higher volatility of the losses



- Names with high spreads have bigger correlation sensitivities

Choice of copula

- Joint survival function:

$$S(t_1, \ldots, t_n) = Q(\tau_1 > t_1, \ldots, \tau_n > t_n)$$

  - *Needs to be specified given marginal distributions.*
  - $S_i(t) = Q(\tau_i > t)$ *or* $F_i(t) = Q(\tau_i \leq t)$ *given from CDS quotes.*

- (Survival) Copula of default times:

$$C(S_1(t_1), \ldots, S_n(t_n)) = S(t_1, \ldots, t_n)$$

  - *C characterizes the dependence between default times.*

Choice of copula

- Factor approaches to joint distributions:
  - *V: low dimensional factor, not observed « latent factor ».*
  - *Conditionally on V, default times are independent.*
  - *Conditional default probabilities:*

$$p_t^{i|V} = Q\left(\tau_i \leq t \mid V\right), \quad q_t^{i|V} = Q\left(\tau_i > t \mid V\right).$$

  - *Conditional joint distribution:*

$$Q(\tau_1 \leq t_1, \ldots, \tau_n \leq t_n \mid V) = \prod_{1 \leq i \leq n} p_{t_i}^{i|V}$$

  - *Joint survival function (implies integration wrt V):*

$$Q(\tau_1 > t_1, \ldots, \tau_n > t_n) = E\left[\prod_{i=1}^{n} q_{t_i}^{i|V}\right]$$

Choice of copula

- Why factor models ?
  - *Standard approach in finance and statistics*
  - *Tackle with large dimensions*
- Need tractable dependence between defaults:
  - *Parsimonious modeling*
    - One factor Gaussian copula: n parameters
    - But constraints on dependence structure
  - *Semi-explicit computations for portfolio credit derivatives*
    - Premiums, Greeks
    - Much quicker than plain Monte-Carlo

Choice of copula

- One factor Gaussian copula:
  - $V, \bar{V}_i,\ i = 1, \ldots, n$ *independent Gaussian,*

$$V_i = \rho_i V + \sqrt{1-\rho_i^2}\bar{V}_i$$

  - *Default times:* $\tau_i = F_i^{-1}(\Phi(V_i))$
  - F_i *marginal distribution function of default times*
  - *Conditional default probabilities:* $p_t^{i|V} = \Phi\left(\frac{-\rho_i V + \Phi^{-1}(F_i(t))}{\sqrt{1-\rho_i^2}}\right)$
- Joint survival function:

$$S(t_1, \ldots, t_n) = \int \left(\prod_{i=1}^{n} \Phi\left(\frac{\rho_i v - \Phi^{-1}(F_i(t_i))}{\sqrt{1-\rho_i^2}}\right)\right) \varphi(v) dv$$

Choice of copula

- Gaussian copula
  - *No tail dependence (if* $|\rho| < 1$*)*
  - *Upper tail dependence*

$$\lim_{u \to 1} Q\left(\tau_i > F_i^{-1}(u) \,\middle|\, \tau_j > F_j^{-1}(u)\right) = \lim_{u \to 1} \frac{C(u,u) + 1 - 2u}{1 - u}$$

- Kendall's tau $\rho_K = \frac{2}{\pi} \arcsin \rho$

$$\rho_K = 4 \iint_{[0,1]^2} C_\rho(u,v) dC_\rho(u,v) - 1$$

- Spearman rho $\rho_S = \frac{6}{\pi} \arcsin(\rho/2)$

$$\rho_S = 12 \iint_{[0,1]^2} uv dC_\rho(u,v) - 3 = 12 \iint_{[0,1]^2} C_\rho(u,v) du dv - 3$$

Choice of copula

- Concordance ordering

$$\rho \leqslant \rho' \Rightarrow C_\rho(u_1, \dots, u_n) \leqslant C_{\rho'}(u_1, \dots, u_n)$$

- $\rho = 0$ independence between default dates
  - $C(u_1, \dots, u_n) = u_1 \times \dots \times u_n$
  - *Product copula*
- $\rho = 1$ comonotonic case:
  - *"perfect correlation" between default dates*
  - $C(u_1, \dots, u_n) = \min(u_1, \dots, u_n)$

Choice of copula

- Student *t* copula
  - *Embrechts, Lindskog & McNeil, Greenberg et al, Mashal & Zeevi, Gilkes & Jobst*

$$\begin{cases} X_i = \rho V + \sqrt{1-\rho^2}\,\bar{V}_i \\ V_i = \sqrt{W} \times X_i \\ \tau_i = F_i^{-1}\left(t_\nu\left(V_i\right)\right) \end{cases}$$

  - $V, \bar{V}_i$ *independent Gaussian variables*
  - $\dfrac{\nu}{W}$ *follows a* χ_ν^2 *distribution*
  - *Conditional default probabilities (two factor model)*

$$p_t^{i|V,W} = \Phi\left(\frac{-\rho V + W^{-1/2} t_\nu^{-1}\left(F_i(t)\right)}{\sqrt{1-\rho^2}}\right)$$

Choice of copula

- Student *t* copula
  - *Kendall's tau:* $\rho_K = \dfrac{2}{\pi} \arcsin \rho$
  - *Tail dependence parameter* $2t_{\nu+1}\left(-\sqrt{\nu+1} \times \sqrt{\dfrac{1-\rho}{1+\rho}}\right)$
  - *correlation parameter* $\rho = 0$ *does not lead to independence*
  - *Correlation parameter = 1, comonotonic case*
  - *Copula increasing with correlation parameter*

Choice of copula

- *Clayton* copula
  - *Schönbucher & Schubert, Rogge & Schönbucher*

$$V_i = \psi\left(-\frac{\ln U_i}{V}\right) \quad \tau_i = F_i^{-1}(V_i) \quad \psi(s) = (1+s)^{-1/\theta}$$

  - *V: Gamma distribution with parameter* θ
  - $U_1, \ldots, U_n$ *independent uniform variables*
  - *Conditional default probabilities (one factor model)*

$$p_t^{i|V} = \exp\left(V\left(1 - F_i(t)^{-\theta}\right)\right)$$

  - *Frailty model: multiplicative effect on default intensity*
  - *Copula:* $C(u_1, \ldots, u_n) = \left(u_1^{-\theta} + \ldots + u_n^{-\theta} - n + 1\right)^{-1/\theta}$

Choice of copula

- Clayton copula:
  - *Archimedean copula*
  - *lower tail dependence:* $\lambda_L = 2^{-1/\theta}$
  - *no upper tail dependence*

- Kendall tau $\rho_K = \dfrac{\theta}{\theta + 2}$
  - *Spearman rho has to be computed numerically*
- C_θ increasing with θ
- $\theta = 0$ independence case
- $\theta = +\infty$ comonotonic case

Choice of copula

- Shock models
  - *Duffie & Singleton, Giesecke, Elouerkhaoui, Lindskog & McNeil, Wong*
- Modeling of default dates: $\tau_i = \min(\bar{\tau}_i, \tau)$
  - $Q(\tau_i = \tau_j) \geq Q\left(\tau \leq \min(\bar{\tau}_i, \bar{\tau}_j)\right) > 0$ *simultaneous defaults.*
  - *Conditionally on* $\tau,\ \tau_i$ *are independent.*

$$Q(\tau_1 \leq t_1, \ldots, \tau_n \leq t_n \mid \tau) = \prod_{1 \leq i \leq n} Q(\tau_i \leq t_i \mid \tau)$$

- Conditional default probabilities (one factor model)

$$p_t^{i|\tau} = 1_{\tau > t} Q(\bar{\tau}_i \leq t) + 1_{\tau \leq t}$$

Choice of copula

- Shock models
- $\tau, \bar{\tau}_i$ exponential distributions with parameters $\lambda, \bar{\lambda}_i$
- Survival copula $\hat{C}$ is *Marshall Olkin* copula

$$\hat{C}(u_i, u_j) = \min(u_i^{1-\alpha_i} u_j, u_i u_j^{1-\alpha_j})$$

$$\alpha_i = \lambda / (\lambda + \bar{\lambda}_i)$$

  - *Kendall tau* $\rho_K^{i,j} = \dfrac{\alpha_i \alpha_j}{\alpha_i + \alpha_j - \alpha_i \alpha_j}$
  - *Spearman rho* $\rho_S^{i,j} = \dfrac{3\alpha_i \alpha_j}{2\alpha_i + 2\alpha_j - \alpha_i \alpha_j}$

Choice of copula

- Shock models
  - *Tail dependence* $\min\left(\alpha_i, \alpha_j\right)$
  - *Symmetric case:* $\alpha_i = \alpha$
  - $\alpha = 0$ *independence case*
  - $\alpha = 1$ *comonotonic case*
  - *Marshall-Olkin copula increasing with* α

Choice of copula

- Example 1: first to default swap
  - *Default leg* $$\int_0^T \sum_{i=1}^{n} M_i B(t) E\left[\prod_{j \neq i}\left(1 - p_t^{j|V}\right)\frac{dp_t^{i|V}}{dt}\right] dt$$
  - *One factor Gaussian* $$p_t^{i|V} = \Phi\left(\frac{-\rho_i V + \Phi^{-1}(F_i(t))}{\sqrt{1-\rho_i^2}}\right)$$
  - *Clayton* $$p_t^{i|V} = \exp\left(V\left(1 - F_i(t)^{-\theta}\right)\right)$$
  - *Marshall Olkin* $$p_t^{i|\tau} = 1_{\tau > t} Q(\bar{\tau}_i \leq t) + 1_{\tau \leq t}$$
  - *Student t* $$p_t^{i|V,W} = \Phi\left(\frac{-\rho V + W^{-1/2} t_\nu^{-1}\left(F_i(t)\right)}{\sqrt{1-\rho^2}}\right)$$
- Ease of implementation

Choice of copula

- Example 2: CDO's
- Accumulated loss at *t*: $L(t) = \sum_{1 \leq i \leq n} N_i(1-\delta_i)N_i(t)$
  - *Where* $N_i(t) = 1_{\tau_i \leq t}$, $N_i(1-\delta_i)$ *loss given default.*
- Characteristic function: $\varphi_{L(t)}(u) = E\left[e^{iuL(t)}\right]$
  - *By conditioning:* $\varphi_{L(t)}(u) = E\left[\prod_{1 \leq j \leq n} \left(1 - p_t^{j|V} + p_t^{j|V} \varphi_{1-\delta_j}(uN_j)\right)\right]$
  - *Distribution of L(t) can be obtained by FFT.*
  - *Only need of conditional probabilities*

Choice of copula

B. Pricing of five-year maturity CDO tranches

	Equity (0–3%)		Mezzanine (3–14%)		Senior (14–100%)	
	SE	MC	SE	MC	SE	MC
0%	8,219.4	8,228.5	816.2	814.3	0.0	0.0
20%	4,321.1	4,325.3	809.4	806.9	13.7	13.7
40%	2,698.8	2,696.7	734.3	731.4	33.4	33.2
60%	1,750.6	1,738.5	641.0	637.8	54.1	53.7
80%	1,077.5	1,067.9	529.5	526.9	77.0	76.6
100%	410.3	406.6	371.2	367.0	110.4	109.6

Premiums in basis points per annum as a function of correlation for 5-year maturity CDO tranches on a portfolio with credit spreads uniformly distributed between 60 and 250bp. The recovery rates are 40%

- *Semi-explicit vs MonteCarlo*
- *One factor Gaussian copula*
- *CDO tranches margins with respect to correlation parameter*

Choice of copula

- First to default swap premium vs number of names
  - *From n=1 to n=50 names*
  - *Unit nominal*
  - *Credit spreads = 80 bp*
  - *Recovery rates = 40 %*
  - *Maturity = 5 years*
  - *Basket premiums in bppa*
  - *Gaussian correlation parameter= 30%*
- Gaussian, Student *t*, Clayton and Marshall-Olkin copulas

Names	Gaussian	Student (6)	Student (12)	Clayton	MO
1	80	80	80	80	80
5	332	339	335	336	244
10	567	578	572	574	448
15	756	766	760	762	652
20	917	924	920	921	856
25	1060	1060	1060	1060	1060
30	1189	1179	1185	1183	1264
35	1307	1287	1298	1294	1468
40	1417	1385	1403	1397	1672
45	1521	1475	1500	1492	1875
50	1618	1559	1591	1580	2079
Kendall	19%			8%	33%

Choice of copula

- From first to last to default swap premiums
  - *10 names, unit nominal*
  - *Spreads of names uniformly distributed between 60 and 150 bp*
  - *Recovery rate = 40%*
  - *Maturity = 5 years*
  - *Gaussian correlation: 30%*
- Same FTD premiums imply consistent prices for protection at all ranks
- Model with simultaneous defaults provides very different results

Rank	Gaussian	Student (6)	Student (12)	Clayton	MO
1	723	723	723	723	723
2	277	278	276	274	160
3	122	122	122	123	53
4	55	55	55	56	37
5	24	24	25	25	36
6	11	10	10	11	36
7	3.6	3.5	4.0	4.3	36
8	1.2	1.1	1.3	1.5	36
9	0.28	0.25	0.35	0.39	36
10	0.04	0.04	0.06	0.06	36
Kendall	19%			19%	NS

Choice of copula

- CDO margins (bp)
  - *With respect to correlation*
  - *Gaussian copula*
  - *Attachment points: 3%, 10%*
  - *100 names*
  - *Unit nominal*
  - *Credit spreads 100 bp*
  - *5 years maturity*

	equity	mezzanine	senior
0 %	5341	560	0.03
10 %	3779	632	4.6
30 %	2298	612	20
50 %	1491	539	36
70 %	937	443	52
100%	167	167	91

Choice of copula

ρ	0%	10%	30%	50%	70%	100%
Gaussian	560	633	612	539	443	167
Clayton	560	637	628	560	464	167
Student (6)	676	676	637	550	447	167
Student (12)	647	647	621	543	445	167
MO	560	284	144	125	134	167

Table 8: mezzanine tranche (bp pa)

ρ	0%	10%	30%	50%	70%	100%
Gaussian	0.03	4.6	20	36	52	91
Clayton	0.03	4.0	18	33	50	91
Student (6)	7.7	7.7	17	34	51	91
Student (12)	2.9	2.9	19	35	52	91
MO	0.03	25	49	62	73	91

Table 9: senior tranche (bp pa)

Conclusion

- Factor models of default times:
  - *Deal easily with a large range of names and dependence structures*
  - *Simple computation of basket credit derivatives and CDO's*
    - Prices and risk parameters
- Gaussian, Clayton and Student *t* copulas provide very similar patterns
  - *Rank correlation and tail dependence not meaningful*
- Shock models quite different