Applying hedging techniques to credit derivatives

Risk Training
Pricing and Hedging Credit Derivatives
London 26 & 27 April 2001

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Credit risk: the global picture

- **Portfolio approaches to credit risk.**
  - **Relating *portfolio approaches* and *credit derivatives*.**
- **Closing the gap between supply and demand of credit risk:**
  - ➢Default Swaps,
  - ➢Dynamic Default Swaps, Basket Credit Derivatives,
  - ➢Credit Spread Options.
  - **The previous means tend to be more integrated.**
  - **Technical innovations favour efficient risk transfer.**
- **The early stages of hedging credit derivatives.**
- **The nature of credit risk.**

Portfolio approaches to credit risk

- **Consider a given portfolio:**
  - **Including credits, lines of credit, corporate bonds, interest rate swaps and forex swaps, OTC options, tranches of CDO's…**
  - **Over various defaultable counterparties.**
  - **Some credit exposures may be partially protected through collateral, credit insurance, prioritisation,…**
- **The main goal is to construct a <u>distribution of losses</u> arising from credit risk (and other financial risks).**
  - **Over one given time horizon (say one year)**
  - **From this distribution, one may consider different *risk indicators*, such as quantiles (VaR measures), Expected shortfall,…**

Portfolio approaches to credit risk

- **Some issues currently addressed (portfolio approaches):**
  - **Construction of Databases:**
    - ➢ default events, credit spreads,
    - ➢ use of external data (credit ratings, expected default frequencies).
  - **Extending the scope of credit risk assessment:**
    - ➢ default risk on non quoted or small counterparties.
    - ➢ Integrating default risk approaches for smaller credits (mortgages, consumer loans) and corporate credit risk.
  - **Theoretical issues:**
    - ➢ Consistency over different time horizons.
    - ➢ Should we use Value at Risk for credit risk ?

Portfolio approaches to credit risk

- **Some issues currently addressed (portfolio approaches):**
  - **Modelling of correlation:**
    - ➢non Gaussian variables,
    - ➢default times, default losses, indicators of credit events,
  - **Wrong way exposure:**
    - ➢correlation between financial variables (such as interest rates, stock indices, exchange rates) and defaults.
  - **Joint modelling of defaults and credit spread risk.**
  - **Relating portfolio approaches based on historical data and market prices on traded default risk**
    - ➢someinconsistencies may appear between from the two points of view (basket default swaps).

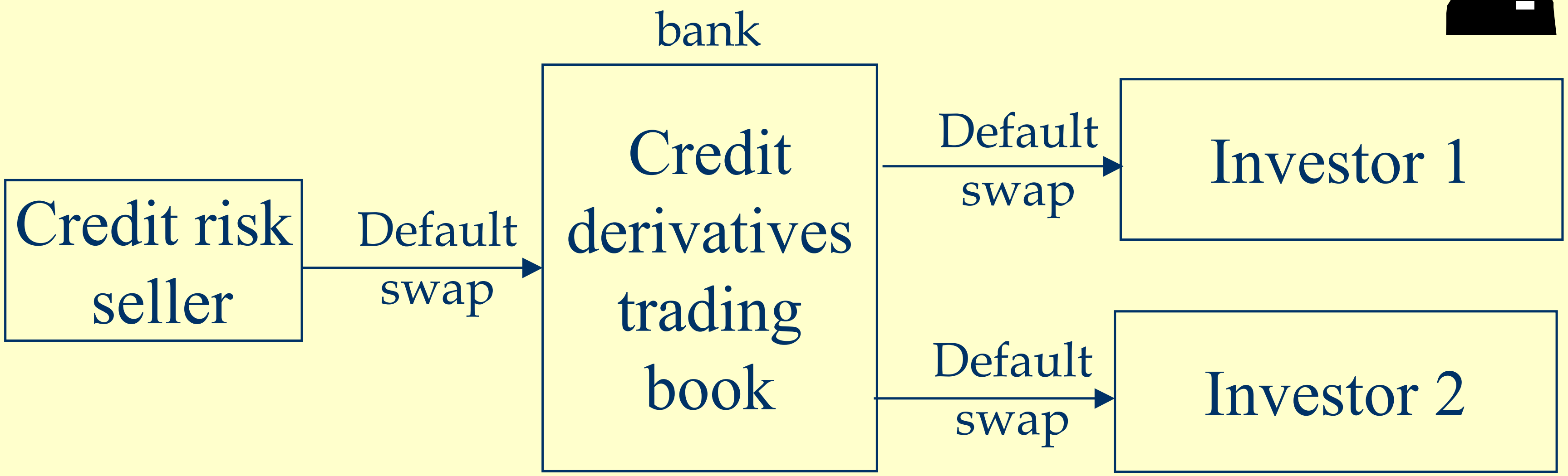
Portfolio approaches to credit risk

- **From *assessment* to *management* of credit risk.**
- **Credit risk profile (Expected and unexpected loss) of portfolios can be modified. Risk/Return ratios can be enhanced.**
  - **By using *credit derivatives***
  - **Through securitization schemes**
  - **By *dynamic management* of credit exposures : we can think of reducing credit exposures when credit spreads rise.**
- **Dynamic approaches to the hedging and management of credit risk are being transposed to the financial industry.**
  - **This will eventually enhance the ability of credit derivatives desks to an efficient management of more sophisticated risks.**

Portfolio approaches to credit risk

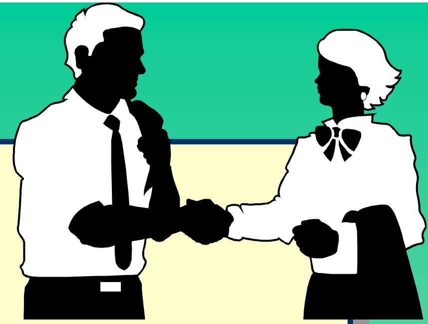
- **From *assessment* to *management* of credit risk.**
- **Understanding the main ideas and techniques regarding dynamic hedging of credit risk in the credit derivatives world may be useful to *credit risk managers*, *capital managers*, *CRO*,…**
  - **As end users of credit protection structures, sellers of credit risk**
  - **To better manage some dynamic aspects of credit risk management, such as variable credit exposure :**
    - risk reduction can be achieved through a dynamic use of standard products (plain CDS) and through more sophisticated derivatives (dynamic CDS, basket CDS).

Closing the gap between supply and demand

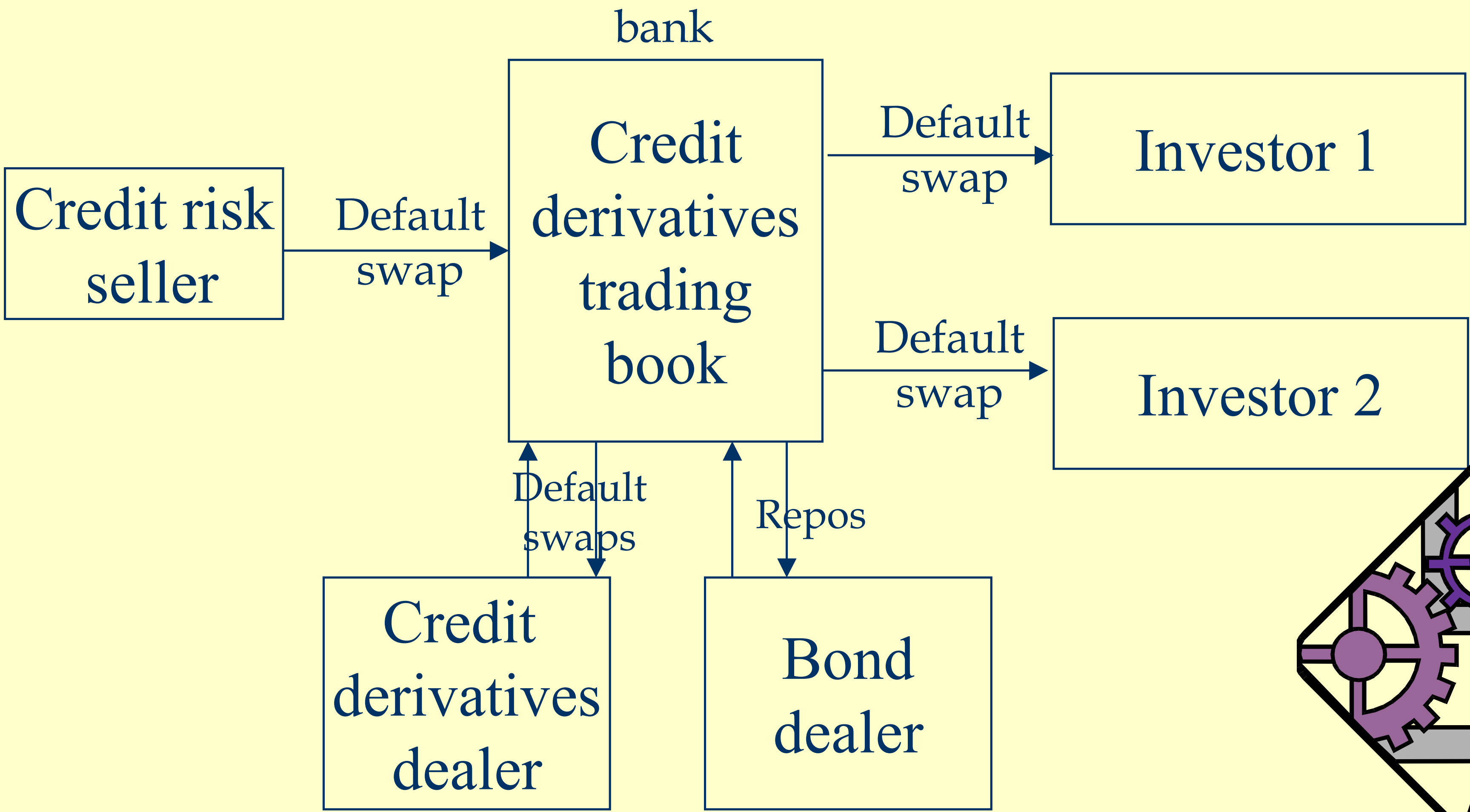


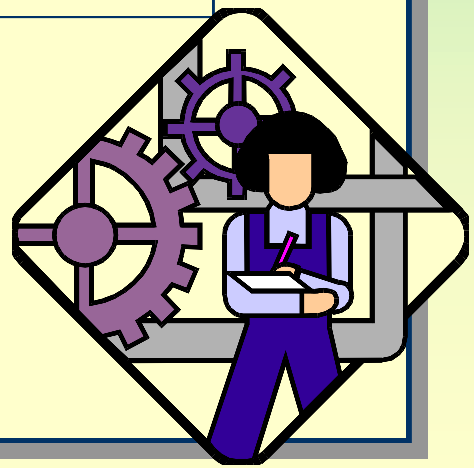
- **Credit risk trades may not be simultaneous.**
  - **Since at one point in time, demand and offer of credit risk may not match.**
  - **It is not required to find customers with exact opposite interest at every new deal.**
    - ➢ Meanwhile, credit risk remains within the balance sheet of the financial intermediary (*high capital at risk*)

Closing the gap between supply and demand

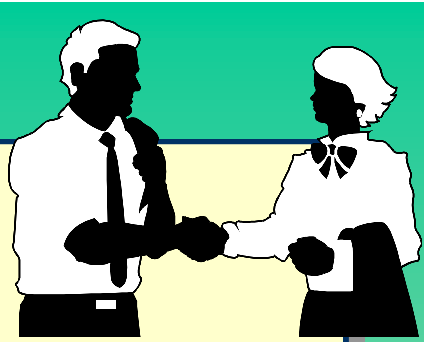


- ***Hedging*** **credit risk enhances ability to transfer credit risk by** ***<u>lowering capital at risk</u>***



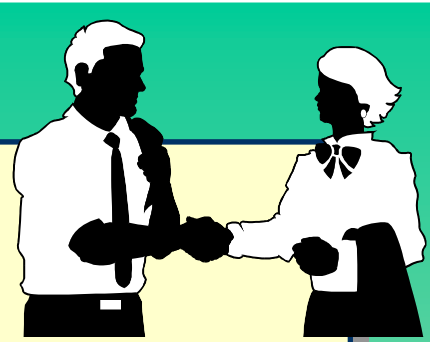


Closing the gap between supply and demand



- **dynamic default swap : « *Structured product* »**
  - ***efficient way* to transfer credit risk**
- **Anatomy of a dynamic default swap**
  - **A dynamic default swap is like a standard default swap but with variable nominal (or exposure).**
  - **However the periodic premium paid for the credit protection remains fixed.**
  - **The protection payment arises at default of *one given single risky counterparty*.**
- **Examples:**
  - **quanto default swaps (credit protection of forex bonds)**
  - **cancellable swaps (cancelled at default time of a third counterparty).**
  - **credit protection of a portfolio of contracts:**
    - vulnerable swaps, OTC options,
    - full protection, excess of loss insurance, partial collateralization

Closing the gap between supply and demand



- Example: ***defaultable interest rate swap***
- Consider a defaultable interest rate swap (with unit nominal)
  - We are default-free, our counterparty is defaultable.
  - We consider a (fixed-rate) *receiver* swap on a standalone basis.
- Recovery assumption, payments in case of default:
  - if default at time τ, compute the default-free value of the swap: PV_τ
  - and get: $$\delta\left(PV_\tau\right)^+ + \left(PV_\tau\right)^- = PV_\tau - (1-\delta)\left(PV_\tau\right)^+$$
  - $0 \leq \delta \leq 1$ recovery rate, $(PV_\tau)^+ = \mathrm{Max}(PV_\tau, 0)$, $(PV_\tau)^- = \mathrm{Min}(PV_\tau, 0)$
  - *In case of default,*
    - we receive default-free value PV_τ
    - *minus*
    - loss equal to $(1-\delta)(PV_\tau)^+$.

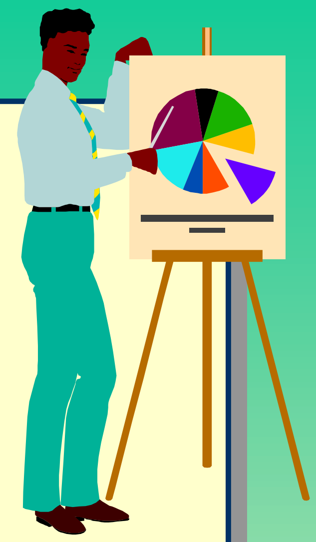
Closing the gap between supply and demand

- **Example:** ***defaultable interest rate swap***
- **Using a *dynamic default swap* to *hedge* credit risk:**
  - **Consider a <u>dynamic default swap</u> paying $(1-\delta)(PV_\tau)^+$ at default time τ (if $\tau \leq T$),**
    - PV_τ is the present value of a default-free swap with *same fixed rate* than defaultable swap.
  - **At default, we receive $(1-\delta)(PV_\tau)^+ + PV_\tau - (1-\delta)(PV_\tau)^+ = PV_\tau$**
  - **Thanks to credit protection, we receive the PV of the default free interest rate swap.**

The nature of credit risk

- **Pricing at the cost of the hedge:**
  - **If some risk can be hedged, its price should be the cost of the hedge.**
  - **Think of a plain vanilla stock index call. Its replication price is 10% (say).**
  - **One given investor is ready to pay for 11% (He feels better of with such an option, then doing nothing). Should he really give this 1% to the market ?**
- **The feasibility of hedging (« *completeness* ») is a fundamental idea.**
  - **If credit instruments can be <u>hedged</u>, pricing dynamic default swaps, basket default swaps based only historical data and portfolio approaches will eventually lead to arbitrage opportunities.**
  - **Good news for knowledgeable individuals. Bad news for the understanding of risks.**

The nature of credit risk

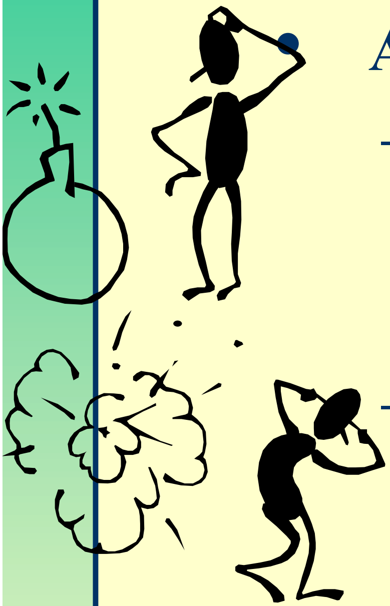


- **Misconceptions about *credit risk*.**
  - **At the early stage of credit risk analysis, a common idea was that credit risk was not hedgeable :**
    - incomplete markets, multiplicity of risk-neutral measures.
    - In firm-value models and complete information, default bonds are (too simplistically) considered as equity barrier options.
- **When some a *defaultable bond* is already <u>traded</u>, then the market can become *complete*.**
  - **If there is also credit spread risk (that is not fully correlated to other financial variables), then we need (at least) <u>two</u> defaultable bonds.**

The early stages of hedging credit derivatives

- **Static arbitrage of plain default swaps with *short selling* underlying defaultable bond**
  - **CDS premiums should be related to credit spreads on floaters.**
- **One step further: hedging non standard maturities assuming smooth credit curves.**
  - **"bond strippers" : allow to compute prices of risky zero-coupon bonds.**
- **Consider a six years maturity CDS hedged with a five years maturity CDS of same nominal.**
  - **Protection at default time.**
  - **Since maturities of credit derivatives do not perfectly match, credit spread risk.**
  - **We may need several maturities of CDS + some assumptions on the dynamics of CDS premiums in order to hedge credit spread risk.**

The early stages of hedging credit derivatives

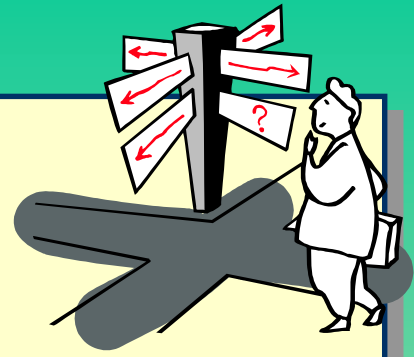


- **Assessing the varieties of risks involved in credit derivatives**
  - **Specific risk or credit spread risk**
    - ➢ *prior to default*, the P&L of a book of credit derivatives is driven by changes in credit spreads.
  - **Default risk**
    - ➢ *in case of default*, <u>if unhedged</u>,
    - ➢ dramatic jumps in the P&L of a book of credit derivatives.
- **Real world issues with hedging plain CDS :**
  - **deliverance of some unknown underlying, possibly short-term or fixed rate long term bond,**
  - **Management of short-selling and repo margins on illiquid bonds,**
  - **« small inconsistencies » due to accrued coupons, accrued premiums.**
  - **illiquid hedging default swaps**

Hedging credit derivatives: overview

- **Hedging default (and recovery) risk : an introduction.**
  - **Short term default swaps v.s. long-term default swaps**
  - **Credit spread transformation risk**
- **One step further: hedging *Dynamic Default Swaps*, *credit spread options*.**
  - **Hedging default risk through dynamics holdings in standard default swaps.**
  - **Hedging credit spread risk by choosing appropriate default swap maturities.**
- **Hedging : *Basket Default Swaps* some specificities**
  - **Uncertainty at default time**

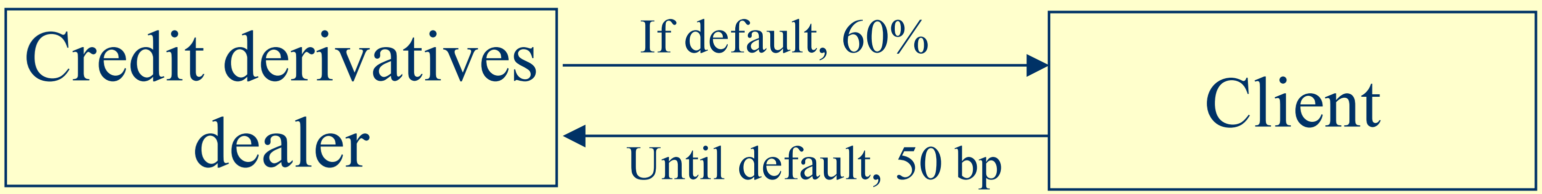
Hedging default risk: an introduction

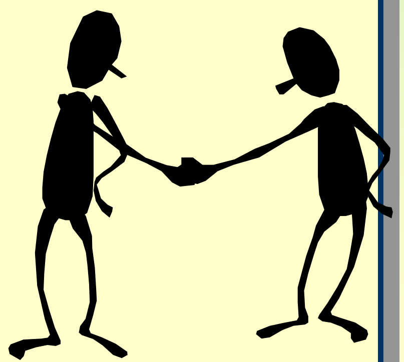


- **Disentangling risks in *credit instruments***
  - ***Interest rate risk*: due to movements in default-free interest rates.**
  - ***Default risk*: default bond price jumps to recovery value at default time.**
  - ***Credit spread risk* (specific risk): variation in defaultable bond prices prior to default, due to changes in credit quality (for instance ratings migration) or changes in risk premiums.**
  - ***Recovery risk*: unknown recovery rate in case of default.**
- **<u>Hedging</u> exotic credit derivatives will imply hedging all sources of risk.**
- **A *new approach* to credit derivatives modelling based on an <u>hedging</u> point of view**

Hedging default risk: an introduction

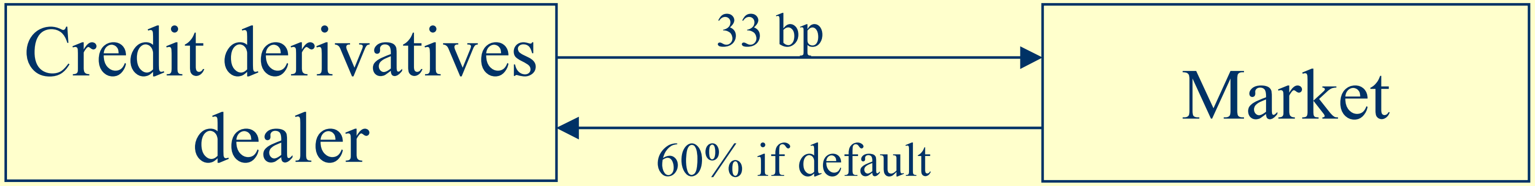
- **Purpose:**
  - **Introduction to dynamic trading of default swaps**
  - **Illustrates how default and credit spread risk arise**
- **Arbitrage between long and short term default swaps**
  - **sell one long-term default swap**
  - **buy a series of short-term default swaps**
- **Example:**
  - **default swaps on a FRN issued by BBB counterparty**
  - **5 years default swap premium : 50bp, recovery rate = 40%**



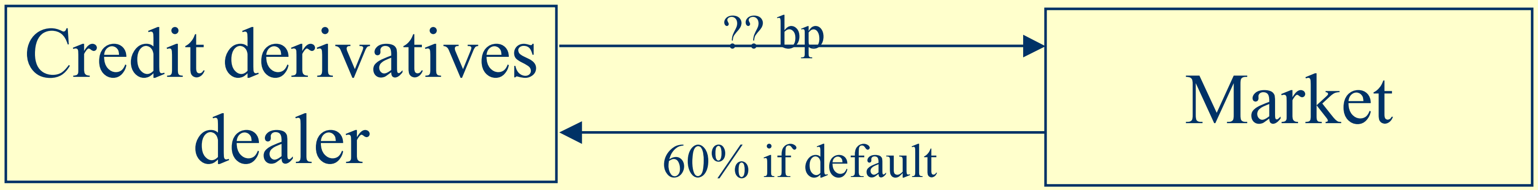


Hedging default risk: an introduction

- **Rolling over short-term default swap**
  - **at inception, one year default swap premium : 33bp**
  - **cash-flows after one year:**

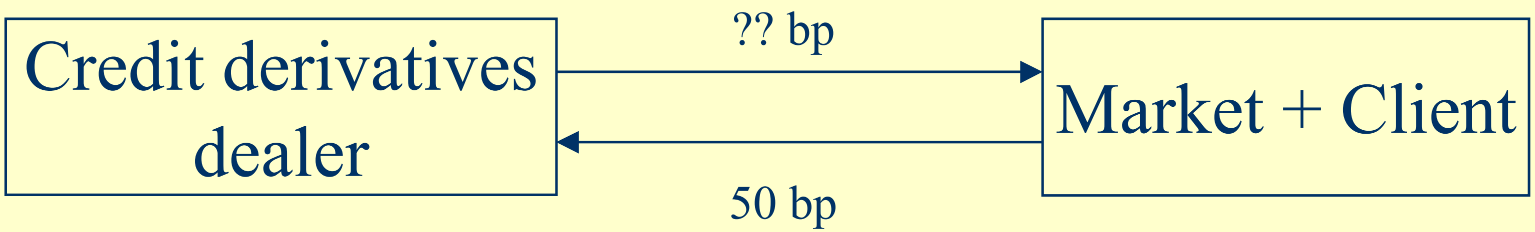


- **Buy a one year default swap at the end of every yearly period, if no default:**
  - **Dynamic strategy,**
  - **<u>future</u> premiums depend on <u>future</u> credit quality**
  - **future premiums are <u>unknown</u>**



Hedging default risk: an introduction

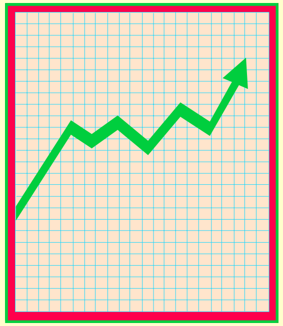
- ***Risk analysis* of rolling over short term against long term default swaps**

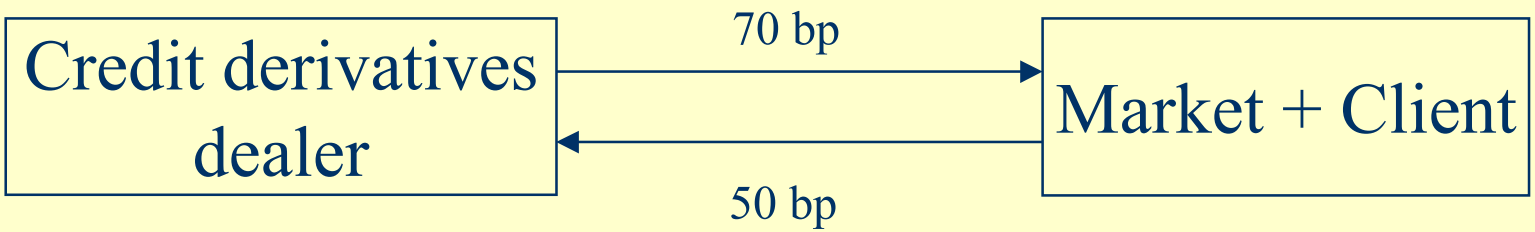


- **Exchanged cash-flows :**
  - **Dealer receives 5 years (fixed) credit spread,**
  - **Dealer pays 1 year (variable) credit spread.**
- **Full one to one protection at default time**
  - **the previous strategy has eliminated one source of risk, that is default risk**
  - **Recovery risk has been eliminated too.**

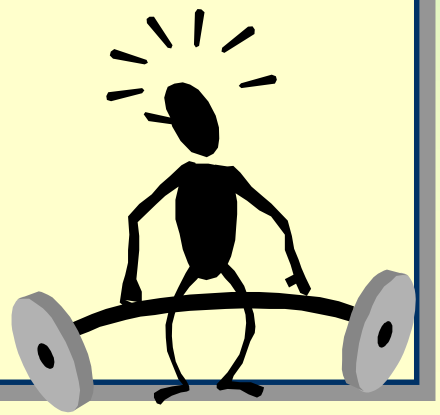
Hedging default risk: an introduction

- **Negative exposure to an increase in short-term default swap premiums**
  - **if short-term premiums increase from 33bp to 70bp**
  - **reflecting a lower (short-term) credit quality**
  - **and no default occurs before the fifth year**



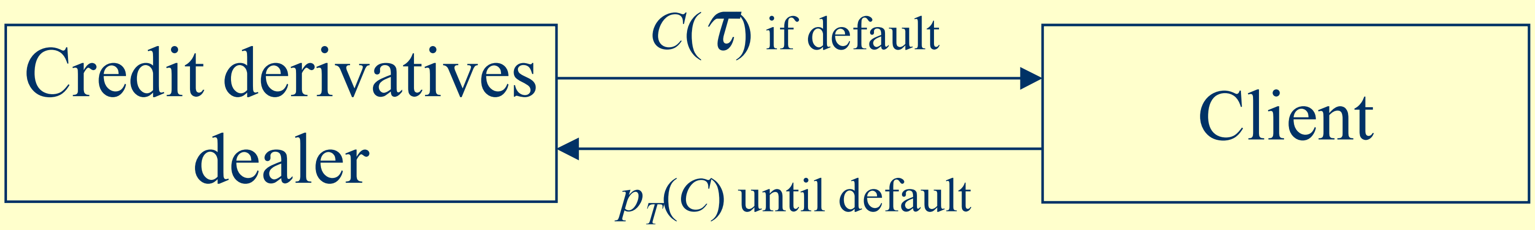


- **Loss due to negative carry**
  - **long position in long term credit spreads**
  - **short position in short term credit spreads**



Hedging default risk: an introduction

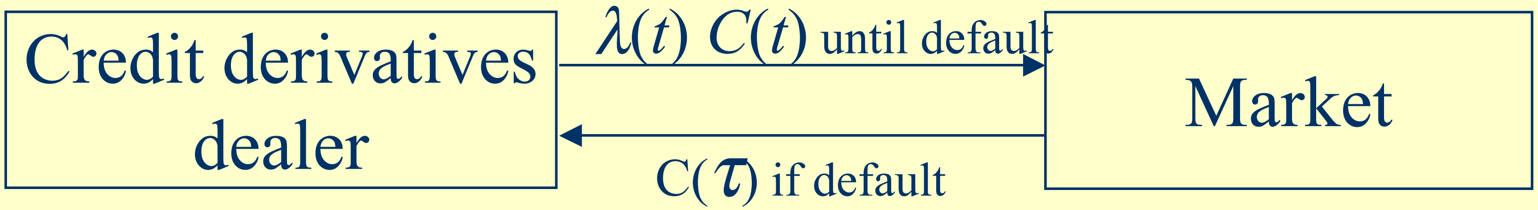
- ***Dynamic Default Swap***
  - client pays to dealer a periodic premium $p_T(C)$ until default time τ, or maturity of the contract T.
  - dealer pays $C(\tau)$ to client at default time τ, if $\tau \leq T$.



- **Hedging side:**
  - Dynamic strategy based on standard default swaps:
  - At time t, hold an amount $C(t)$ of standard default swaps
  - $\lambda(t)$ denotes the periodic premium at time t for a short-term default swap

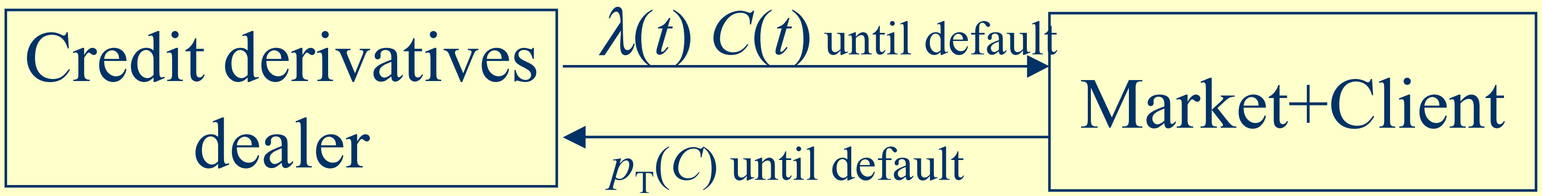
Hedging default risk: an introduction

- **<u>Hedging side</u>:**



  - **Amount of standard default swaps equals the (variable) <u>credit exposure</u> on the dynamic default swap.**

- **Net position is a "*basis swap*":**



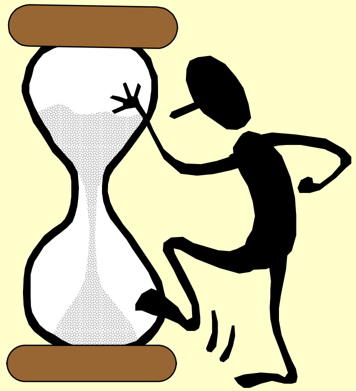
- **The client transfers credit spread risk to the credit derivatives dealer**

One step further: Hedging dynamic default swaps

- **Hedging *credit risk***
  - **Uniqueness of *equivalent martingale measure.***
- **PV of plain and dynamic default swaps**
- **Hedging dynamic default swaps**
  - **Hedging *default risk***
  - **Explaining *theta* effects**
  - **Hedging default risk *and* credit spread risk**
- **Hedging Credit spread options**

hedging credit risk

- **"firm-value" models :**
  - **Modelling of firm's assets**
  - **First time passage <u>below</u> a critical threshold**



- **risk-intensity based models**
  - **Default arrivals are no longer <u>predictable</u>**
  - **Model conditional local probabilities of default $\lambda(t)\, dt$**
  - **τ : default date, $\lambda(t)$ *risk intensity* or *hazard rate***

$$\lambda(t)dt = P\left[\tau \in [t, t+dt[\,\middle|\, \tau > t\right]$$

- **We need a <u>hedging based approach</u> to pricing.**

Hedging credit risk

- **<u>Uniqueness</u> of *equivalent martingale measure***
  - **Assume deterministic default-free interest rates**
  - **$r(t)$ default-free short rate, τ, default time**
  - **$I(t)=1_{\{\tau>t\}}$ indicator function. $I(t)$ jumps from 1 to 0 at time τ.**
  - **$H_t = \sigma\,(I(s), s \leq t)$: natural filtration of τ.**
  - **$P(\tau \in [t,t+dt[\;|\;H_t) = E(I(t)-I(t+dt)|\;H_t) = \lambda(t)I(t)dt$,**
  - **λ (historical) *default intensity* (w.r.t H_t):**
    - *Girsanov theorem*: Under any equivalent probability Q, the risk-intensity of τ becomes $\lambda(t)\,\phi(t)$ with $\phi(t)>0$.
  - **$Q(\tau \in [t,t+dt[\;|\;H_t) = E^Q(I(t)-I(t+dt)|\;H_t) = \lambda(t)\phi(t)I(t)dt$,**
- **Risky discount bond with maturity T : pays $1_{\{\tau>T\}}$ at time T**

Hedging credit risk

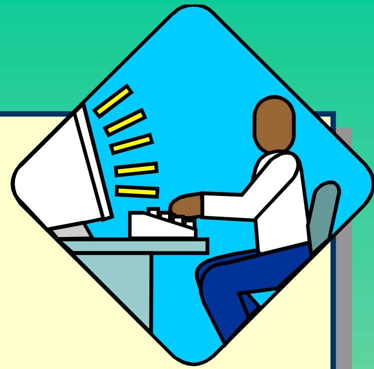
- $\bar{B}(t,T) \in H_t$ *t*-time price of **risky discount bond**.
- **Lemma:** Let $Z \in H_t$. Then Z is constant on $\{\tau > t\}$.
  - Proof: $\{\tau > t\}$ is an atom of H_t. Every random variable is constant on atoms
- $\bar{B}(t,T)$ is constant on $\{\tau > t\}$ and $\bar{B}(t,T) = 0$ on $\{\tau \le t\}$.
- $\Rightarrow \bar{B}(t,T) = c(t,T)1_{\{\tau > t\}} = c(t,T)I(t)$ where $c(t,T)$ is deterministic.
  - Then, $d\bar{B}(t,T) = c_t(t,T)I(t)dt + c(t,T)dI(t)$
- Let Q be an *equivalent martingale measure*.
  - Then $E^Q\left[d\bar{B}(t,T) \middle| H_t\right] \overset{(2)}{=} I(t) \times \left(c_t(t,T) - c(t,T)\lambda(t)\phi(t)\right)dt$
- On the other hand, $E^Q\left[d\bar{B}(t,T) \middle| H_t\right] \overset{(1)}{=} r(t)\bar{B}(t,T)dt = r(t)c(t,T)I(t)dt$

$$(1)+(2) \Rightarrow \boxed{\lambda(t)\phi(t) \overset{(3)}{=} -r(t) + d\ln c(t,T)/dt}, \text{ on } \{\tau > t\}$$

Hedging credit risk

- **Thus $\phi(t)$ and then *Q* are identified (uniqueness) from** $\overline{B}(t,T)$
  - **Let us denote:** $\overline{r}(t) = r(t) + \lambda(t)\phi(t)$
  - **Thus:** $d\ln c(t,T)/dt = \overline{r}(t)dt$ **with *c*(*T*,*T*)=1**
  - **Which provides:** $c(t,T) = \exp - \int_t^T \overline{r}(s)ds$ **(predefault price).**
- ***Summary* of results:**
  - $\overline{r}(t)$ **defaultable short rate,** $\boxed{\lambda(t)\phi(t) = \overline{r}(t) - r(t)}$ **risk-neutral intensity**
  - **Defaultable bond:** $\overline{B}(t,T) = 1_{\{\tau > t\}} \exp - \int_t^T \big(r(s) + \lambda(s)\phi(s)\big)ds$
  - **risk-neutral measure *Q*, with intensity of default $\lambda(t)\phi(t)$: *pricing*.**
  - **Historical measure *P*, with intensity of default $\lambda(t)$: *portfolio approaches***

PV of credit contracts



- **Risky discount bond price (no recovery):**

$$\bar{B}(t,T) = E_t\left[1_{\{\tau>T\}} \exp -\int_t^T r(s)ds\right] = 1_{\{\tau>t\}} E_t\left[\exp -\int_t^T (r+\lambda)(s)ds\right]$$

  - **λ: risk-neutral intensity**

- **More generally let X_T be a payoff paid at T, if $\tau>T$:**

$$PV_X(t) = E_t\left[X_T 1_{\{\tau>T\}} \exp -\int_t^T r(s)ds\right] = 1_{\{\tau>t\}} E_t\left[X_T \exp -\int_t^T (r+\lambda)(s)ds\right]$$

- $\exp -\int_t^T (r+\lambda)(s)ds$ ***stochastic risky discount factor***

PV of plain default swaps (continuous premiums)

- **Time u -PV of a plain default swap:**
  - **Maturity T, *continuously* paid premium p, recovery rate δ**
  - **Risk-free short rate r, default intensity λ**
  - **E_u expectation conditional on information carried by financial prices.**
  - **$r+\lambda$ is the « risky » short rate : payoffs discounted at a higher rate**
  - **Similar to an index amortizing swap (payments only if no prepayment).**

- **PV of *default payment leg*:**

$$1_{\{\tau>u\}} E_u\left[\int_u^T \left(\exp-\int_u^t (r+\lambda)(s)ds\right)\times(1-\delta)\lambda(t)dt\right]+1_{\{\tau\le u\}}(1-\delta)\exp\int_\tau^u r(s)ds$$

- **PV of *premium payment leg*:**

$$1_{\{\tau>u\}}\, p\times E_u\left[\int_u^T \left(\exp-\int_u^t (r+\lambda)(s)ds\right)dt\right]$$

PV of dynamic default swaps (continuous premiums)

- **Time *u* -PV of a dynamic default swap**
  - **Payment $C(\tau)$ at default time if $\tau<T$:**
- **PV of *default payment leg***

$$1_{\{\tau>u\}}E_u\left[\int_u^T\left(\exp-\int_u^t(r+\lambda)(s)ds\right)C(t)\lambda(t)dt\right]+1_{\{\tau\geq u\}}C(\tau)\exp\int_\tau^u r(s)ds$$

  - **This embeds the plain default swap case where $C(\tau)=1-\delta$**
- **PV of *premium payment leg***

$$1_{\{\tau>u\}}p\times E_u\left[\int_u^T\left(\exp-\int_u^t(r+\lambda)(s)ds\right)dt\right]$$

  - **Same as in the case of plain default swap**

Hedging dynamic default swaps

- **Exotic credit derivatives can be *hedged* against default:**
  - **Constrains the amount of underlying standard default swaps.**
  - **Variable amount of standard default swaps.**
  - **Full protection at default time by construction of the hedge.**
  - **No more *discontinuity* in the P&L at default time.**
  - **Model-free approach.**
- **Credit spread exposure has to be hedged by *other means*:**
  - **Appropriate *choice of maturity* of underlying default swap**
  - **Use of CDS with different maturities.**
  - **Computation of sensitivities with respect to changes in credit spreads are model dependent.**

Hedging dynamic default swaps

- **PV at time *u* of a digital default swap**

$$PV(u) = 1_{\{\tau>u\}} E_u\left[\int_u^T \left(\exp - \int_u^t (r+\lambda)(s)ds\right) \times (\lambda(t) - p)dt\right] + 1_{\{\tau \le u\}} \exp \int_\tau^u r(t)dt$$

  - **At default time τ, PV switches from** $E_u\left[\int_u^T \left(\exp - \int_u^t (r+\lambda)(s)ds\right) \times (\lambda(t) - p)dt\right]$
  - **to one (default payment). If digital default swap at the money, d*PV*(τ)=1**
- **PV at time *u* of a dynamic default swap with payment *C*: $PV_C(u)$**

$$1_{\{\tau>u\}} E_u\left[\int_u^T \left(\exp - \int_u^t (r+\lambda)(s)ds\right) \times (\lambda(t)C(t) - p_C)dt\right] + 1_{\{\tau \le u\}} C(\tau) \exp \int_\tau^u r(t)dt$$

  - **At default time τ, PV switches from pre-default market value PV(τ-) to *C*(τ)**
- **To hedge default risk, we hold ($C(u)$-$PV_C(u)$) digital default swaps**
  - **Variation of PV at default time on the hedging portfolio:**

$$\left(C(\tau) - PV_C(\tau^-)\right) dPV(\tau) = C(\tau) - PV_C(\tau^-)$$

- **Hedging default risk is <u>model free</u>. No *recovery risk*.**

Explaining theta effects in the P&L dynamics

- ***Different aspects*** **of "carrying" credit contracts through time.**
  - **Analyse the risk-neutral dynamics of the P&L.**
- **Consider a *short* position in a dynamic default swap.**
- ***Pre-default* Present value of the contract provided by:**

$$PV(u) = E_u\left[\int_u^T \left(\exp - \int_u^t (r+\lambda)(s)ds\right) \times \left(p_T - \lambda(t)C(t)\right)dt\right]$$

- ***Net expected capital gain*** **(conditional on no default):**

$$E_u\left[PV(u+du) - PV(u)\right] = \left(r(u)+\lambda(u)\right)PV(u)du + \left(\lambda(u)C(u) - p_T\right)du$$

- ***Accrued cash-flows*** **(received premiums):** $p_T du$
  - **By summation, Incremental P&L (if no default between *u* and *u+du*):**

$$r(u)PV(u)du + \lambda(u)\left(C(u) + PV(u)\right)du$$

Explaining theta effects in the P&L dynamics

- **<u>Apparent</u>** ***extra return*** **effect :** $\lambda(u)\big(C(u)+PV(u)\big)du$
  - **But, probability of default between *u* and *u*+*du*: $\lambda(u)du$.**
  - **Losses in case of default:**
    - Commitment to pay: $C(u)$ + loss of PV of the credit contract: PV(u)
    - PV(u) consists in **<u>unrealised</u>** capital gains or losses in the credit derivatives book that "disappear" in case of default.
  - **Expected loss charge:** $\lambda(u)\big(C(u)+PV(u)\big)du$
- **Under risk-neutral probability, *in average* P&L <u>does</u> increase at rate $r(u)$!**
- **Hedging aspects:**
  - **If we hold $C(u)+PV(u)$ short-term digital default swaps, we are protected at default-time (no jump in the P&L).**
  - **Premiums to be paid:** $\lambda(u)\big(C(u)+PV(u)\big)du$
  - **The hedged P&L increases at rate $r(u)$ (mimics savings account).**

Hedging default risk and credit spread risk

- **Denote by $I(u)=1_{\{\tau>u\}}$, $dI(u)$ = variation of jump part.**
- **Digital default swap:**
  - **PV prior to default:** $PV^b(u) = E_u\left[\int_u^T \left(\exp - \int_u^t (r+\lambda)(s)ds\right) \times (\lambda(u) - p)dt\right]$
  - **PV after default:** $PV^a(u) = \exp \int_\tau^u r(t)dt$
  - **PV whenever:** $PV(u) = I(u)PV^b(u) + (1 - I(u))PV^a(u)$

$$dPV(u) = \left(PV^b(u) - PV^a(u)\right)dI(u) + I(u)dPV^b(u) + \left(1 - I(u)\right)dPV^a(u)$$

Discontinuous part
default risk

Continuous part (credit spread risk)

- **Discontinuous part : constrains the amount of hedging default swaps**
  - ***After hedging default risk, no jump in the PV at default time.***
- **Hedging continuous part (see below)**

Hedging Default risk and credit spread risk

- **Hedging continuous part**
  - **Assume some state variable following diffusion processes (i.e. no jumps in credit spreads).**
  - **Pre-default PV of dynamic default swaps, plain CDS:**

$$E_u\left[\int_u^T\left(\exp-\int_u^t (r+\lambda)(s)ds\right)\times\left(\lambda(t)C(t)-p_C\right)dt\right]$$

  - **Provided as a solution of linear PDE.**
- ***Credit spread risk* (« continuous » part) is hedged by delta analysis:**
  - **Compute the sensitivities of dynamic default swap to be hedged and of hedging CDS w.r.t state variables.**
  - **Choose amount of hedging CDS so that portfolio sensitivity =0.**

Hedging Default risk and credit spread risk

- **Example: hedging CDS with non standard maturities.**
  - **Maturity *T*, premium *p*, pre-default PV:**

$$PV_T^b(u) = E_u\left[\int_u^T \left(\exp - \int_u^t (r+\lambda)(s)ds\right) \times \left(p - \lambda(u)(1-\delta)\right)dt\right]$$

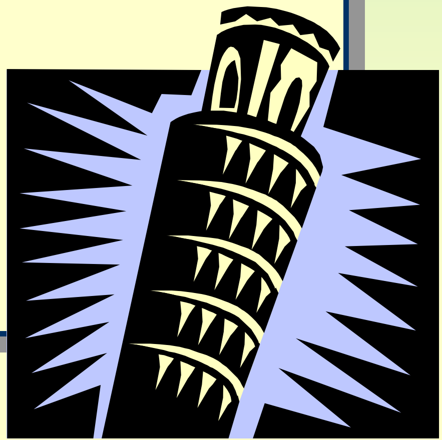
  - **PV jumps from $PV_T^b(u)$ to -(1- δ) at default time.**
  - **Hedging instruments: at the money traded CDS (PV(u)=0)**
  - **Total amount of hedging CDS:** $\dfrac{1-\delta+PV_T^b(u)}{1-\delta} \approx 1$
  - ***Small recovery risk.***
  - **Hedging credit spread risk:**
    - choose amount of hedging CDS so that the sensitivities of maturity *T* CDS and hedging CDS w.r.t to credit spreads are equal
    - Need of two hedging CDS (two constraints)

Hedging Default risk and credit spread risk

- **Hedging *default risk* only constrains the amount of underlying standard default swap.**
  - **Maturity of underlying default swap is arbitrary.**
- **Choose maturity (of underlying CDS) to be protected against credit spread risk**
  - **PV of dynamic default swaps and standard default swaps are sensitive to the level of credit spreads**
  - **Sensitivity of standard default swaps to a shift in credit spreads increases with maturity**
  - **Choose maturity of underlying default swap in order to equate sensitivities.**
    - All the computations are ***model dependent***.
    - Previous approach involves changing the maturity of underlying through time.

Hedging Default risk and credit spread risk

- **Alternative approach: choose two given maturities**
- ***Several maturities* of underlying default swaps may be used to match sensitivities.**
  - For example, in the case of **defaultable** interest rate swap, the nominal amount of default swaps $(PV_\tau)^+$ is usually small.
  - ***Single*** default swap with nominal $(PV_\tau)^+$ has a ***smaller sensitivity*** to credit spreads than ***defaultable interest rate swap***, even for long maturities.
  - **Short** and **long** positions in default swaps are required to hedge ***credit spread risk***.



Hedging credit spread options

- **Option to enter a given default swap with premium *p*, maturity *T'* at exercise date *T*.**
  - **Call option provides positive payoff if credit spreads increase.**
    - Credit spread risk
  - **If default prior to *T*, cancellation of the option**
    - Default risk
- **The PV is of the form** $PV(u) = 1_{\{\tau > u\}} PV^b(u)$
  - **Hedge default risk by holding an amount of $PV^b(u)$ default swaps.**
  - **$PV^b(u)$ is usually small compared with payments involved in default swaps.**
  - **$PV^b(u)$ depends on risk-free and risky curves (mainly on credit spreads).**
  - **Credit spread risk is also hedged through default swaps.**
- **Our previous framework for hedging default risk and credit spread risk still holds.**

Hedging Basket default swaps: some specificities

- **Consider a basket of *M* defaultable bonds**
  - **multiple counterparties**
- **First to default swaps**
  - **protection against the first default**
- ***N* out of *M* default swaps (*N* < *M*)**
  - **protection against the first *N* defaults**
- **Hedging and valuation of basket default swaps**
  - **involves the joint (multivariate) modelling of default arrivals of issuers in the basket of bonds.**
  - **Modelling accurately the dependence between default times is a critical issue.**

Hedging Basket default swaps: some specificities

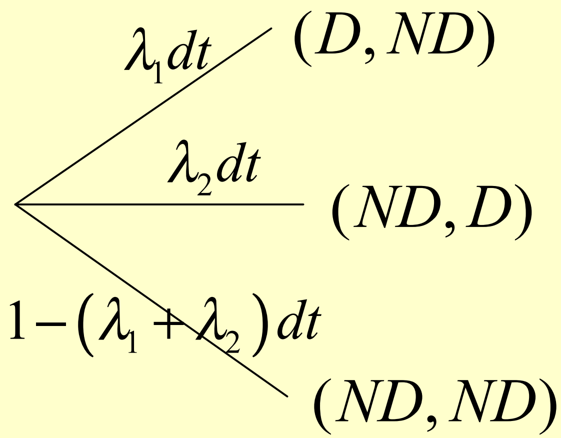
- Hedging Default Risk in Basket Default Swaps
- Example: first to default swap from a basket of two risky bonds.
  - If the first default time occurs before maturity,
  - The seller of the first to default swap pays the non recovered fraction of the defaulted bond.
  - Prior to that, he receives a periodic premium.
- Assume that the two bonds cannot default simultaneously
  - We moreover assume that default on one bond has *no effect* on the credit spread of the remaining bond.
- How can the seller be protected *at default time* ?
  - The only way to be protected at default time is to hold two default swaps with the *same nominal* than the *nominal* of the bonds.
  - The *maturity* of underlying default swaps does not matter.

Hedging Basket default swaps: some specificities

- **Some notations :**
  - **τ_1, τ_2 default times of counterparties 1 and 2,**
  - **$\mathcal{H}_t$ available information at time *t*,**
  - ***P* historical probability,**
  - **λ_1 , λ_2 : (historical) risk intensities:**
    - $$P\left[\tau_i \in [t, t+dt[\middle| H_t\right] = \lambda_i dt,\ i = 1, 2$$
- **Assumption : « Local » independence between default events**
  - **Probability of 1 and 2 defaulting altogether:**
    - $$P\left[\tau_1 \in [t, t+dt[, \tau_2 \in [t, t+dt[\middle| H_t\right] = \lambda_1 dt \times \lambda_2 dt \text{ in } (dt)^2$$
  - **Local independence: simultaneous joint defaults can be neglected**

Hedging Basket default swaps: some specificities

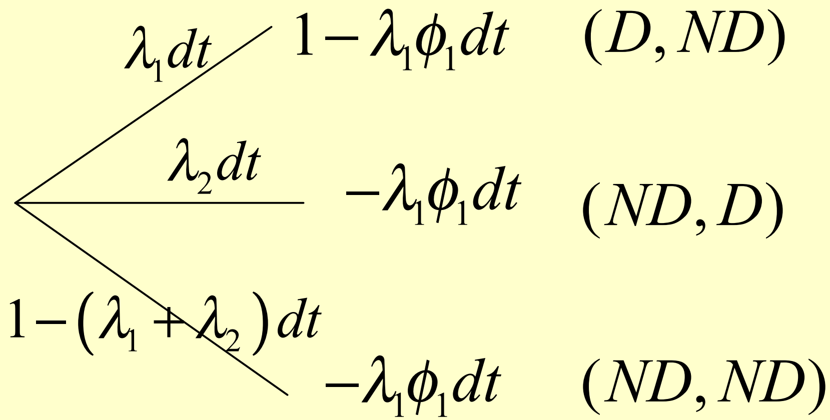
- **Building up a tree:**
  - **Four possible states: (*D*,*D*), (*D*,*ND*), (*ND*,*D*), (*ND*,*ND*)**
  - **Under no simultaneous defaults assumption $p_{(D,D)}=0$**
  - **Only three possible states: (*D*,*ND*), (*ND*,*D*), (*ND*,*ND*)**
  - **Identifying (historical) tree probabilities:**



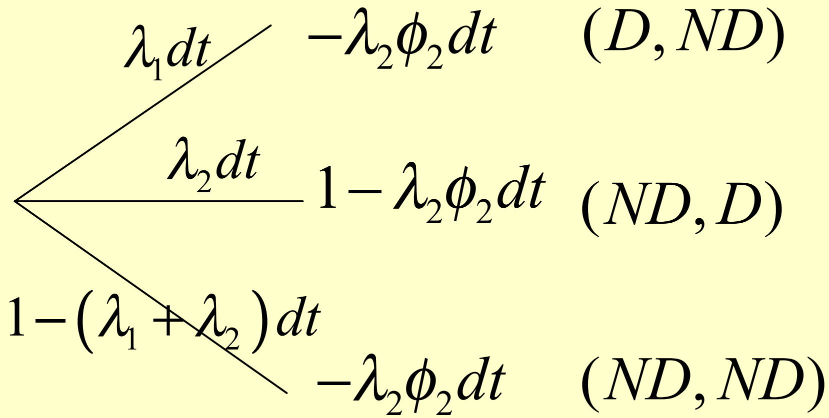
$$\begin{cases} p_{(D,D)}=0 \Rightarrow p_{(D,ND)}=p_{(D,D)}+p_{(D,ND)}=p_{(D,.)}=\lambda_1 dt \\ p_{(D,D)}=0 \Rightarrow p_{(ND,D)}=p_{(D,D)}+p_{(ND,D)}=p_{(.,D)}=\lambda_2 dt \\ p_{(ND,ND)}=1-p_{(D,.)}-p_{(.,D)} \end{cases}$$

Hedging Basket default swaps: some specificities

- **Cash flows of (digital) CDS on counterparty 1:**
  - **$\lambda_1 \phi_1 dt$ CDS premium, ϕ_1 default risk premium**



- **Cash flows of (digital) CDS on counterparty 2:**



Hedging <u>Basket default swaps</u>: some specificities

- **Cash flows of (digital) first to default swap (with premium p_F):**

  - $\lambda_1 dt$: $1 - p_F dt \quad (D, ND)$
  - $\lambda_2 dt$: $1 - p_F dt \quad (ND, D)$
  - $1 - (\lambda_1 + \lambda_2) dt$: $-p_F dt \quad (ND, ND)$

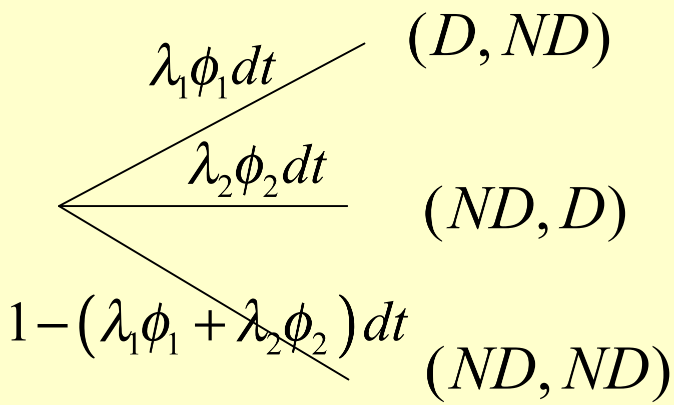
- **Cash flows of holding CDS 1 + CDS 2:**

  - $\lambda_1 dt$: $1 - (\lambda_1 \phi_1 + \lambda_2 \phi_2) dt \quad (D, ND)$
  - $\lambda_2 dt$: $1 - (\lambda_1 \phi_1 + \lambda_2 \phi_2) dt \quad (ND, D)$
  - $1 - (\lambda_1 + \lambda_2) dt$: $-(\lambda_1 \phi_1 + \lambda_2 \phi_2) dt \quad (ND, ND)$

- **Absence of arbitrage opportunities imply:**
  - $p_F = \lambda_1 \phi_1 + \lambda_2 \phi_2$
  - **Perfect hedge of first to default swap by holding 1 CDS 1 + 1 CDS 2**

Hedging Basket default swaps: some specificities

- **Three possible states: (D,ND), (ND,D), (ND,ND)**
- **Three tradable assets: CDS1, CDS2, risk-free asset**
  - ➢The market is still « complete »
- **Risk-neutral probabilities**
  - **Used for computing prices**
  - **Consistent pricing of traded instruments**
  - **Uniquely determined from CDS premiums**
  - $p_{(D,D)}=0$, $p_{(D,ND)}=\lambda_1 \phi_1 dt$, $p_{(ND,D)}=\lambda_2 \phi_2 dt$, $p_{(ND,ND)}=1-(\lambda_1 \phi_1+\lambda_2 \phi_2)\, dt$



Hedging Basket default swaps: some specificities

- ***hedge ratios*** **for first to default swaps**
- **Consider a first to default swap associated with a basket of two defaultable loans.**
  - **Hedging portfolios based on standard underlying default swaps**
  - **Hedge ratios if:**
    - simultaneous default events
    - *Jumps* of credit spreads at default times
- **Simultaneous default events:**
  - **If counterparties default *altogether*, holding the *complete* set of default swaps is a conservative (and thus expensive) hedge.**
  - **In the *extreme* case where default *always* occur altogether, we only need a single default swap on the loan with largest nominal.**
  - **In other cases, holding a *fraction* of underlying default swaps does not hedge default risk (if *only one* counterparty defaults).**

Hedging Basket default swaps: some specificities

- hedge ratios for first to default swaps:
- What occurs if there is a *jump in the credit spread* of the second counterparty after default of the first ?
  - default of first counterparty means *bad news* for the second.
- If hedging with short-term default swaps, no capital gain at default.
  - Since PV of short-term default swaps is not *sensitive* to credit spreads.
- This is not the case if hedging with long term default swaps.
  - If credit spreads jump, PV of long-term default swaps jumps.
- Then, the amount of hedging default swaps can be reduced.
  - This reduction is *model-dependent.*

Hedging and Risk Management of Basket and Dynamic Default Swaps: conclusion

- **hazard rate based models :**
  - **default is a sudden, *non predictable* event,**
  - **that causes a sharp jump in defaultable bond prices.**
  - **Most dynamic default swaps and basket default derivatives have payoffs that are *linear* (at default) in the prices of defaultable bonds.**
  - **Thus, good news: default risk and recovery risk can be *hedged*.**
  - **More realistic approach to default.**
  - ***Hedge ratios* are robust with respect to default risk.**
  - **Credit spread risk can be hedged too, but model risk.**

Hedging and Risk Management of Basket and Dynamic Default Swaps: conclusion

- **Looking for a better understanding of credit derivatives**
  - **payments in case of default,**
  - **volatility of credit spreads.**
- **Bridge between risk-neutral valuation and the cost of the hedge approach.**
- **<u>dynamic</u> hedging strategy based on *standard default swaps.***
  - **hedge ratios in order to get protection at default time.**
  - **hedging default risk is *model-independent.***
  - **importance of quantitative models for a better management of the P&L and the <u>residual</u> <u>risks</u>.**