Applying hedging techniques to credit derivatives

Risk Training
Pricing and Hedging Credit Derivatives
London 26 & 27 April 2001

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• Portfolio approaches to credit risk.
  – Relating portfolio approaches and credit derivatives.

• Closing the gap between supply and demand of credit risk:
  ➢ Default Swaps,
  ➢ Dynamic Default Swaps, Basket Credit Derivatives,
  ➢ Credit Spread Options.
  – The previous means tend to be more integrated.
  – Technical innovations favour efficient risk transfer.

• The early stages of hedging credit derivatives.

• The nature of credit risk.
Consider a given portfolio:

- Including credits, lines of credit, corporate bonds, interest rate swaps and forex swaps, OTC options, tranches of CDO’s…
- Over various defaultable counterparties.
- Some credit exposures may be partially protected through collateral, credit insurance, prioritisation,…

The main goal is to construct a distribution of losses arising from credit risk (and other financial risks).

- Over one given time horizon (say one year)
- From this distribution, one may consider different risk indicators, such as quantiles (VaR measures), Expected shortfall,…
Portfolio approaches to credit risk

• Some issues currently addressed (portfolio approaches):
  – Construction of Databases:
    ➢ default events, credit spreads,
    ➢ use of external data (credit ratings, expected default frequencies).
  – Extending the scope of credit risk assessment:
    ➢ default risk on non quoted or small counterparties.
    ➢ Integrating default risk approaches for smaller credits (mortgages, consumer loans) and corporate credit risk.
  – Theoretical issues:
    ➢ Consistency over different time horizons.
    ➢ Should we use Value at Risk for credit risk?
Some issues currently addressed (portfolio approaches):

- Modelling of correlation:
  - non Gaussian variables,
  - default times, default losses, indicators of credit events,
- Wrong way exposure:
  - correlation between financial variables (such as interest rates, stock indices, exchange rates) and defaults.
- Joint modelling of defaults and credit spread risk.
- Relating portfolio approaches based on historical data and market prices on traded default risk
  - some inconsistencies may appear between from the two points of view (basket default swaps).
Portfolio approaches to credit risk

• From assessment to management of credit risk.
• Credit risk profile (Expected and unexpected loss) of portfolios can be modified. Risk/Return ratios can be enhanced.
  – By using credit derivatives
  – Through securitization schemes
  – By dynamic management of credit exposures: we can think of reducing credit exposures when credit spreads rise.
• Dynamic approaches to the hedging and management of credit risk are being transposed to the financial industry.
  – This will eventually enhance the ability of credit derivatives desks to an efficient management of more sophisticated risks.
Portfolio approaches to credit risk

- From *assessment* to *management* of credit risk.
- Understanding the main ideas and techniques regarding dynamic hedging of credit risk in the credit derivatives world may be useful to *credit risk managers, capital managers, CRO,…*
  - As end users of credit protection structures, sellers of credit risk
  - To better manage some dynamic aspects of credit risk management, such as variable credit exposure:
    - risk reduction can be achieved through a dynamic use of standard products (plain CDS) and through more sophisticated derivatives (dynamic CDS, basket CDS).
• Credit risk trades may not be **simultaneous**.
  – Since at one point in time, demand and offer of credit risk may not match.
  – It is not required to find customers with exact opposite interest at every new deal.

  ➢ **Meanwhile**, credit risk remains within the balance sheet of the financial intermediary (**high capital at risk**)
**Hedging** credit risk enhances ability to transfer credit risk by *lowering capital at risk*.
• **dynamic default swap**: « *Structured product* »
  – *efficient way* to transfer credit risk

• **Anatomy of a dynamic default swap**
  – A dynamic default swap is like a standard default swap but with variable nominal (or exposure).
  – However the periodic premium paid for the credit protection remains fixed.
  – The protection payment arises at default of *one given single risky counterparty*.

• **Examples:**
  – *quanto default swaps* (credit protection of forex bonds)
  – *cancellable swaps* (cancelled at default time of a third counterparty).
  – credit protection of a portfolio of contracts:
    - *vulnerable swaps*, OTC options,
    - *full protection*, *excess of loss insurance*, *partial collateralization*
Example: *defaultable interest rate swap*

Consider a defaultable interest rate swap (with unit nominal)
- We are default-free, our counterparty is defaultable.
- We consider a (fixed-rate) *receiver* swap on a standalone basis.

Recovery assumption, payments in case of default:
- if default at time $\tau$, compute the default-free value of the swap: $PV_\tau$
- and get: $\delta(PV_\tau)^+ + (PV_\tau)^- = PV_\tau - (1-\delta)(PV_\tau)^+$
- $0 \leq \delta \leq 1$ recovery rate, $(PV_\tau)^+ = \text{Max}(PV_\tau, 0)$, $(PV_\tau)^- = \text{Min}(PV_\tau, 0)$
- *In case of default,*
  - we receive default-free value $PV_\tau$
  - minus
  - loss equal to $(1-\delta)(PV_\tau)^+$. 
Example: defaultable interest rate swap

Using a dynamic default swap to hedge credit risk:

- Consider a dynamic default swap paying \((1-\delta)(PV_\tau)^+\) at default time \(\tau\) (if \(\tau \leq T\)),
  
  \[
  PV_\tau \text{ is the present value of a default-free swap with same fixed rate than defaultable swap.}
  \]

- At default, we receive \((1-\delta)(PV_\tau)^+ + PV_\tau - (1-\delta)(PV_\tau)^+ = PV_\tau\)

- Thanks to credit protection, we receive the PV of the default free interest rate swap.
The nature of credit risk

• Pricing at the cost of the hedge:
  – If some risk can be hedged, its price should be the cost of the hedge.
  – Think of a plain vanilla stock index call. Its replication price is 10% (say).
  – One given investor is ready to pay for 11% (He feels better of with such an option, then doing nothing). Should he really give this 1% to the market?

• The feasibility of hedging («completeness») is a fundamental idea.
  – If credit instruments can be hedged, pricing dynamic default swaps, basket default swaps based only historical data and portfolio approaches will eventually lead to arbitrage opportunities.
  – Good news for knowledgeable individuals. Bad news for the understanding of risks.
The nature of credit risk

• **Misconceptions about credit risk.**
  – At the early stage of credit risk analysis, a common idea was that credit risk was not hedgeable:
    - incomplete markets, multiplicity of risk-neutral measures.
    - In firm-value models and complete information, default bonds are (too simplistically) considered as equity barrier options.

• **When some a defaultable bond is already traded, then the market can become complete.**
  – If there is also credit spread risk (that is not fully correlated to other financial variables), then we need (at least) two defaultable bonds.
The early stages of hedging credit derivatives

- Static arbitrage of plain default swaps with short selling underlying defaultable bond
  - CDS premiums should be related to credit spreads on floaters.
- One step further: hedging non standard maturities assuming smooth credit curves.
  - “bond strippers”: allow to compute prices of risky zero-coupon bonds.
- Consider a six years maturity CDS hedged with a five years maturity CDS of same nominal.
  - Protection at default time.
  - Since maturities of credit derivatives do not perfectly match, credit spread risk.
  - We may need several maturities of CDS + some assumptions on the dynamics of CDS premiums in order to hedge credit spread risk.
Assessing the varieties of risks involved in credit derivatives

- Specific risk or credit spread risk
  - prior to default, the P&L of a book of credit derivatives is driven by changes in credit spreads.

- Default risk
  - in case of default, if unhedged,

**Real world issues with hedging plain CDS:**

- deliverance of some unknown underlying, possibly short-term or fixed rate long term bond,
- Management of short-selling and repo margins on illiquid bonds,
- « small inconsistencies » due to accrued coupons, accrued premiums.
- illiquid hedging default swaps
Hedging credit derivatives: overview

- Hedging default (and recovery) risk: an introduction.
  - Short term default swaps v.s. long-term default swaps
  - Credit spread transformation risk
- One step further: hedging *Dynamic Default Swaps*, credit spread options.
  - Hedging default risk through dynamics holdings in standard default swaps.
  - Hedging credit spread risk by choosing appropriate default swap maturities.
- Hedging: *Basket Default Swaps* some specificities
  - Uncertainty at default time
Hedging default risk: an introduction

• Disentangling risks in credit instruments
  – **Interest rate risk**: due to movements in default-free interest rates.
  – **Default risk**: default bond price jumps to recovery value at default time.
  – **Credit spread risk** (specific risk): variation in defaultable bond prices prior to default, due to changes in credit quality (for instance ratings migration) or changes in risk premiums.
  – **Recovery risk**: unknown recovery rate in case of default.

• **Hedging** exotic credit derivatives will imply hedging all sources of risk.

• A *new approach* to credit derivatives modelling based on an **hedging** point of view
Hedging default risk: an introduction

**Purpose:**
- Introduction to dynamic trading of default swaps
- Illustrates how default and credit spread risk arise

**Arbitrage between long and short term default swaps**
- sell one long-term default swap
- buy a series of short-term default swaps

**Example:**
- default swaps on a FRN issued by BBB counterparty
- 5 years default swap premium: 50bp, recovery rate = 40%

Credit derivatives dealer

If default, 60%

Until default, 50 bp

Client
Hedging default risk: an introduction

- **Rolling over short-term default swap**
  - at inception, one year default swap premium: 33bp
  - cash-flows after one year:

  ![Diagram showing the rolling over of short-term default swap]

  - Credit derivatives dealer → Market
  - 33 bp
  - 60% if default

- **Buy a one year default swap at the end of every yearly period, if no default:**
  - Dynamic strategy,
  - future premiums depend on future credit quality
  - future premiums are unknown

  ![Diagram showing the purchase of a one year default swap]

  - Credit derivatives dealer → Market
  - ?? bp
  - 60% if default
**Risk analysis** of rolling over short term against long term default swaps

- Exchanged cash-flows:
  - Dealer receives 5 years (fixed) credit spread,
  - Dealer pays 1 year (variable) credit spread.

- **Full one to one protection at default time**
  - the previous strategy has **eliminated** one source of risk, that is **default risk**
  - **Recovery risk** has been eliminated too.
Hedging default risk: an introduction

- Negative exposure to an increase in short-term default swap premiums
  - if short-term premiums increase from 33bp to 70bp
  - reflecting a lower (short-term) credit quality
  - and no default occurs before the fifth year

- Loss due to negative carry
  - long position in long term credit spreads
  - short position in short term credit spreads
Hedging default risk: an introduction

• **Dynamic Default Swap**
  – client pays to dealer a periodic premium $p_T(C)$ until default time $\tau$, or maturity of the contract $T$.
  – dealer pays $C(\tau)$ to client at default time $\tau$, if $\tau \leq T$.

\[
\text{Credit derivatives}\quad C(\tau) \text{ if default}\quad \text{dealer}\quad p_T(C) \text{ until default}\quad \text{Client}
\]

• **Hedging side:**
  – **Dynamic** strategy based on **standard** default swaps:
  – At time $t$, hold an amount $C(t)$ of standard default swaps
  – $\lambda(t)$ denotes the periodic premium at time $t$ for a short-term default swap
Hedging default risk: an introduction

**Hedging side:**

- Amount of standard default swaps equals the (variable) credit exposure on the dynamic default swap.

**Net position is a “basis swap”:**

- The client transfers credit spread risk to the credit derivatives dealer.
One step further: Hedging dynamic default swaps

- Hedging credit risk
  - Uniqueness of equivalent martingale measure.
- PV of plain and dynamic default swaps
- Hedging dynamic default swaps
  - Hedging default risk
  - Explaining theta effects
  - Hedging default risk and credit spread risk
- Hedging Credit spread options
hedging credit risk

• “firm-value” models:
  – Modelling of firm’s assets
  – First time passage below a critical threshold

• risk-intensity based models
  – Default arrivals are no longer predictable
  – Model conditional local probabilities of default $\lambda(t) \, dt$
  – $\tau$: default date, $\lambda(t)$ risk intensity or hazard rate

$$\lambda(t)\,dt = P[\tau \in [t, t+dt] | \tau > t]$$

• We need a hedging based approach to pricing.
Hedging credit risk

- **Uniqueness of equivalent martingale measure**
  - Assume deterministic default-free interest rates
  - \( r(t) \) default-free short rate, \( \tau \), default time
  - \( I(t)=1_{\{\tau >t\}} \) indicator function. \( I(t) \) jumps from 1 to 0 at time \( \tau \).
  - \( H_t = \sigma (I(s), s \leq t) \): natural filtration of \( \tau \).
  - \( P( \tau \in [t,t+dt] \mid H_t) = E(I(t)-I(t+dt) \mid H_t) = \lambda(t)I(t)dt \),
  - \( \lambda \) (historical) default intensity (w.r.t \( H_t \)):
    - **Girsanov theorem**: Under any equivalent probability \( Q \), the risk-intensity of \( \tau \) becomes \( \lambda(t) \phi(t) \) with \( \phi(t)>0 \).
  - \( Q( \tau \in [t,t+dt] \mid H_t) = E^Q(I(t)-I(t+dt) \mid H_t) = \lambda(t)\phi(t)I(t)dt \)

- **Risky discount bond** with maturity \( T \): pays \( 1_{\{\tau >T\}} \) at time \( T \)
Hedging credit risk

- $\bar{B}(t,T) \in H_t$, $t$-time price of risky discount bond.
- Lemma: Let $Z \in H_t$. Then $Z$ is constant on \{τ > t\}.
  
  - Proof: \{τ > t\} is an atom of $H_t$. Every random variable is constant on atoms

- $\bar{B}(t,T)$ is constant on \{τ > t\} and $\bar{B}(t,T) = 0$ on \{τ ≤ t\}.

- $\Rightarrow \bar{B}(t,T) = c(t,T)1_{\{τ > t\}} = c(t,T)I(t)$ where $c(t,T)$ is deterministic.
  
  - Then, $d\bar{B}(t,T) = c_i(t,T)I(t)dt + c(t,T)dI(t)$

- Let $Q$ be an equivalent martingale measure.
  
  - Then $E^Q\left[d\bar{B}(t,T)\mid H_t\right]^{(2)} = I(t)\times(c_i(t,T) - c(t,T)\lambda(t)φ(t))dt$

- On the other hand, $E^Q\left[d\bar{B}(t,T)\mid H_t\right]^{(1)} = r(t)\bar{B}(t,T)dt = r(t)c(t,T)I(t)dt$

$(1) + (2) \Rightarrow \lambda(t)φ(t) = -r(t) + d\ln c(t,T)/dt$, on \{τ > t\}
Hedging credit risk

- Thus $\phi(t)$ and then $Q$ are identified (uniqueness) from $\bar{B}(t,T)$
  - Let us denote: $\bar{r}(t) = r(t) + \lambda(t)\phi(t)$
  - Thus: $d \ln c(t,T) / dt = \bar{r}(t) dt$ with $c(T,T)=1$
  - Which provides: $c(t,T) = \exp{-\int_{t}^{T} \bar{r}(s) ds}$ (predefault price).

- Summary of results:
  - $\bar{r}(t)$ defaultable short rate, $\lambda(t)\phi(t) = \bar{r}(t) - r(t)$ risk-neutral intensity
  - Defaultable bond: $\bar{B}(t,T) = 1_{\{\tau > t\}} \exp{-\int_{t}^{T} (r(s) + \lambda(s)\phi(s)) ds}$
  - risk-neutral measure $Q$, with intensity of default $\lambda(t)$: pricing.
  - Historical measure $P$, with intensity of default $\lambda(t)$: portfolio approaches
PV of credit contracts

- Risky discount bond price (no recovery):

\[
\overline{B}(t, T) = E_t \left[ 1_{\{\tau>T\}} \exp \left( \int_{t}^{T} r(s)ds \right) \right] = 1_{\{\tau>t\}} E_t \left[ \exp \left( \int_{t}^{T} (r + \lambda)(s)ds \right) \right]
\]

- \(\lambda\): risk-neutral intensity

- More generally let \(X_T\) be a payoff paid at \(T\), if \(\tau>T\):

\[
PV_{X}(t) = E_t \left[ X_T 1_{\{\tau>T\}} \exp \left( \int_{t}^{T} r(s)ds \right) \right] = 1_{\{\tau>t\}} E_t \left[ X_T \exp \left( \int_{t}^{T} (r + \lambda)(s)ds \right) \right]
\]

- \(\exp \left( \int_{t}^{T} (r + \lambda)(s)ds \right)\) stochastic risky discount factor
PV of plain default swaps (continuous premiums)

- **Time $u$ -PV of a plain default swap:**
  - Maturity $T$, continuously paid premium $p$, recovery rate $\delta$
  - Risk-free short rate $r$, default intensity $\lambda$
  - $E_u$ expectation conditional on information carried by financial prices.
  - $r + \lambda$ is the « risky » short rate: payoffs discounted at a higher rate
  - Similar to an index amortizing swap (payments only if no prepayment).

- **PV of default payment leg:**
  $$1_{\{\tau > u\}} E_u \left[ \int_u^T \left( e^{-\int_u^t (r + \lambda)(s)ds} \right) \times (1 - \delta) \lambda(t) dt \right] + 1_{\{\tau \leq u\}} (1 - \delta) \exp \int_{\tau}^u r(s) ds$$

- **PV of premium payment leg:**
  $$1_{\{\tau > u\}} p \times E_u \left[ \int_u^T \left( e^{-\int_u^t (r + \lambda)(s)ds} \right) dt \right]$$
PV of dynamic default swaps (continuous premiums)

- Time $u$ - PV of a dynamic default swap
  - Payment $C(\tau)$ at default time if $\tau < T$:

- PV of default payment leg

$$1_{\{\tau > u\}} E_u \left[ \int_u^T \left( \exp - \int_u^t (r + \lambda)(s) ds \right) C(t) \lambda(t) ds \right] + 1_{\{\tau \geq u\}} C(\tau) \exp \int_{\tau}^u r(s) ds$$

  - This embeds the plain default swap case where $C(\tau) = 1 - \delta$

- PV of premium payment leg

$$1_{\{\tau > u\}} p \times E_u \left[ \int_u^T \left( \exp - \int_u^t (r + \lambda)(s) ds \right) dt \right]$$

  - Same as in the case of plain default swap
Exotic credit derivatives can be hedged against default:

- Constrains the amount of underlying standard default swaps.
- Variable amount of standard default swaps.
- Full protection at default time by construction of the hedge.
- No more discontinuity in the P&L at default time.
- Model-free approach.

Credit spread exposure has to be hedged by other means:

- Appropriate choice of maturity of underlying default swap
- Use of CDS with different maturities.
- Computation of sensitivities with respect to changes in credit spreads are model dependent.
Hedging dynamic default swaps

- **PV at time** $u$ **of a digital default swap**

$$ PV(u) = 1_{\{\tau > u\}} E_u \left[ \int_u^T \left( \exp - \int_u^t (r + \lambda) (s) ds \right) \times (\lambda(t) - p) dt \right] + 1_{\{\tau \leq u\}} \exp \int_{\tau}^u r(t) dt $$

  - **At default time** $\tau$, **PV switches from** $E_u \left[ \int_u^T \left( \exp - \int_u^t (r + \lambda) (s) ds \right) \times (\lambda(t) - p) dt \right]$ **to one (default payment).** If digital default swap at the money, $dPV(\tau) = 1$

- **PV at time** $u$ **of a dynamic default swap with payment** $C$: $PV_C(u)$

$$ 1_{\{\tau > u\}} E_u \left[ \int_u^T \left( \exp - \int_u^t (r + \lambda) (s) ds \right) \times (\lambda(t)C(t) - p_C) dt \right] + 1_{\{\tau \leq u\}} C(\tau) \exp \int_{\tau}^u r(t) dt $$

  - **At default time** $\tau$, **PV switches from pre-default market value** $PV(\tau)$ **to** $C(\tau)$

- **To hedge default risk, we hold** $(C(u) - PV_C(u))$ **digital default swaps**

  - **Variation of PV at default time on the hedging portfolio:**

$$ \left( C(\tau) - PV_C(\tau^-) \right) dPV(\tau) = C(\tau) - PV_C(\tau^-) $$

- **Hedging default risk is model free. No recovery risk.**
Explaining theta effects in the P&L dynamics

- **Different aspects** of “carrying” credit contracts through time.
  - Analyse the risk-neutral dynamics of the P&L.
- Consider a **short** position in a dynamic default swap.
- **Pre-default** Present value of the contract provided by:
  \[
P V (u) = \mathbb{E}_u \left[ \int_u^T \left( \exp \left( \int_u^t (r + \lambda(s)) ds \right) \times (p_T - \lambda(t)C(t)) dt \right) \right]
\]
- **Net expected capital gain** (conditional on no default):
  \[
  E_u [PV(u + du) - PV(u)] = (r(u) + \lambda(u))PV(u)du + (\lambda(u)C(u) - p_T) du
  \]
- **Accrued cash-flows** (received premiums): \( p_T du \)
  - By summation, Incremental P&L (if no default between \( u \) and \( u + du \)):
    \[
    r(u)PV(u)du + \lambda(u)(C(u) + PV(u)) du
    \]
Explaining theta effects in the P&L dynamics

- **Apparent extra return effect:** \( \lambda(u)(C(u) + PV(u))du \)
  - But, probability of default between \( u \) and \( u+du \): \( \lambda(u)du \).
  - Losses in case of default:
    - Commitment to pay: \( C(u) + \text{loss of PV of the credit contract: PV}(u) \)
    - \( PV(u) \) consists in **unrealised** capital gains or losses in the credit derivatives book that “disappear” in case of default.
  - Expected loss charge: \( \lambda(u)(C(u) + PV(u))du \)

- **Under risk-neutral probability, in average P&L does increase at rate \( r(u) \)!

- **Hedging aspects:**
  - If we hold \( C(u) + PV(u) \) short-term digital default swaps, we are protected at default-time (**no jump in the P&L**).
  - Premiums to be paid: \( \lambda(u)(C(u) + PV(u))du \)
  - The hedged P&L increases at rate \( r(u) \) (mimics savings account).
Hedging default risk and credit spread risk

Denote by $I(u) = 1_{\{\tau > u\}}$, $dI(u) = \text{variation of jump part}$.

Digital default swap:
- $\text{PV prior to default: } PV^b(u) = E_u \left[ \int_u^T \left( \exp - \int_u^t (r + \lambda)(s) ds \right) \times (\lambda(u) - p) dt \right]$
- $\text{PV after default: } PV^a(u) = \exp \int r(t) dt$
- $\text{PV whenever: } PV(u) = I(u)PV^b(u) + (1 - I(u))PV^a(u)$

$\text{dPV}(u) = \left( PV^b(u) - PV^a(u) \right) dI(u) + I(u)dPV^b(u) + (1 - I(u))dPV^a(u)$

Discontinuous part : constrains the amount of hedging default swaps

After hedging default risk, no jump in the PV at default time.

Hedging continuous part (see below)
Hedging Default risk and credit spread risk

• Hedging continuous part
  – Assume some state variable following diffusion processes (i.e. no jumps in credit spreads).
  – Pre-default PV of dynamic default swaps, plain CDS:
    \[
    E_u \left[ \int_u^T \left( \exp \left( \int_u^t (r + \lambda)(s) ds \right) \times (\lambda(t)C(t) - p_c) \right) dt \right]
    \]
    – Provided as a solution of linear PDE.

• Credit spread risk («continuous » part) is hedged by delta analysis:
  – Compute the sensitivities of dynamic default swap to be hedged and of hedging CDS w.r.t state variables.
  – Choose amount of hedging CDS so that portfolio sensitivity =0.
Hedging Default risk and credit spread risk

• Example: hedging CDS with non standard maturities.
  – Maturity $T$, premium $p$, pre-default PV:

$$PV^b_T(u) = E_u \left[ \int_u^T \exp - \int_u^t (r + \lambda(s)) ds \right] \times (p - \lambda(u)(1 - \delta)) dt \right]$$

  – PV jumps from $PV^b_T(u)$ to $-(1 - \delta)$ at default time.
  – Hedging instruments: at the money traded CDS ($PV(u) = 0$)

  – Total amount of hedging CDS: $\frac{1 - \delta + PV^b_T(u)}{1 - \delta} \approx 1$
  – Small recovery risk.
  – Hedging credit spread risk:
    - choose amount of hedging CDS so that the sensitivities of maturity $T$ CDS and hedging CDS w.r.t to credit spreads are equal
    - Need of two hedging CDS (two constraints)
Hedging Default risk and credit spread risk

- Hedging *default risk* only constrains the *amount* of underlying standard default swap.
  - *Maturity* of underlying default swap is arbitrary.
- Choose *maturity* (of underlying CDS) to be protected against *credit spread risk*
  - PV of dynamic default swaps and standard default swaps are sensitive to the level of credit spreads
  - Sensitivity of standard default swaps to a shift in credit spreads increases with maturity
  - Choose maturity of underlying default swap in order to *equate sensitivities*.
    - All the computations are *model dependent*.
    - Previous approach involves changing the maturity of underlying through time.
Hedging Default risk and credit spread risk

- Alternative approach: choose two given maturities
- Several maturities of underlying default swaps may be used to match sensitivities.
  - For example, in the case of defaultable interest rate swap, the nominal amount of default swaps \((PV_\tau)^+\) is usually small.
  - Single default swap with nominal \((PV_\tau)^+\) has a smaller sensitivity to credit spreads than defaultable interest rate swap, even for long maturities.
  - Short and long positions in default swaps are required to hedge credit spread risk.
Hedging credit spread options

- Option to enter a given default swap with premium $p$, maturity $T'$ at exercise date $T$.
  - Call option provides positive payoff if credit spreads increase.
    - Credit spread risk
  - If default prior to $T$, cancellation of the option
    - Default risk

- The PV is of the form $PV(u) = 1_{\{\tau > u\}} PV^b(u)$
  - Hedge default risk by holding an amount of $PV^b(u)$ default swaps.
  - $PV^b(u)$ is usually small compared with payments involved in default swaps.
  - $PV^b(u)$ depends on risk-free and risky curves (mainly on credit spreads).
  - Credit spread risk is also hedged through default swaps.

- Our previous framework for hedging default risk and credit spread risk still holds.
Hedging Basket default swaps: some specificities

- Consider a basket of $M$ defaultable bonds
  - multiple counterparties
- First to default swaps
  - protection against the first default
- $N$ out of $M$ default swaps ($N < M$)
  - protection against the first $N$ defaults
- Hedging and valuation of basket default swaps
  - involves the joint (multivariate) modelling of default arrivals of issuers in the basket of bonds.
  - Modelling accurately the dependence between default times is a critical issue.
Hedging Basket default swaps: some specificities

- Hedging Default Risk in Basket Default Swaps
- Example: first to default swap from a basket of two risky bonds.
  - If the first default time occurs before maturity,
  - The seller of the first to default swap pays the non recovered fraction of the defaulted bond.
  - Prior to that, he receives a periodic premium.
- Assume that the two bonds cannot default simultaneously
  - We moreover assume that default on one bond has no effect on the credit spread of the remaining bond.
- How can the seller be protected at default time?
  - The only way to be protected at default time is to hold two default swaps with the same nominal than the nominal of the bonds.
  - The maturity of underlying default swaps does not matter.
Hedging Basket default swaps: some specificities

• Some notations:
  — $\tau_1, \tau_2$ default times of counterparties 1 and 2,
  — $\mathcal{H}_t$ available information at time $t$,
  — $P$ historical probability,
  — $\lambda_1, \lambda_2$: (historical) risk intensities:

\[
P[\tau_i \in [t, t + dt | H_t] = \lambda_i dt, \ i = 1, 2
\]

• Assumption: « Local » independence between default events
  — Probability of 1 and 2 defaulting altogether:

\[
P[\tau_1 \in [t, t + dt], \tau_2 \in [t, t + dt | H_t] = \lambda_1 dt \times \lambda_2 dt \text{ in } (dt)^2
\]
  — Local independence: simultaneous joint defaults can be neglected
Hedging *Basket default swaps*: some specificities

- **Building up a tree:**
  - Four possible states: \((D,D), (D,ND), (ND,D), (ND,ND)\)
  - Under no simultaneous defaults assumption \(p_{(D,D)}=0\)
  - Only three possible states: \((D,ND), (ND,D), (ND,ND)\)
  - Identifying (historical) tree probabilities:

\[
\begin{align*}
\lambda_1 dt & \quad (D, ND) \\
\lambda_2 dt & \quad (ND, D) \\
1 - (\lambda_1 + \lambda_2) dt & \quad (ND, ND) \\
\end{align*}
\]

\[
\begin{cases}
p_{(D,D)} = 0 \Rightarrow p_{(D,ND)} = p_{(D,D)} + p_{(D,ND)} = p_{(D,.)} = \lambda_1 dt \\
p_{(D,D)} = 0 \Rightarrow p_{(ND,D)} = p_{(D,D)} + p_{(ND,D)} = p_{(.,D)} = \lambda_2 dt \\
p_{(ND,ND)} = 1 - p_{(D,.)} - p_{(.,D)} 
\end{cases}
\]
Hedging Basket default swaps: some specificities

- Cash flows of (digital) CDS on counterparty 1:
  - $\lambda_1 \phi_1 \, dt$ CDS premium, $\phi_1$ default risk premium

\[
\begin{align*}
\lambda_1 dt & : 1 - \lambda_1 \phi_1 dt & (D, ND) \\
\lambda_2 dt & : -\lambda_1 \phi_1 dt & (ND, D) \\
1 - (\lambda_1 + \lambda_2) dt & : -\lambda_1 \phi_1 dt & (ND, ND)
\end{align*}
\]

- Cash flows of (digital) CDS on counterparty 2:

\[
\begin{align*}
\lambda_1 dt & : -\lambda_2 \phi_2 dt & (D, ND) \\
\lambda_2 dt & : 1 - \lambda_2 \phi_2 dt & (ND, D) \\
1 - (\lambda_1 + \lambda_2) dt & : -\lambda_2 \phi_2 dt & (ND, ND)
\end{align*}
\]
Hedging Basket default swaps: some specificities

- Cash flows of (digital) first to default swap (with premium $p_F$):
  \[
  \begin{align*}
  \lambda_1 dt & \quad 1 - p_F dt \quad (D, ND) \\
  \lambda_2 dt & \quad 1 - p_F dt \quad (ND, D) \\
  1 - (\lambda_1 + \lambda_2) dt & \quad -p_F dt \quad (ND, ND)
  \end{align*}
  \]

- Cash flows of holding CDS 1 + CDS 2:
  \[
  \begin{align*}
  \lambda_1 dt & \quad 1 - (\lambda_1 \phi_1 + \lambda_2 \phi_2) dt \quad (D, ND) \\
  \lambda_2 dt & \quad 1 - (\lambda_1 \phi_1 + \lambda_2 \phi_2) dt \quad (ND, D) \\
  1 - (\lambda_1 + \lambda_2) dt & \quad - (\lambda_1 \phi_1 + \lambda_2 \phi_2) dt \quad (ND, ND)
  \end{align*}
  \]

- Absence of arbitrage opportunities imply:
  - $p_F = \lambda_1 \phi_1 + \lambda_2 \phi_2$
  - Perfect hedge of first to default swap by holding 1 CDS 1 + 1 CDS 2
Hedging Basket default swaps: some specificities

- Three possible states: \((D, ND)\), \((ND, D)\), \((ND, ND)\)
- Three tradable assets: CDS1, CDS2, risk-free asset
  - The market is still « complete »
- Risk-neutral probabilities
  - Used for computing prices
  - Consistent pricing of traded instruments
  - Uniquely determined from CDS premiums
  - \(p_{(D,D)} = 0, \ p_{(D,ND)} = \lambda_1 \phi_1 dt, \ p_{(ND,D)} = \lambda_2 \phi_2 dt, \ p_{(ND,ND)} = 1 - (\lambda_1 \phi_1 + \lambda_2 \phi_2) dt\)

\[
\begin{align*}
&D, ND \\
\lambda_1 \phi_1 dt & \quad \lambda_2 \phi_2 dt \\
ND, D & \quad ND, ND \\
1 - (\lambda_1 \phi_1 + \lambda_2 \phi_2) dt
\end{align*}
\]
**Hedging Basket default swaps: some specificities**

- **hedge ratios** for first to default swaps
- Consider a first to default swap associated with a basket of two defaultable loans.
  - Hedging portfolios based on standard underlying default swaps
  - Hedge ratios if:
    - **simultaneous** default events
    - Jumps of credit spreads at default times
- **Simultaneous default events:**
  - If counterparties default **altogether**, holding the complete set of default swaps is a conservative (and thus expensive) hedge.
  - In the **extreme** case where default **always** occur altogether, we only need a single default swap on the loan with largest nominal.
  - In other cases, holding a fraction of underlying default swaps **does not hedge default risk** (if **only one** counterparty defaults).
Hedging Basket default swaps: some specificities

- hedge ratios for first to default swaps:
- What occurs if there is a jump in the credit spread of the second counterparty after default of the first?
  - default of first counterparty means bad news for the second.
- If hedging with short-term default swaps, no capital gain at default.
  - Since PV of short-term default swaps is not sensitive to credit spreads.
- This is not the case if hedging with long term default swaps.
  - If credit spreads jump, PV of long-term default swaps jumps.
- Then, the amount of hedging default swaps can be reduced.
  - This reduction is model-dependent.
• **hazard rate** based models:
  – default is a sudden, *non predictable* event,
  – that causes a sharp **jump** in defaultable bond prices.
  – Most dynamic default swaps and basket default derivatives have payoffs that are *linear* (at default) in the prices of defaultable bonds.
  – Thus, good news: **default risk and recovery risk** can be **hedged**.
  – More **realistic** approach to default.
  – **Hedge ratios** are **robust** with respect to default risk.
  – **Credit spread risk** can be hedged too, but **model risk**.
Hedging and Risk Management of Basket and Dynamic Default Swaps: conclusion

- Looking for a better understanding of credit derivatives
  - payments in case of default,
  - volatility of credit spreads.
- Bridge between risk-neutral valuation and the cost of the hedge approach.
- **Dynamic** hedging strategy based on *standard default swaps*.
  - hedge ratios in order to get protection at default time.
  - hedging default risk is *model-independent*.
  - importance of quantitative models for a better management of the P&L and the **residual risks**.