Hedging and Risk Management of Basket and Dynamic Default Swaps

Risk 2001

Jean-Paul LAURENT
Professor, ISFA Actuarial School, University of Lyon,
Scientific Advisor, BNP PARIBAS, Fixed Income Research and Strategies

Correspondence
laurent.jeanpaul@online.fr or jean-paul.laurent@univ-lyon1.fr
Web page : http://laurent.jeanpaul.free.fr/

Hedging and Risk Management of Basket and Dynamic Default Swaps: Topics to be addressed

- **Dynamic hedging strategies involving credit default swaps**

- **Joint hedging of default risk and credit spread risk**

- **Hedging variable credit exposure**

- **Hedging credit spread options**

- **Hedging basket credit derivatives**

- *Reference paper*: **"On the edge of completeness"**, RISK, October 1999.

Credit risk: the global picture

- **Portfolio approaches to credit risk.**
- **Closing the gap between supply and demand (of credit risk): from pass-through to credit derivatives**
  - ➢Securitization.
  - ➢Default Swaps .
  - ➢Dynamic Default Swaps, Basket Credit Derivatives.
  - ➢Credit Spread Options
    - – **The previous means tend to be more integrated.**
    - – **Technical innovations favour efficient risk transfer.**
- **The early stages of hedging credit derivatives.**
- **The nature of credit risk.**

- **Distribution of losses**
  - Consider some time horizon (say one year).
  - A given portfolio of credits, lines of credit, corporate bonds, interest rate swaps and forex swaps dealt with various defaultable counterparties, OTC options, tranches of CDO's.
  - Some of the credit exposures may be partially protected through collateral, credit insurance, prioritisation,…
  - The aim is to construct a <u>distribution of losses</u> arising from credit risk (and other financial risks).
  - From this distribution, one may consider different *risk indicators*, such as quantiles (VaR measures), Expected shortfall,…

Portfolio approches to credit risk

- **Topics currently addressed (portfolio approaches):**
  - **Databases on default events, credit spreads, the use of external data (credit ratings, expected default frequencies).**
  - **Assessing default risk on non quoted or small counterparties.**
  - **Integrating default risk approaches for smaller credits (mortgages, consumer loans) and**
  - **wrong way exposure, correlation between financial variables (such as interest rates, stock indices, exchange rates) and defaults.**
  - **The use of Value at Risk for credit risk : a well diversified portfolio may have a greater VaR then**
  - **What should be the good criteria for portfolio optimisation**

Portfolio approaches to credit risk

- **Topics currently addressed (portfolio approaches):**
- **Modelling of correlation**
  - **non Gaussian variables,**
  - **default times, default losses, indicators of credit events,**
- **consistency over different time horizons,**
- **joint modelling of defaults and credit spread risk.**
- **Relating portfolio approaches based on historical data and market prices on traded default risk**
  - **some inconsistencies may appear between prices of basket default swaps from the two points of view.**

Portfolio approaches to credit risk

- **From *assessment* to *management* of credit risk.**

- <u>**Credit risk profile**</u> **(Expected and unexpected loss) of portfolios can be modified. Risk/Return ratios can be <u>enhanced</u>.**

  - **By using *credit derivatives***

  - **Through securitization schemes**

  - **By *dynamic management* of <u>credit exposures</u> : we can think of reducing credit exposures when credit spreads rise.**

- **Dynamic approaches to the hedging and management of credit risk are being transposed to the financial industry.**

  - **This will eventually enhance the ability of credit derivatives desks to an efficient management of more sophisticated risks.**

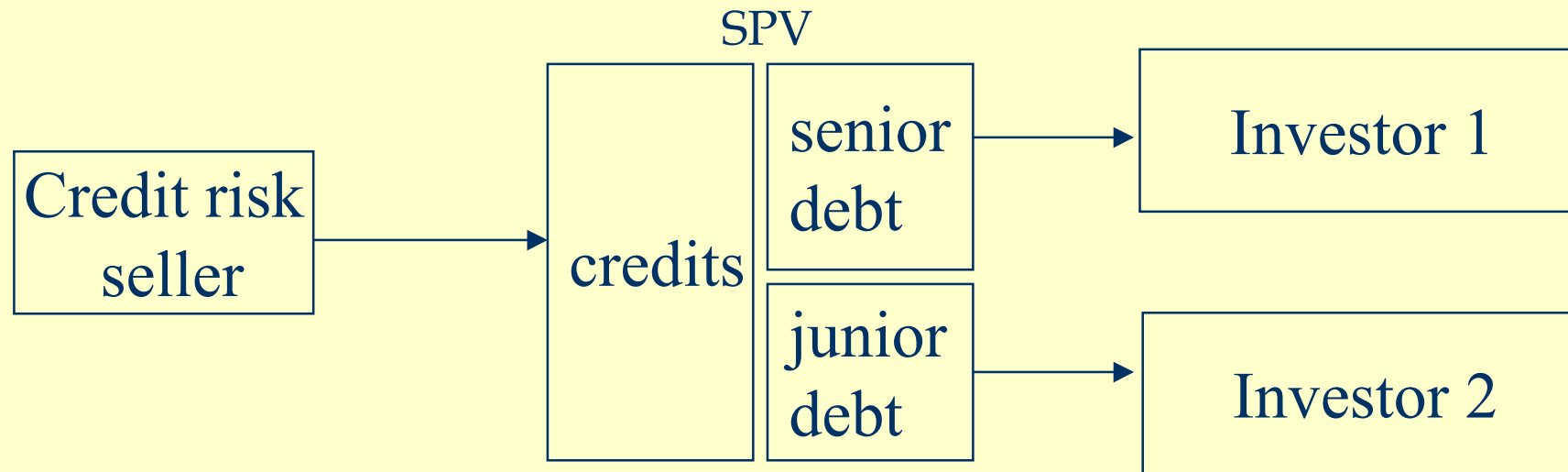## Portfolio approaches to credit risk

- **From *assessment* to *management* of credit risk.**

- **Understanding the main ideas and techniques regarding dynamic hedging of credit risk in the credit derivatives world may be useful to *credit risk managers*, *capital managers*, *CRO*,…**

  - **As end users of credit protection structures, sellers of credit risk**

  - **To better manage some dynamic aspects of credit risk management, such as variable credit exposure :**

    - risk reduction can be achieved through a dynamic use of standard products (plain CDS) and through more sophisticated derivatives (dynamic CDS, basket CDS).

Closing the gap between supply and demand

- **From pass-through to credit derivatives**
- **Securitization of credit risk:**

SPV

```
Credit risk          ┌────────┬────────┐         ┌──────────────┐
seller     ────────▶ │        │ senior │ ──────▶ │ Investor 1   │
                     │        │ debt   │         └──────────────┘
                     │ credits├────────┤
                     │        │ junior │ ──────▶ ┌──────────────┐
                     │        │ debt   │         │ Investor 2   │
                     └────────┴────────┘         └──────────────┘
```

- – **Credit risk seller can transfer loans to SPV or instead use default swaps**
- – **All credit trades are <u>simultaneous</u>.**
- – **<u>No</u> residual risk remains within SPV.**

- **Securitization of credit risk**
  - **Financial intermediaries provide structuring and arrangement advice.**
    - ➢Low capital at risk for financial intermediaries
  - **credit models are used to assess credit risks**
    - ➢No need of <u>hedging</u> models
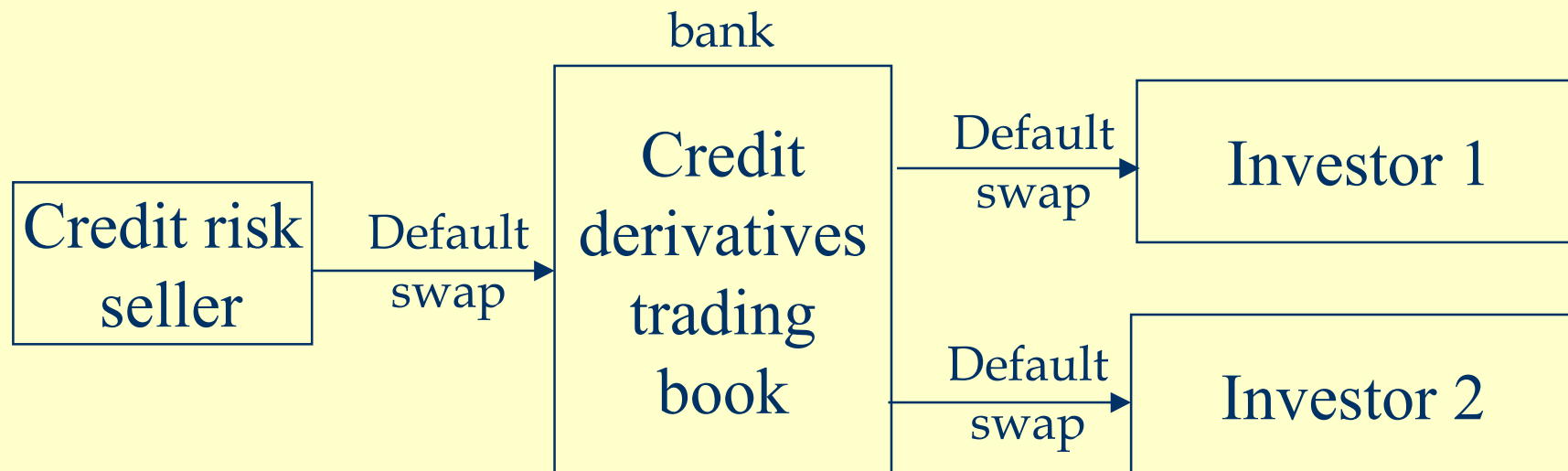
Closing the gap between supply and demand

- **There is room for financial intermediation of credit risk**
  - **The transfers of credit risk between commercial banks and investors may not be <u>simultaneous</u>.**
  - **Since at one point in time, demand and offer of credit risk may not match.**
    - ➤ <u>Meanwhile</u>, credit risk remains within the balance sheet of the financial intermediary.
  - **It is not further required to find customers with exact opposite interest at every new deal.**
    - ➤ <u>Residual risks</u> remain within the balance sheet of the financial intermediary.
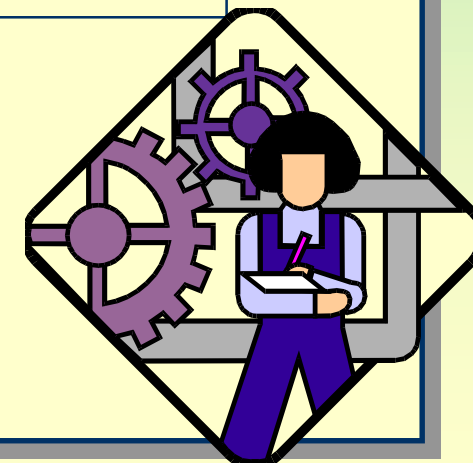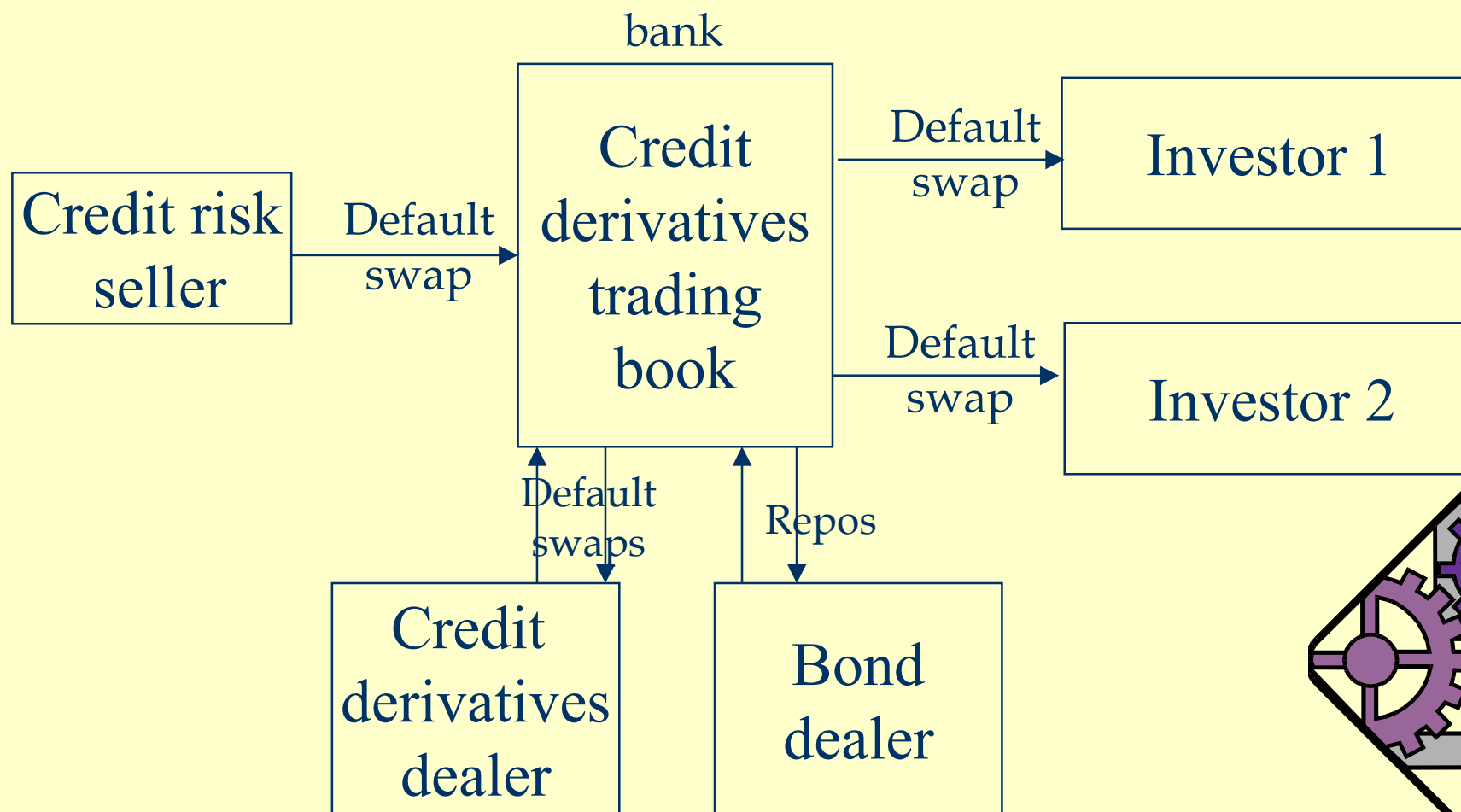
Closing the gap between supply and demand

- **Emphasis on:**
  - portfolio effects: correlation between default events
  - posting collateral
  - computation of capital at risk, risk assessment

- **Main issues:**
  - capital at risk can be high
 - what is the competitive advantage of investment banks

bank

```
                                              ┌──────────────────┐
                    Credit         Default    │   Investor 1     │
┌──────────────┐   derivatives ──── swap ───→ └──────────────────┘
│ Credit risk  │── Default ──→ trading
│   seller     │    swap        book          ┌──────────────────┐
└──────────────┘                ──── Default ─→│   Investor 2     │
                                      swap     └──────────────────┘
```

- *Hedging* **credit risk enhances ability to transfer credit risk by** <u>*lowering capital at risk*</u>

bank

```
                    ┌──────────────┐   Default   ┌──────────────┐
                    │    Credit    │─── swap ───▶│  Investor 1  │
┌────────────┐ Default│ derivatives │            └──────────────┘
│ Credit risk│─ swap─▶│   trading   │
│   seller   │      │     book     │   Default   ┌──────────────┐
└────────────┘      │              │─── swap ───▶│  Investor 2  │
                    └──────────────┘             └──────────────┘
                      Default      Repos
                       swaps
                  ┌───────────┐  ┌───────────┐
                  │  Credit   │  │   Bond    │
                  │derivatives│  │  dealer   │
                  │  dealer   │  │           │
                  └───────────┘  └───────────┘
```

Closing the gap between supply and demand
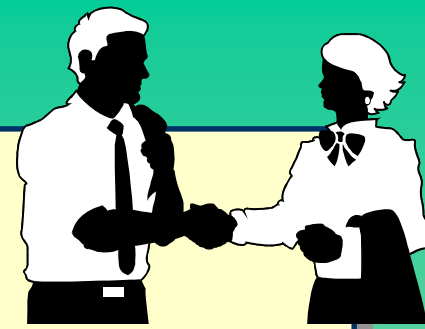
- **Anatomy of a general dynamic default swap**
  - A dynamic default swap is like a standard default swap but with variable nominal (or exposure)
  - However the periodic premium paid for the credit protection remains fixed.
  - The protection payment arises at default of one given single risky counterparty.
- **Examples**
  - ➤ **cancellable swaps**
  - ➤ quanto default swaps
  - ➤ credit protection of vulnerable swaps, OTC options.
  - ➤ credit protection of a portfolio of contracts (full protection, excess of loss insurance, partial collateralization)
- **Efficient ways to transfer credit risk**

- **Example: defaultable interest rate swap**
- **Consider a defaultable interest rate swap (with unit nominal)**
  - We are <u>default-free</u>, our counterparty is <u>defaultable</u>.
  - We consider a (fixed-rate) *receiver* swap on a <u>standalone</u> basis.
- **Recovery assumption, payments in case of default:**
  - if default at time τ, compute the <u>default-free</u> value of the swap: PV_τ
  - and get:

  $$\delta(PV_\tau)^+ + (PV_\tau)^- = PV_\tau - (1-\delta)(PV_\tau)^+$$

  - $0 \leq \delta \leq 1$ recovery rate, $(PV_\tau)^+ = \text{Max}(PV_\tau, 0)$, $(PV_\tau)^- = \text{Min}(PV_\tau, 0)$

  - *In case of default,*
    - ➢ we <u>receive</u> default-free value PV_τ
    - ➢ *minus*
    - ➢ <u>loss</u> equal to $(1-\delta)(PV_\tau)^+$.

Closing the gap between supply and demand

- **Example: defaultable interest rate swap**

- **Using a hedging instrument rather than a credit reserve**

  - Consider a <u>dynamic default swap</u> paying $(1-\delta)(PV_\tau)^+$ at default time τ (if $\tau \le T$), where PV_τ is the present value of a default-free swap with *same fixed rate than defaultable swap.*

 - At default, we receive $(1-\delta)(PV_\tau)^+ + PV_\tau - (1-\delta)(PV_\tau)^+ = PV_\tau$

The nature of credit risk

- **Pricing at the cost of the hedge:**
  - If some risk can be hedged, its price should be the cost of the hedge.
  - Think of a plain vanilla stock index call. Its replication price is 10% (say).
  - One given investor is ready to pay for 11% (He feels better of with such an option, then doing nothing). Should he really give this 1% to the market ?

- **The feasibility of hedging (« *completeness* ») is a fundamental idea.**
  - If credit instruments can be <u>hedged</u>, pricing dynamic default swaps, basket default swaps based only historical data and portfolio approaches will eventually lead to arbitrage opportunities.
  - Good news for knowledgeable individuals. Bad news for the understanding of risks.
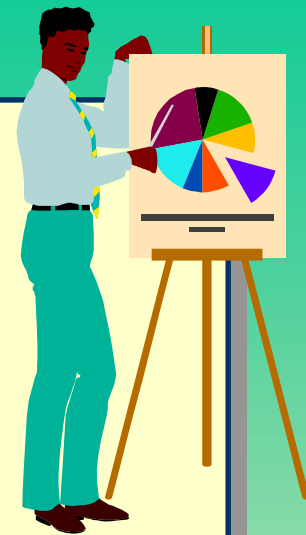
- **Misconceptions about *credit risk*.**
  - At the early stage of credit risk analysis, a common idea was that credit risk was not hedgeable :
    - ➢ incomplete markets, multiplicity of risk-neutral measures.
    - ➢ In firm-value models and complete information, default bonds are (too simplistically) considered as equity barrier options.
- **When some a *defaultable bond* is already <u>traded</u>, then the market can become *complete*.**
  - If there is also credit spread risk (that is not fully correlated to other financial variables), then we need (at least) <u>two</u> defaultable bonds.

The early stages of hedging credit derivatives

- **Static arbitrage of plain default swaps with *short selling* underlying defaultable bond**
  - CDS premiums should be related to credit spreads on floaters.
- **One step further: hedging non standard maturities assuming smooth credit curves.**
  - "bond strippers" : allow to compute prices of risky zero-coupon bonds.
- **Consider a six years maturity CDS hedged with a five years maturity CDS of same nominal.**
  - Protection at default time.
  - Since maturities of credit derivatives do not perfectly match, credit spread risk.
  - We may need several maturities of CDS + some assumptions on the dynamics of CDS premiums in order to hedge credit spread risk.

The early stages of hedging credit derivatives
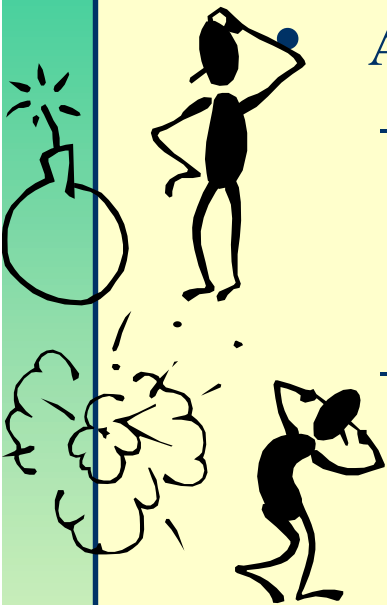
- **Assessing the varieties of risks involved in credit derivatives**
  - **Specific risk or credit spread risk**
    - ➢ *prior to default*, the P&L of a book of credit derivatives is driven by changes in credit spreads.
  - **Default risk**
    - ➢ *in case of default*, <u>if unhedged,</u>
    - ➢ dramatic jumps in the P&L of a book of credit derivatives.
- **Real world issues with hedging plain CDS :**
  - **deliverance of some unknown underlying, possibly short-term or fixed rate long term bond,**
  - **Management of short-selling and repo margins on illiquid bonds,**
  - **« small inconsistencies » due to accrued coupons, accrued premiums.**
  - **illiquid hedging default swaps**

Hedging credit derivatives: overview

- **Hedging default (and recovery) risk : an introduction.**
  - **Short term default swaps v.s. long-term default swaps**
  - **Credit spread <u>transformation risk</u>**
- **One step further: hedging *Dynamic Default Swaps*, *credit spread options*.**
  - **Hedging default risk through <u>dynamics holdings</u> in standard default swaps.**
  - **Hedging credit spread risk by choosing appropriate default swap maturities.**
- **Hedging : *Basket Default Swaps* some specificities**
  - **Uncertainty at default time**

Hedging default risk: an introduction
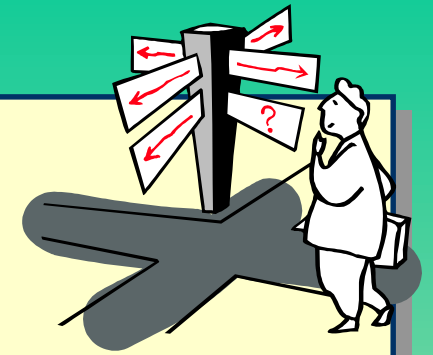
- **Disentangling risks in *credit instruments***
  - *Interest rate risk*: due to movements in default-free interest rates.
  - *Default risk*: default bond price jumps to recovery value at default time.
  - *Credit spread risk* (specific risk): variation in defaultable bond prices prior to default, due to changes in credit quality (for instance ratings migration) or changes in risk premiums.
  - *Recovery risk*: unknown recovery rate in case of default.
- **Hedging** exotic credit derivatives will imply hedging all sources of risk.
- A *new approach* to credit derivatives modelling based on an **hedging** point of view

Hedging default risk: an introduction

- **Purpose:**
  - **Introduction to dynamic trading of default swaps**
  - **Illustrates how default and credit spread risk arise**

- **Arbitrage between long and short term default swaps**
  - **sell one long-term default swap**
  - **buy a series of short-term default swaps**

- **Example:**
  - **default swaps on a FRN issued by BBB counterparty**
  - **5 years default swap premium : 50bp, recovery rate = 60%**

| Credit derivatives dealer	If default, 60% →	Client
	← Until default, 50 bp	
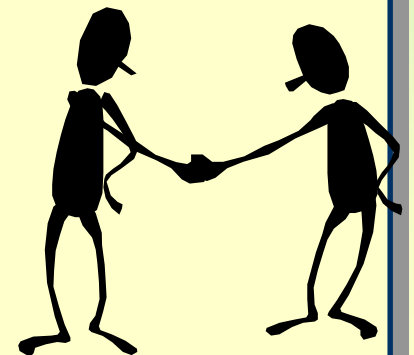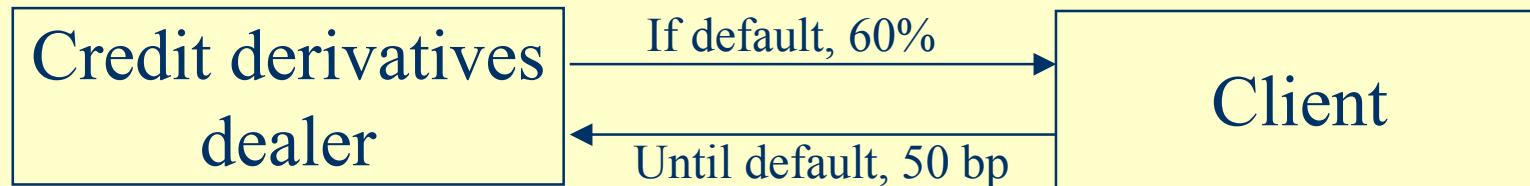
Hedging default risk: an introduction

- **Rolling over short-term default swap**
  - at inception, one year default swap premium : 33bp
  - cash-flows after one year:

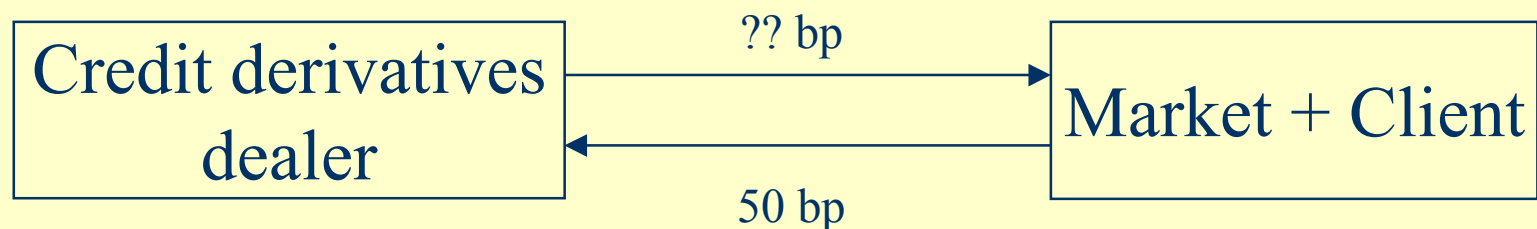| Credit derivatives dealer	→ 33 bp →	Market
	← 60% if default ←	
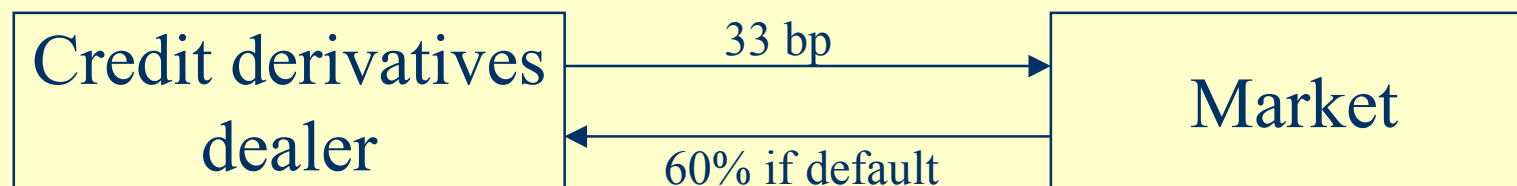
- **Buy a one year default swap at the end of every yearly period, if no default:**
  - Dynamic strategy,
  - <u>future</u> premiums depend on <u>future</u> credit quality
 - future premiums are <u>unknown</u>

Credit derivatives dealer	→ ?? bp →	Market
	← 60% if default ←	
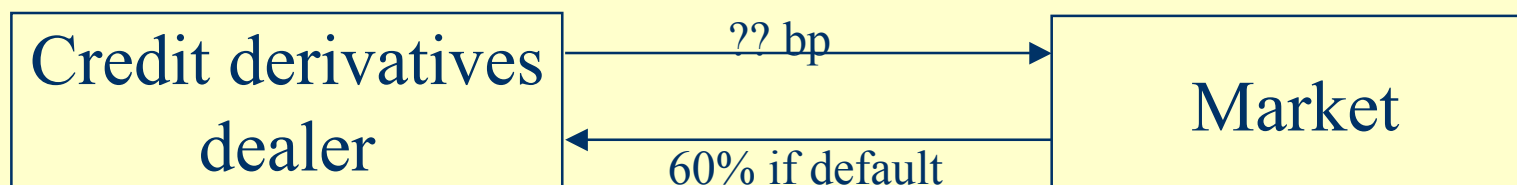
Hedging default risk: an introduction

- *Risk analysis* **of rolling over short term against long term default swaps**

```
┌──────────────────────┐        ?? bp         ┌──────────────────────┐
│  Credit derivatives  │ ──────────────────►  │    Market + Client   │
│       dealer         │ ◄──────────────────  │                      │
└──────────────────────┘        50 bp         └──────────────────────┘
```

- **Exchanged cash-flows :**
  - **Dealer receives 5 years (fixed) credit spread,**
  - **Dealer pays 1 year (variable) credit spread.**
- **Full one to one protection at default time**
  - **the previous strategy has <u>eliminated</u> one source of risk, that is <u>default risk</u>**
  - **<u>Recovery risk</u> has been eliminated too.**

Hedging default risk: an introduction

- **Negative exposure to an <u>increase</u> in <u>short-term</u> default swap premiums**
  - if short-term premiums increase from 33bp to 70bp
  - reflecting a lower (short-term) credit quality
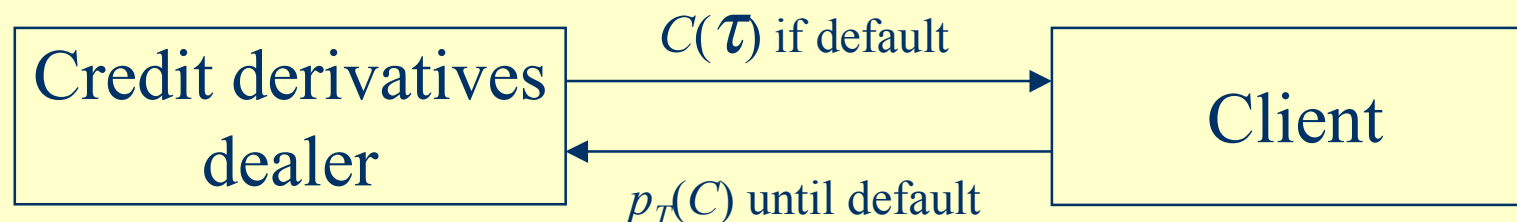 - <u>and</u> no default occurs before the fifth year

```
Credit derivatives     --- 70 bp --->     Market + Client
dealer                 <-- 50 bp ---
```
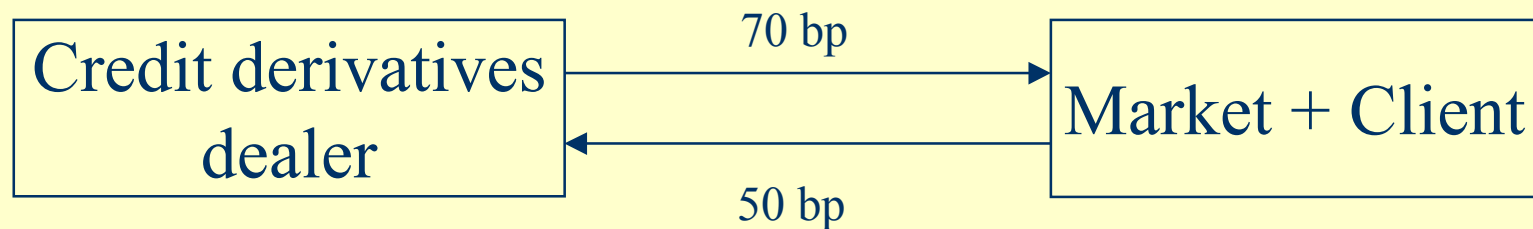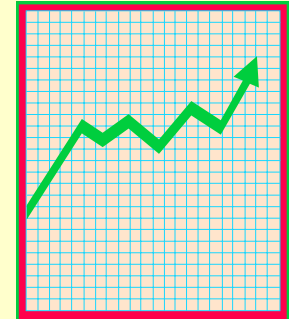
- **Loss due to negative carry**
  - long position in long term credit spreads
  - short position in short term credit spreads

Hedging default risk: an introduction

- ***Dynamic Default Swap***
  - client pays to dealer a periodic premium $p_T(C)$ until default time $\tau,$ or maturity of the contract $T.$
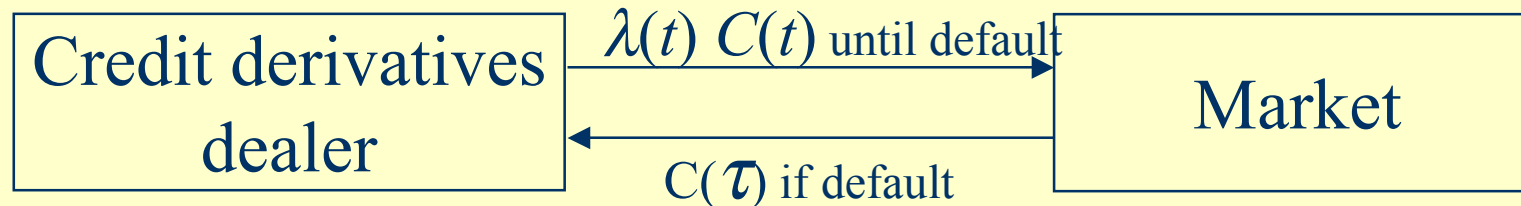 - dealer pays $C(\tau)$ to client at default time $\tau,$ if $\tau \leq T.$

$$\boxed{\text{Credit derivatives dealer}} \xrightarrow{\quad C(\tau) \text{ if default} \quad} \boxed{\text{Client}}$$
$$\xleftarrow{\quad p_T(C) \text{ until default} \quad}$$

- **Hedging side:**
  - **Dynamic** strategy based on **standard** default swaps:
  - At time t, hold an amount $C(t)$ of standard default swaps
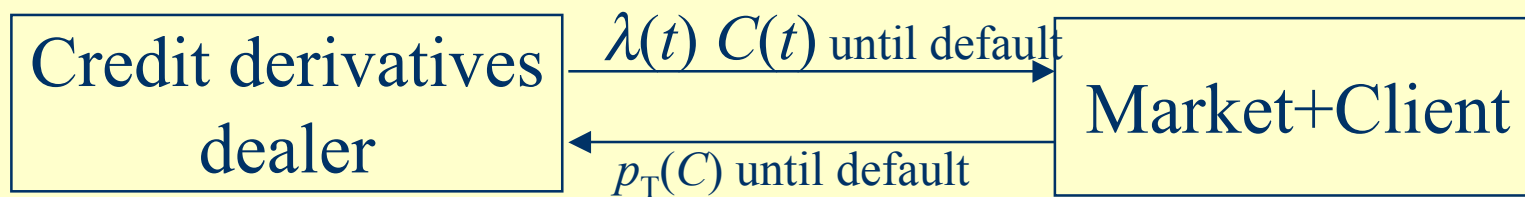 - $\lambda(t)$ denotes the periodic premium at time t for a short-term default swap

Hedging default risk: an introduction

- **Hedging side:**

| Credit derivatives dealer | $\lambda(t)\, C(t)$ until default $\longrightarrow$
 $\longleftarrow$ $C(\tau)$ if default | Market |

  - **Amount of standard default swaps equals the (variable) <u>credit exposure</u> on the dynamic default swap.**

- **Net position is a "*basis swap*":**

| Credit derivatives dealer | $\lambda(t)\, C(t)$ until default $\longrightarrow$
 $\longleftarrow$ $p_T(C)$ until default | Market+Client |

- **The client transfers credit spread risk to the credit derivatives dealer**

One step further: Hedging dynamic default swaps

- **Hedging *credit risk***
  - Uniqueness of *equivalent martingale measure.*

- **PV of plain and dynamic default swaps**

- **Hedging dynamic default swaps**
  - Hedging *default risk*
  - Explaining *theta* effects
  - Hedging default risk *and* credit spread risk

- **Hedging Credit spread options**

hedging credit risk

- **"firm-value" models :**
  - Modelling of firm's assets
  - First time passage <u>below</u> a critical threshold

- **risk-intensity based models**
  - Default arrivals are no longer <u>predictable</u>
  - Model conditional local probabilities of default $\lambda(t)\ dt$
  - τ : default date, $\lambda(t)$ *risk intensity* or *hazard rate*

$$\lambda(t)dt = P\big[\tau \in [t, t + dt[\,|\,\tau > t\big]$$

- **We need a <u>hedging based approach</u> to pricing.**
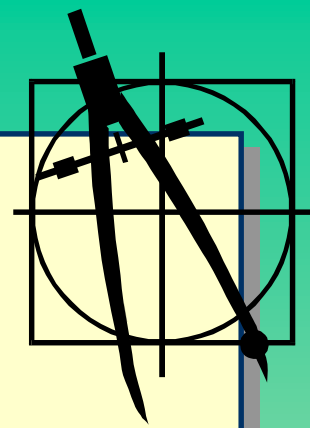
- **Uniqueness** of *equivalent martingale measure*
  - **Assume deterministic default-free interest rates**
  - $r(t)$ **default-free short rate,** τ, **default time**
  - $I(t)=1_{\{\tau>t\}}$ **indicator function.** $I(t)$ **jumps from 1 to 0 at time** τ.

  - $H_t = \sigma(I(s), s \leq t)$: **natural filtration of** τ.
  - $P(\tau \in [t,t+dt[\mid H_t) = E(I(t)-I(t+dt) \mid H_t) = \lambda(t)I(t)dt,$
  - λ **(historical)** *default intensity (*wrt H_t**):**

    > *Girsanov theorem*: Under any equivalent probability Q, the risk-intensity of τ becomes $\lambda(t)\phi(t)$ with $\phi(t)>0$.
  - $Q(\tau \in [t,t+dt[\mid H_t) = E^Q(I(t)-I(t+dt) \mid H_t) = \lambda(t)\phi(t)I(t)dt,$
- **Risky discount bond with maturity** T **: pays** $1_{\{\tau>t\}}$ **at time** T

Hedging credit risk
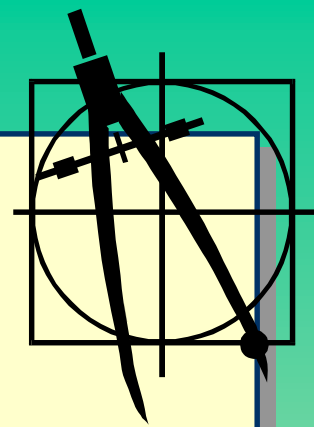
- $\bar{B}(t,T) \in H_t$ **t-time price of risky discount bond.**

- **Lemma: Let $Z \in H_t$. Then Z is constant on $\{\tau > t\}$.**

  - **Proof: $\{\tau > t\}$ is an atom of H_t. Every random variable is constant on atoms**

- **$\bar{B}(t,T)$ is constant on $\{\tau > t\}$ and $\bar{B}(t,T) = 0$ on $\{\tau \leq t\}$.**

- $\Rightarrow \bar{B}(t,T) = c(t,T)1_{\{\tau > t\}} = c(t,T)I(t)$ **where $c(t,T)$ is deterministic.**

  - **Then,** $d\bar{B}(t,T) = c_t(t,T)I(t)dt + c(t,T)dI(t)$

- **Let Q be an *equivalent martingale measure*.**

  - **Then** $E^Q\left[d\bar{B}(t,T)\big|H_t\right] \overset{(2)}{=} I(t) \times \left(c_t(t,T) - c(t,T)\lambda(t)\phi(t)\right)dt$

- **On the other hand,** $E^Q\left[d\bar{B}(t,T)\big|H_t\right] \overset{(1)}{=} r(t)\bar{B}(t,T)dt = r(t)c(t,T)I(t)dt$

$$(1)+(2) \Rightarrow \boxed{\lambda(t)\phi(t) \overset{(3)}{=} -r(t) + d\ln c(t,T)/dt}, \text{ on } \{\tau > t\}$$
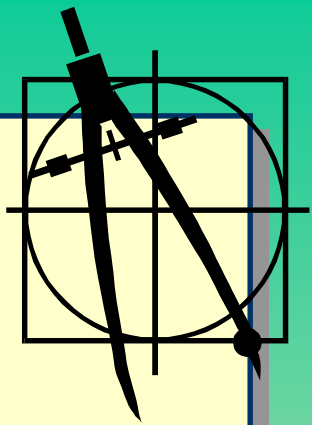
- **Thus $\phi(t)$ and then Q are identified (uniqueness) from $\bar{B}(t,T)$**

  - **Let us denote:** $\bar{r}(t) = r(t) + \lambda(t)\phi(t)$

  - **Thus:** $d\ln c(t,T)/dt = \bar{r}(t)dt$ **with $c(T,T)=1$**

  - **Which provides:** $c(t,T) = \exp-\int_t^T \bar{r}(s)ds$ **(predefault price).**

- ***Summary* of results:**

  - $\bar{r}(t)$ **defaultable short rate,** $\boxed{\lambda(t)\phi(t) = \bar{r}(t) - r(t)}$ **risk-neutral intensity**

  - **Defaultable bond:** $\bar{B}(t,T) = 1_{\{\tau > t\}}\exp-\int_t^T \big(r(s) + \lambda(s)\phi(s)\big)ds$

  - **risk-neutral measure Q, with intensity of default $\lambda(t)\phi(t)$: *pricing*.**

  - **Historical measure P, with intensity of default $\lambda(t)$: *portfolio approaches***

- **Risky discount bond price (no recovery):**

$$\overline{B}(t,T) = E_t\left[1_{\{\tau>T\}}\exp-\int_t^T r(s)ds\right] = 1_{\{\tau>t\}}E_t\left[\exp-\int_t^T (r+\lambda)(s)ds\right]$$
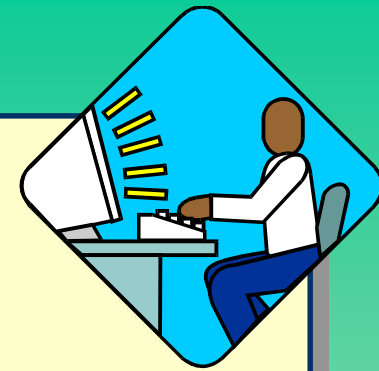
  - λ: **risk-neutral intensity**

- **More generally let X_T be a payoff paid at T, if $\tau>T$:**

$$PV_X(t) = E_t\left[X_T 1_{\{\tau>T\}}\exp-\int_t^T r(s)ds\right] = 1_{\{\tau>t\}}E_t\left[X_T \exp-\int_t^T (r+\lambda)(s)ds\right]$$

- $\exp-\int_t^T (r+\lambda)(s)ds$ *stochastic risky discount factor*

- **Time u -PV of a plain default swap:**
  - **Maturity T, *continuously* paid premium p, recovery rate δ**
  - **Risk-free short rate r, default intensity λ**
  - *E_u expectation conditional on information carried by financial prices.*
  - **$r + \lambda$ is the « risky » short rate : payoffs discounted at a higher rate**
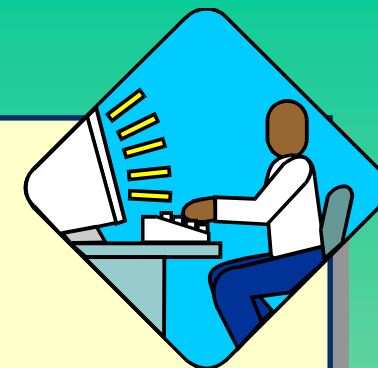 - **Similar to an index amortizing swap (payments only if no prepayment).**

- **PV of *default payment leg*:**

$$1_{\{\tau>u\}} E_u \left[\int_u^T \left(\exp - \int_u^t (r+\lambda)(s)\,ds \right) \times (1-\delta)\lambda(t)\,dt \right] + 1_{\{\tau \le u\}} (1-\delta) \exp \int_\tau^u r(s)\,ds$$

- **PV of *premium payment leg*:**

$$1_{\{\tau>u\}} p \times E_u \left[\int_u^T \left(\exp - \int_u^t (r+\lambda)(s)\,ds \right) dt \right]$$

- **Time u -PV of a dynamic default swap**
  - Payment $C(\tau)$ at default time if $\tau < T$:

- **PV of *default payment leg***

$$1_{\{\tau>u\}} E_u\left[\int_u^T\left(\exp-\int_u^t (r+\lambda)(s)ds\right)C(t)\lambda(t)ds\right]+1_{\{\tau\geq u\}}C(\tau)\exp\int_\tau^u r(s)ds$$

  - This embeds the plain default swap case where $C(\tau)=1-\delta$

- **PV of *premium payment leg***

$$1_{\{\tau>u\}} p\times E_u\left[\int_u^T\left(\exp-\int_u^t (r+\lambda)(s)ds\right)dt\right]$$

  - Same as in the case of plain default swap

Hedging dynamic default swaps

- **Exotic credit derivatives can be *hedged* against <u>default</u>:**
  - Constrains the <u>amount</u> of underlying <u>standard</u> default swaps.
  - <u>Variable</u> amount of standard default swaps.
  - <u>Full protection</u> at default time by construction of the hedge.
  - No more *<u>discontinuity</u>* in the P&L at default time.
  - <u>Model-free</u> <u>approach.</u>
- **<u>Credit spread exposure</u> has to be hedged by *other means*:**
  - Appropriate *choice of maturity* of underlying default swap
  - Use of CDS with different maturities.
  - Computation of sensitivities with respect to changes in credit spreads are <u>model</u> <u>dependent.</u>

Hedging dynamic default swaps

- **PV at time *u* of a digital default swap**

$$PV(u) = 1_{\{\tau>u\}} E_u \left[\int_u^T \left(\exp - \int_u^t (r+\lambda)(s)\,ds \right) \times (\lambda(t)-p)\,dt \right] + 1_{\{\tau\leq u\}} \exp \int_\tau^u r(t)\,dt$$

  - At default time τ, PV switches from

$$E_u \left[\int_u^T \left(\exp - \int_u^t (r+\lambda)(s)\,ds \right) \times (\lambda(t)-p)\,dt \right]$$

  - to one (default payment). If digital default swap at the money, d*PV*(τ)=1

- **PV at time *u* of a dynamic default swap with payment *C*: *PV$_C$*(*u*)**

$$1_{\{\tau>u\}} E_u \left[\int_u^T \left(\exp - \int_u^t (r+\lambda)(s)\,ds \right) \times (\lambda(t)C(t)-p_C)\,dt \right] + 1_{\{\tau\leq u\}} C(\tau) \exp \int_\tau^u r(t)\,dt$$

  - At default time τ, PV switches from pre-default market value PV(τ-) to $C(\tau)$
- **To hedge default risk, we hold (*C*(*u*)-*PV$_C$*(*u*)) digital default swaps**
  - Variation of PV at default time on the hedging portfolio:

$$\left(C(\tau) - PV_C(\tau^-) \right) dPV(\tau) = C(\tau) - PV_C(\tau^-)$$

- **Hedging default risk is <u>model free</u>. No *recovery risk*.**

Explaining theta effects in the P&L dynamics

- ***Different aspects*** of "**carrying**" credit contracts through time.
  - Analyse the risk-neutral dynamics of the P&L.
- Consider a *short* position in a dynamic default swap.
- *Pre-default* Present value of the contract provided by:

$$PV(u) = E_u\left[\int_u^T\left(\exp-\int_u^t(r+\lambda)(s)ds\right)\times\left(p_T-\lambda(t)C(t)\right)dt\right]$$

- *Net **expected** capital gain* (conditional on no default):

$$E_u\left[PV(u+du)-PV(u)\right] = \left(r(u)+\lambda(u)\right)PV(u)du + \left(\lambda(u)C(u)-p_T\right)du$$

- *Accrued cash-flows* (received premiums): $p_T du$
  - By summation, Incremental P&L (if no default between *u* and *u+du*):

$$r(u)PV(u)du + \lambda(u)\left(C(u)+PV(u)\right)du$$

Explaining theta effects in the P&L dynamics

- **<u>Apparent</u> *extra return* effect :** $\lambda(u)\big(C(u) + PV(u)\big)du$
  - **But, probability of default between *u* and *u+du*: $\lambda(u)du$.**
  - **Losses in case of default:**
    - ➤ Commitment to pay: $C(u)$ + loss of PV of the credit contract: PV(u)
    - ➤ PV(u) consists in **<u>unrealised</u>** capital gains or losses in the credit derivatives book that "disappear" in case of default.
  - **Expected loss charge:** $\lambda(u)\big(C(u) + PV(u)\big)du$
- **Under risk-neutral probability, *in average* P&L <u>does</u> increase at rate *r(u)*!**
- **Hedging aspects:**
  - **If we hold $C(u) + PV(u)$ short-term digital default swaps, we are protected at default-time (no jump in the P&L).**
  - **Premiums to be paid:** $\lambda(u)\big(C(u) + PV(u)\big)du$
  - **The hedged P&L increases at rate *r(u)* (mimics savings account).**

Hedging default risk *and* credit spread risk

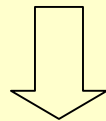- **Denote by $I(u) = 1_{\{\tau > u\}}$, $dI(u)$ = variation of jump part.**

- **Digital default swap:**
  - **PV prior to default:** $PV^b(u) = E_u\left[\int_u^T\left(\exp-\int_u^t(r+\lambda)(s)ds\right)\times(\lambda(u)-p)dt\right]$
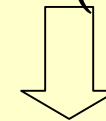
  - **PV after default:** $PV^a(u) = \exp\int_\tau^u r(t)dt$

  - **PV whenever:** $PV(u) = I(u)PV^b(u) + (1-I(u))PV^a(u)$

$$dPV(u) = \left(PV^b(u) - PV^a(u)\right)dI(u) + I(u)dPV^b(u) + (1-I(u))dPV^a(u)$$

Discontinuous part
default risk

Continuous part (credit spread risk)

- **Discontinuous part : constrains the amount of hedging default swaps**
  - *After hedging default risk, __no jump__ in the PV at default time.*

- **Hedging continuous part (see below)**

Hedging Default risk _and_ credit spread risk

- **Hedging continuous part**
  - **Assume some state variable following diffusion processes (i.e. no jumps in credit spreads).**
  - **Pre-default PV of dynamic default swaps, plain CDS:**

$$E_u\left[\int_u^T\left(\exp-\int_u^t (r+\lambda)(s)ds\right)\times\left(\lambda(t)C(t)-p_C\right)dt\right]$$

  - **Provided as a solution of linear PDE.**

- **_Credit spread risk_ («continuous » part) is hedged by delta analysis:**
  - **Compute the sensitivities of dynamic default swap to be hedged and of hedging CDS w.r.t state variables.**
  - **Choose amount of hedging CDS so that portfolio sensitivity =0.**

Hedging Default risk _and_ credit spread risk

- **Example: hedging CDS with non standard maturities.**
  - **Maturity T, premium p, pre-default PV:**

$$PV_T^b(u) = E_u\left[\int_u^T\left(\exp-\int_u^t(r+\lambda)(s)ds\right)\times\left(p-\lambda(u)(1-\delta)\right)dt\right]$$

  - **PV jumps from $PV_T^b(u)$ to $-(1-\delta)$ at default time.**
  - **Hedging instruments: at the money traded CDS (PV(u)=0)**

  - **Total amount of hedging CDS:** $\dfrac{1-\delta+PV_T^b(u)}{1-\delta}\approx 1$
  - _Small recovery risk._
  - **Hedging credit spread risk:**
    - ➢ choose amount of hedging CDS so that the sensitivies of maturity T CDS and hedging CDS w.r.t to credit spreads are equal
    - ➢ Need of two hedging CDS (two constraints)

Hedging Default risk <u>and</u> credit spread risk

- **Hedging *default risk* only constrains the <u>amount</u> of underlying standard default swap.**
  - **<u>Maturity</u> of underlying default swap is arbitrary.**
- **Choose <u>maturity</u> (of underlying CDS) to be protected against credit spread risk**
  - **PV of dynamic default swaps and standard default swaps are sensitive to the level of credit spreads**
  - **Sensitivity of standard default swaps to a shift in credit spreads increases with maturity**
  - **Choose maturity of underlying default swap in order to <u>equate sensitivities</u>.**
    - All the <u>computations</u> are *model dependent*.
    - Previous approach involves changing the maturity of underlying through time.

Hedging Default risk <u>and</u> credit spread risk

- **Alternative approach: choose two given maturities**
- *Several maturities* **of underlying default swaps may be used to match sensitivities.**

  - For example, in the case of **defaultable** interest rate swap, the nominal amount of default swaps $(PV_\tau)^+$ is usually small.

  - *Single* default swap with nominal $(PV_\tau)^+$ has a *smaller sensitivity* to <u>credit</u> <u>spreads</u> than *defaultable interest rate swap*, even for long maturities.

  - <u>Short</u> and <u>long</u> positions in default swaps are required to hedge *credit spread risk*.
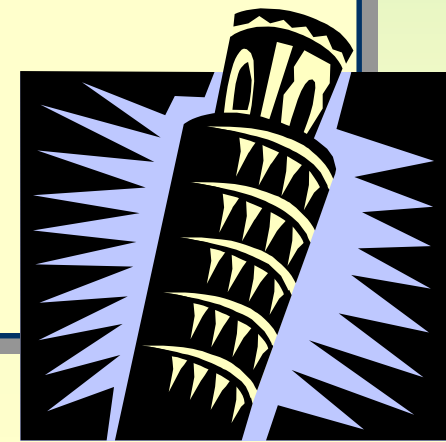
Hedging credit spread options

- **Option to enter a given default swap with premium _p_, maturity _T'_ at exercise date _T_.**
  - **Call option provides positive payoff if credit spreads increase.**
    - ➤ Credit spread risk
  - **If default prior to _T_, cancellation of the option**
    - ➤ Default risk

- **The PV is of the form** $PV(u) = 1_{\{\tau > u\}} PV^b(u)$
  - **Hedge default risk by holding an amount of _PV^b(u)_ default swaps.**
  - **_PV^b(u)_ is usually small compared with payments involved in default swaps.**
  - **_PV^b(u)_ depends on risk-free and risky curves (mainly on credit spreads).**
  - **Credit spread risk is also hedged through default swaps.**

- **Our previous framework for hedging default risk and credit spread risk still holds.**

Hedging __Basket default swaps__: some specificities

- **Consider a basket of *M* defaultable bonds**
  - __multiple__ **counterparties**

- **First to default swaps**
  - **protection against the first default**

- ***N* out of *M* default swaps (*N* < *M*)**
  - **protection against the first *N* defaults**

- **Hedging and valuation of basket default swaps**
  - **involves the joint (__multivariate__) modelling of default arrivals of issuers in the basket of bonds.**
  - **Modelling accurately the __dependence__ between default times is a critical issue.**

Hedging *Basket default swaps*: *some specificities*

- **Hedging Default Risk in Basket Default Swaps**
- **Example: first to default swap from a basket of two risky bonds.**
  - If the first default time occurs before maturity,
  - The seller of the first to default swap pays the non recovered fraction of the defaulted bond.
  - Prior to that, he receives a periodic premium.
- **Assume that the two bonds cannot default simultaneously**
  - We moreover assume that default on one bond has *no effect* on the credit spread of the remaining bond.
- **How can the seller be protected *at default time* ?**
  - The only way to be protected at default time is to hold two default swaps with the *same nominal* than the *nominal* of the bonds.
  - The *maturity* of underlying default swaps does not matter.

Hedging Basket default swaps: some specificities

- *hedge ratios* for first to default swaps
- **Consider a first to default swap associated with a basket of two defaultable loans.**
  - **Hedging portfolios based on standard underlying default swaps**
  - **Uncertain hedge ratios if:**
    - ➤ simultaneous default events
    - ➤ *Jumps* of credit spreads at default times
- **Simultaneous default events:**
  - **If counterparties default *altogether*, holding the *complete* set of default swaps is a conservative (and thus expensive) hedge.**
  - **In the *extreme* case where default *always* occur altogether, we only need a single default swap on the loan with largest nominal.**
  - **In other cases, holding a *fraction* of underlying default swaps does not hedge default risk (if *only one* counterparty defaults).**

Hedging *Basket default swaps*: *some specificities*

- **hedge ratios for first to default swaps**

- What occurs if there is a *jump in the credit spread* of the second counterparty after <u>default</u> of the first ?

  - **default of first counterparty means *bad news* for the second.**

- If hedging with short-term default swaps, <u>no capital gain</u> at default.

  - **Since PV of short-term default swaps is not *sensitive* to credit spreads.**

- **This is not the case if hedging with long term default swaps.**

  - **If credit spreads <u>jump</u>, PV of long-term default swaps <u>jumps</u>.**

- **Then, the amount of hedging default swaps can be <u>reduced</u>.**

  - **This reduction is *model-dependent*.**

Hedging and Risk Management of Basket and Dynamic Default Swaps: conclusion

- **hazard rate based models :**
  - default is a sudden, *non predictable* event,
  - that causes a sharp jump in defaultable bond prices.
  - Most dynamic default swaps and basket default derivatives have payoffs that are *linear* (at default) in the prices of defaultable bonds.
  - Thus, good news: **default risk** and **recovery risk** can be *hedged*.
  - More realistic approach to default.
  - *Hedge ratios* are robust with respect to default risk.
  - **Credit spread risk** can be hedged too, but model risk.

Hedging and Risk Management of Basket and Dynamic Default Swaps: conclusion

- **Looking for a better understanding of credit derivatives**
  - payments in case of default,
  - volatility of credit spreads.

- **Bridge between risk-neutral valuation and the cost of the hedge approach.**

- <u>**dynamic**</u> **hedging strategy based on** *standard default swaps.*
  - hedge ratios in order to get protection at default time.
  - hedging default risk is *model-independent.*
  - importance of quantitative models for a better management of the P&L and the <u>residual</u> <u>risks</u>.