On the Edge of Completeness

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On the Edge of Completeness: Purpose and main ideas

• Purpose:
  − risk-analysis of exotic credit derivatives:
    ➢ credit contingent contracts, basket default swaps.
  − pricing and hedging exotic credit derivatives.

• Main ideas:
  − distinguish between credit spread volatility and default risk.
  − dynamic hedge of exotic default swaps with standard default swaps.

On the Edge of completeness: Overview

- Trading credit risk: closing the gap between supply and demand
- Modelling credit derivatives: the state of the art
- A new approach to credit derivatives modelling:
  - closing the gap between pricing and hedging
  - disentangling default risk and credit spread risk
Trading credit risk: Closing the gap between supply and demand

- From stone age to the new millennium:
  - Technical innovations in credit derivatives are driven by economic forces.
  - Transferring risk from commercial banks to institutional investors:
    - Securitization.
    - Default Swaps: portfolio and hedging issues.
    - Credit Contingent Contracts, Basket Credit Derivatives.
  - The previous means tend to be more integrated.
Securitization of credit risk:

- Credit risk seller
- Investor 1
- Investor 2
- Senior debt
- Junior debt
- SPV

Simplified scheme:
- No residual risk remains within SPV.
- All credit trades are simultaneous.
Trading Credit Risk: Closing the gap between supply and demand

- Financial intermediaries provide structuring and arrangement advice.
  - Credit risk seller can transfer loans to SPV or instead use default swaps
- Good news: low capital at risk for investment banks

- Good times for modelling credit derivatives
  - No need of hedging models
  - Credit pricing models are used to ease risk transfer
  - Need to assess the risks of various tranches
Trading Credit Risk:  
Closing the gap between supply and demand

• There is room for financial intermediation of credit risk
  – The transfers of credit risk between commercial banks and investors may not be simultaneous.
  – Since at one point in time, demand and offer of credit risk may not match.
    ➢ Meanwhile, credit risk remains within the balance sheet of the financial intermediary.
  – It is not further required to find customers with exact opposite interest at every new deal.
    ➢ Residual risks remain within the balance sheet of the financial intermediary.
Credit risk management without hedging default risk

- **Emphasis on:**
  - portfolio effects: correlation between default events
  - posting collateral
  - computation of capital at risk, risk assessment

- **Main issues:**
  - capital at risk can be high
  - what is the competitive advantage of investment banks

**Diagram:**
- Credit risk seller
- Default swap
- Credit derivatives trading book
- Bank
- Default swap
- Investor 1
- Default swap
- Investor 2
Trading against other dealers enhances ability to transfer credit risk by lowering capital at risk.
New ways to transfer credit risk: credit contingent contracts

- **Anatomy of a general credit contingent contract**
  - A credit contingent contract is like a standard default swap but with variable nominal (or exposure).
  - However, the periodic premium paid for the credit protection remains fixed.
  - The protection payment arises at default of one given single risky counterparty.

- **Examples**
  - cancellable swaps
  - quanto default swaps
  - credit protection of vulnerable swaps, OTC options (stand-alone basis)
  - credit protection of a portfolio of contracts (full protection, excess of loss insurance, partial collateralization)
New ways to transfer credit risk:

**Basket default derivatives**

- Consider a basket of $M$ risky bonds
  - multiple counterparties
- First to default swaps
  - protection against the first default
- $N$ out of $M$ default swaps ($N < M$)
  - protection against the first $N$ defaults
- Hedging and valuation of basket default derivatives
  - involves the joint (multivariate) modelling of default arrivals of issuers in the basket of bonds.
  - Modelling accurately the dependence between default times is a critical issue.
Modelling credit derivatives: the state of the art

- Modelling credit derivatives: Where do we stand?
- Financial industry approaches
  - Plain default swaps and risky bonds
  - Credit risk management approaches
- The Noah’s arch of credit risk models
  - “Firm-value” models
  - Risk-intensity based models
  - Looking desperately for a hedging based approach to pricing.
Modelling credit derivatives: Where do we stand?

Plain default swaps

- Static arbitrage of plain default swaps with short selling underlying bond
  - plain default swaps hedged using underlying risky bond
  - “bond strippers”: allow to compute prices of risky zero-coupon bonds
  - repo risk, squeeze risk, liquidity risk, recovery rate assumptions

- Computation of the P&L of a book of default swaps
  - Involves the computation of a P&L of a book of default swaps
  - The P&L is driven by changes in the credit spread curve and by the occurrence of default.
Assessing the varieties of risks involved in credit derivatives

- Specific risk or credit spread risk
  - *prior to default*, the P&L of a book of credit derivatives is driven by changes in credit spreads.

- Default risk
  - *in case of default, if unhedged*,
“firm-value” models:
- Modelling of firm’s assets
- First time passage below a critical threshold

risk-intensity based models
- Default arrivals are no longer predictable
- Model conditional local probabilities of default $\lambda(t) \, dt$
- $\tau$: default date, $\lambda(t)$ risk intensity or hazard rate

$$\lambda(t) dt = P[\tau \in [t, t + dt] | \tau > t]$$

Lack of a hedging based approach to pricing.
- Misunderstanding of hedging against default risk and credit spread risk
A new approach to credit derivatives modelling based on an hedging point of view

• **Rolling over** the hedge:
  – Short term default swaps vs. long-term default swaps
  – Credit spread transformation risk

• **Credit contingent contracts, basket default swaps**
  – Hedging default risk through dynamics holdings in standard default swaps
  – Hedging credit spread risk by choosing appropriate default swap maturities
  – Closing the gap between pricing and hedging

• **Practical hedging issues**
  – Uncertainty at default time
  – Managing net residual premiums
Purpose:
- Introduction to dynamic trading of default swaps
- Illustrates how default and credit spread risk arise

Arbitrage between long and short term default swap
- sell one long-term default swap
- buy a series of short-term default swaps

Example:
- default swaps on a FRN issued by BBB counterparty
- 5 years default swap premium: 50bp, recovery rate = 60%

Credit derivatives dealer

If default, 60%

Until default, 50 bp

Client
**Long-term Default Swaps v.s. Short-term Default Swaps**

**Rolling over the hedge**

- **Rolling over short-term default swap**
  - at inception, one year default swap premium: 33bp
  - cash-flows after one year:
    
    - **Credit derivatives**
    - 33 bp
    - Market
    - 60% if default

- **Buy a one year default swap at the end of every yearly period, if no default:**
  - Dynamic strategy,
  - future premiums depend on future credit quality
  - future premiums are unknown

- **Credit derivatives**
- ?? bp
- Market
- 60% if default
Long-term Default Swaps v.s. Short-term Default Swaps
Rolling over the hedge

- **Risk analysis** of rolling over short term against long term default swaps

  - Exchanged cash-flows:
    - Dealer receives 5 years (fixed) credit spread,
    - Dealer pays 1 year (variable) credit spread.

- **Full one to one protection at default time**
  - the previous strategy has eliminated one source of risk, that is default risk
Long-term Default Swaps v.s. Short-term Default Swaps
Rolling over the hedge

- negative exposure to an increase in short-term default swap premiums
  - if short-term premiums increase from 33bp to 70bp
  - reflecting a lower (short-term) credit quality
  - and no default occurs before the fifth year

Credit derivatives dealer

| 70 bp |
| 50 bp |

Market + Client

- Loss due to negative carry
  - long position in long term credit spreads
  - short position in short term credit spreads
Rolling over the hedge: portfolio of homogeneous loans

- Consider a portfolio of homogeneous loans
  - same unit nominal, non amortising
  - \( \tau_i \): default time of counterparty \( i \)
  - same default time distribution (same hazard rate \( \lambda(t) \)):
    \[
P[\tau_i \in [t, t + dt] | \tau_i > t] = \lambda(t) dt
    \]
  - \( F_t \): available information at time \( t \)
  - Conditional independence between default events \( \{\tau_i \in [t, t + dt]\} \)
    
    \[
P[\tau_i, \tau_j \in [t, t + dt] | F_t] = P[\tau_i \in [t, t + dt] | F_t] \times P[\tau_j \in [t, t + dt] | F_t]
    \]
    - equal to zero or to \( \lambda^2(t)(dt)^2 \), i.e no simultaneous defaults.
    - Remark that indicator default variables \( 1_{\{\tau_i \in [t, t + dt]\}} \) are (conditionally) independent and equally distributed.
Denote by \( N(t) \) the outstanding amount of the portfolio (i.e. the number of non defaulted loans) at time \( t \).

By law of large numbers, 
\[
\frac{1}{N(t)} \sum_{\tau_i \in [t, t+dt]} 1 \rightarrow \lambda(t) dt
\]

Since 
\[
N(t + dt) - N(t) = - \sum_{\tau_i \in [t, t+dt]} 1
\]
we get, 
\[
\frac{N(t + dt) - N(t)}{N(t)} = -\lambda(t) dt
\]
The outstanding nominal decays as 
\[
N(t) = N(0) \exp \left( \int_0^t \lambda(s) ds \right)
\]
Assume zero recovery; Total default loss from \( t \) to \( t + dt \):
\[
N(t) - N(t + dt)
\]
Cost of default per outstanding loan:
\[
\frac{N(t) - N(t + dt)}{N(t)} = \lambda(t) dt
\]
Rolling over the hedge: portfolio of homogeneous loans

- Cost of default per outstanding loan = $\lambda(t)dt$ is known at time $t$.
- **Insurance** diversification approach holds.
- **Fair premium** for a short term insurance contract on a single loan (i.e. a short term default swap) has to be equal to $\lambda(t)dt$.
- Relates hazard rate and short term default swap premiums.

**Expanding on rolling over the hedge**

- Let us be short in 5 years (say) default swaps written on all individual loans.
  - $p_{5Y} dt$, periodic premium per loan.
- Let us buy the short term default swaps on the outstanding loans.
  - Corresponding premium per loan: $\lambda(t)dt$.
- Cash-flows related to default events $N(t)-N(t+dt)$ perfectly offset.
Rolling over the hedge: portfolio of homogeneous loans

- Net (premium) cash-flows between $t$ and $t+dt$: 
  \[ N(t)[p_{5Y} - \lambda(t)]dt \]

- Where \( N(t) = N(0)\exp{-\int_0^t \lambda(s)ds} \)

  ➢ Payoff similar to an “index amortising swap”.

- At inception, \( p_{5Y} \) must be such that the risk-neutral expectation of the discounted net premiums equals zero:

- Pricing equation for the long-term default swap premium \( p_{5Y} \):
  \[
  E\left[ \int_0^T \left( \exp{-\int_0^t r(s)ds} \right) \times N(t)(p_{5Y} - \lambda(t))dt \right] = 0
  \]

  ➢ where \( r(t) \) is the short rate at time \( t \).

- Premiums received when selling long-term default swaps: 
  \( N(t)p_{5Y}dt \)

- Premiums paid on “hedging portfolio”: 
  \( N(t)\lambda(t)dt \)
Convexity effects and the cost of the hedge

- Net premiums paid \( N(t)[p_{5Y} - \lambda(t)]dt \)

What happens if short term premiums \( \lambda(t) \) become more volatile?

- Net premiums become negative when \( \lambda(t) \) is high.
- Meanwhile, the outstanding amount \( N(t) \) tends to be small, mitigating the losses.
- Contrarily when \( \lambda(t) \) is small, the dealer experiments positive cash-flows \( p_{5Y} - \lambda(t) \) on a larger amount \( N(t) \).

The more volatile \( \lambda(t) \), the smaller the average cost of the hedge and thus the long term premium \( p_{5Y} \).
Hedging exotic default swaps: main features

- Exotic credit derivatives can be *hedged* against default:
  - Constrains the *amount* of underlying *standard* default swaps.
  - *Variable* amount of standard default swaps.
  - *Full protection* at default time by construction of the hedge.
  - No more *discontinuity* in the P&L at default time.
  - “Safety-first” criteria: *main source of risk* can be hedged.
  - Model-free approach.

- Credit spread exposure has to be hedged by *other means*:
  - Appropriate *choice of maturity* of underlying default swap
  - Computation of sensitivities with respect to changes in credit spreads are *model dependent*. 
Credit contingent contracts

- client pays to dealer a periodic premium \( p_T(C) \) until default time \( \tau \), or maturity of the contract \( T \).
- dealer pays \( C(\tau) \) to client at default time \( \tau \), if \( \tau \leq T \).

Hedging side:

- **Dynamic** strategy based on standard default swaps:
- At time \( t \), hold an amount \( C(t) \) of standard default swaps
- \( \lambda(t) \) denotes the periodic premium at time \( t \) for a short-term default swap
**Hedging side:**

- Amount of standard default swaps equals the (variable) credit exposure on the credit contingent contract.

**Net position is a “basis swap”:**

- The client transfers credit spread risk to the credit derivatives dealer.

\[
\lambda(t) C(t) \text{ until default} \quad \text{Market+Client} \\
C(\tau) \text{ if default} \\
\]
• What is the cost of hedging default risk?
• Discounted value of hedging default swap premiums:

\[
E \left[ \int_0^T \left( \exp - \int_0^t \left( r + \lambda \right)(s) ds \right) \lambda(t)C(t) dt \right]
\]

Discounting term

Premier paid at time t on protection portfolio

• Equals the discounted value of premiums received by the seller:

\[
E \left[ \int_0^T \left( \exp - \int_0^t \left( r + \lambda \right)(s) ds \right) p_T dt \right]
\]
Consider a defaultable interest rate swap (with unit nominal)
- We are default-free, our counterparty is defaultable (default intensity $\lambda(t)$).
- We consider a (fixed-rate) receiver swap on a standalone basis.

Recovery assumption, payments in case of default.
- if default at time $\tau$, compute the default-free value of the swap: $PV_\tau$
- and get: $\delta(PV_\tau)^+ + (PV_\tau)^- = PV_\tau - (1-\delta)(PV_\tau)^+$
- $0 \leq \delta \leq 1$ recovery rate, $(PV_\tau)^+ = \text{Max}(PV_\tau,0)$, $(PV_\tau)^- = \text{Min}(PV_\tau,0)$
- In case of default,
  - we receive default-free value $PV_\tau$
  - minus
  - loss equal to $(1-\delta)(PV_\tau)^+$. 
Case study: defaultable interest rate swap

• Defaultable and default-free swap
  – Present value of defaultable swap = Present value of default-free swap (with same fixed rate) – Present value of the loss.
  – To compensate for default, fixed rate of defaultable swap (with given market value) is greater than fixed rate of default-free swap (with same market value).
  – Let us remark, that default immediately after negotiating a defaultable swap results in a positive jump in the P&L, because recovery is based on default-free value.

• To account for the possibility of default, we may constitute a credit reserve.
  – Amount of credit reserve equals expected Present Value of the loss
  – This accounts for the expected loss but does not hedge against realized loss.
Case study: defaultable interest rate swap

- Using a hedging instrument rather than a credit reserve
  - Consider a credit contingent contract that pays \((1-\delta)(PV_\tau)^+\) at default time \(\tau\) (if \(\tau \leq T\)), where \(PV_\tau\) is the present value of a default-free swap with \textit{same fixed rate} than defaultable swap.
  - Such a credit contract + a defaultable swap \textit{synthesises} a default-free swap (at a fixed rate equal to the \textit{initial} fixed rate):
    - At default, we receive \((1-\delta)(PV_\tau)^+ + PV_\tau - (1-\delta)(PV_\tau)^+ = PV_\tau\)
    - The \textbf{upfront} premium for this credit protection is equal to the Present Value of the \textbf{loss} \((1-\delta)(PV_\tau)^+\) given default:
      \[
      E\left[\int_0^T \left( \exp - \int_0^t (r + \lambda(u) du) \right) \lambda(t)(1-\delta)(PV_t)^+ \ dt \right]
      \]
Case study: defaultable interest rate swap
Interpreting the cost of the hedge

- **Average cost of default on a large portfolio of swaps**
  - Large number of **homogeneous defaultable** receiver swaps:
    - Same fixed rate and maturity; initial nominal value $N(0)=1$
    - Independent default dates and same default intensity $\lambda(t)$.
  - **Outstanding nominal amount**: $N(t) = \exp\left(-\int_0^t \lambda(s) ds\right)$
  - Nominal amount defaulted in $[t, t+dt]$:
    \[ N(t) - N(t+dt) = \lambda(t) dt \exp\left(-\int_0^t \lambda(s) ds\right) \]
  - **Cost of default in $[t, t+dt]$**: $(N(t) - N(t+dt)) \ (1-\delta)(PV_t)^+$
    - Where $PV_t$: present value of receiver swap with unit nominal.
  - **Aggregate cash-flows do not** depend on *default risk*.
  - Aggregate cash-flows are those of an index amortising swap
  - Standard discounting provides previous slide pricing equation
Randomly exercised swaption:
- Assume for simplicity no recovery ($\delta=0$).
- Interpret default time as a random time $\tau$ with intensity $\lambda(t)$.
- At that time, defaulted counterparty “exercises” a swaption, i.e. decides whether to cancel the swap according to its present value.
- PV of default-losses equals price of that randomly exercised swaption

American Swaption
- PV of American swaption equals the supremum over all possible stopping times of randomly exercised swaptions.
  - The upper bound can be reached for special default arrival dates:
  - $\lambda(t)=0$ above exercise boundary and $\lambda(t)=\infty$ on exercise boundary
Case study: defaultable interest rate swap

• Previous hedge leads to (small) jumps in the P&L:
  – Consider a 5,1% fixed rate defaultable receiver swap with PV=3%.
  – Buy previous credit contingent contract at market price.
    ➢ Due to credit protection, we hold a synthetic default-free 5,1% swap.
    ➢ Total PV remains equal to 3%.
  – Assume that default immediate default: \( \tau = 0^+ \).
  – Clearly a 5,1% default free swap has PV>3%, thus occurring a positive jump in P&L.

• Jumps in the P&L due to extra default insurance:
  – To hedge the previous credit contingent contract:
  – At time 0, we hold an amount of short term default swap that is equal to the Present Value of a default-free 5,1% swap
  – This amount is greater than 3%, the current Present Value.
Case study: defaultable interest rate swap

- Alternative hedging approach:
  - Fixed rate of default-free swap with 3% PV = 5% (say)
    - Consider a credit contingent contract that pays at default time:
    - Present value of a default free 5% swap minus recovered value on the 5,1% defaultable swap.
    - at default time, holder of defaultable swap + credit contract receives:
      - recovery value on 5,1% defaultable swap + PV of default free 5% swap - recovered value on 5,1% defaultable swap
      - = PV of default free 5% swap
    - Assume credit contract has a periodic annual premium denoted by $p$.
    - Prior to default time, defaultable swap + credit contract pays:
      - Default-free swap cash-flows with fixed rate = 5,1%-p
    - $p$ must be equal to 10bp = 5,1%-5%, otherwise arbitrage with 5% default-free swap.
Case study: defaultable interest rate swap

- Credit contingent contract transforms 5.1% defaultable swap into a 5% default free swap with the same PV.
  - If default occurs immediately, no jump in the hedged P&L.
  - To hedge the default payment on the credit contingent contract, we must hold default swaps providing payments of:
    - PV of default free 5% swap - recovery on 5.1% defaultable swap:
      \[ PV_\tau(5\%) - \delta PV_\tau(5.1\%)^+ - PV_\tau(5.1\%)^- \]
    - \( PV_\tau(5.1\%) \) is close to \( PV_\tau(5\%) \) (here 3\%=PV of defaultable swap).
    - Required payment on hedging default swap close to \((1- \delta) PV_\tau(5.1\%)^+\)
      \[ \text{Plain default swap pays } 1- \delta \text{ at default time.} \]
- Nominal amount of hedging default swap almost equal to \( PV_\tau(5.1\%)^+ \)
Hedging Default risk and credit spread risk in Credit Contingent Contracts

- **Purpose**: joint hedge of default risk and credit spread risk
- **Hedging default risk** only constrains the amount of underlying standard default swap.
  - Maturity of underlying default swap is arbitrary.
- **Choose maturity to be protected against credit spread risk**
  - PV of credit contingent contracts and standard default swaps are sensitive to the level of credit spreads
  - Sensitivity of standard default swaps to a shift in credit spreads increases with maturity
  - Choose maturity of underlying default swap in order to *equate sensitivities.*
Hedging credit spread risk

- Example:
  - dependence of simple default swaps on defaultable forward rates.
  - Consider a $T$-maturity default swap with continuously paid premium $p$. Assume zero-recovery (digital default swap).
  - PV (at time 0) of a long position provided by:
    \[
    PV = E \left[ \int_0^T \left( \exp \int_0^t (r + \lambda(s)) ds \right) \times (\lambda(t) - p) dt \right]
    \]
    - where $r(t)$ is the short rate and $\lambda(t)$ the default intensity.
    - Assume that $r(.)$ and $\lambda(.)$ are independent.
    - $B(0,t)$: price at time 0 of a $t$-maturity default-free discount bond
    - $f(0,t)$: corresponding forward rate
    \[
    B(0, t) = E \left[ \exp - \int_0^t r(u) du \right] = \exp - \int_0^t f(0, u) du
    \]
Hedging credit spread risk

Let $\overline{B}(0, t)$ be the defaultable discount bond price and $\bar{f}(0, t)$ the corresponding instantaneous forward rate:

$$\overline{B}(0, t) = E\left[\exp - \int_0^t (r + \lambda)(u)du \right] = \exp - \int_0^t f(0, u)du$$

Simple expression for the PV of the $T$-maturity default swap:

$$PV(T) = \int_0^T \overline{B}(0, t)\left(\bar{f}(0, t) - f(0, t) - p\right)dt$$

The derivative of default swap present value with respect to a shift of defaultable forward rate $\bar{f}(0, t)$ is provided by:

$$\frac{\partial PV}{\partial \bar{f}}(t) = PV(t) - PV(T) + \overline{B}(0, t)$$

$\Rightarrow$ $PV(t)$-$PV(T)$ is usually small compared with $\overline{B}(0, t)$. 
Hedging credit spread risk

- Similarly, we can compute the sensitivities of plain default swaps with respect to *default-free forward curves* \( f(0,t) \).
- And thus to *credit spreads*.
- Same approach can be conducted with the *credit contingent contract* to be hedged.
  - *All the computations are model dependent.*
- *Several maturities* of underlying default swaps can be used to match sensitivities.
  - For example, in the case of *defaultable* interest rate swap, the nominal amount of default swaps \((PV_\tau)^+\) is usually small.
  - *Single* default swap with nominal \((PV_\tau)^+\) has a *smaller sensitivity* to credit spreads than *defaultable interest rate swap*, even for long maturities.
  - *Short* and *long* positions in default swaps are required to hedge *credit spread risk*. 
Explaining theta effects with and without hedging

- **Different aspects** of “carrying” credit contracts through time.
  - Assume “historical” and “risk-neutral” intensities are equal.
- Consider a short position in a credit contingent contract.
- Present value of the deal provided by:
  \[
  PV(u) = E_u \left[ \int_u^T \exp \left( \int_u^t (r + \lambda)(s)ds \right) \times (p_T - \lambda(t)C(t))dt \right]
  \]
- (after computations) **Net expected capital gain:**
  \[
  E_u \left[ PV(u + du) - PV(u) \right] = (r(u) + \lambda(u))PV(u)du + (\lambda(u)C(u) - p_T)du
  \]
- **Accrued cash-flows (received premiums):** \(p_T du\)
  - By summation, Incremental P&L (if no default between \(u\) and \(u+du\)):
    \[
    r(u)PV(u)du + \lambda(u)(C(u) + PV(u))du
    \]
Explaining theta effects with and without hedging

- **Apparent extra return effect**: \( \lambda(u)(C(u) + PV(u))du \)
  - But, probability of default between \( u \) and \( u+du \): \( \lambda(u)du \).
  - Losses in case of default:
    - Commitment to pay: \( C(u) \)
    - Loss of PV of the credit contract: \( PV(u) \)
    - \( PV(u) \) consists in *unrealised* capital gains or losses in the credit derivatives book that “disappear” in case of default.
  - Expected loss charge: \( \lambda(u)(C(u) + PV(u))du \)

- **Hedging aspects**:
  - If we hold \( C(u) + PV(u) \) short-term digital default swaps, we are protected at default-time (no jump in the P&L).
  - Premiums to be paid: \( \lambda(u)(C(u) + PV(u))du \)
  - Same average rate of return, but smoother variations of the P&L.
Hedging Default Risk in Basket Default Swaps

- **Example:** first to default swap from a basket of two risky bonds.
  - If the first default time occurs before maturity,
  - The seller of the first to default swap pays the non recovered fraction of the defaulted bond.
  - Prior to that, he receives a periodic premium.

- **Assume that the two bonds cannot default simultaneously**
  - We moreover assume that default on one bond has no effect on the credit spread of the remaining bond.

- **How can the seller be protected at default time?**
  - The only way to be protected at default time is to hold two default swaps with the same nominal than the nominal of the bonds.
  - The maturity of underlying default swaps does not matter.
Real World hedging and risk-management issues

• uncertainty at default time
  – illiquid default swaps
  – recovery risk
  – simultaneous default events

• Managing net premiums
  – Maturity of underlying default swaps
  – Lines of credit
  – Management of the carry
  – Finite maturity and discrete premiums
  – Correlation between hedging cash-flows and financial variables
• Consider a first to default swap associated with a basket of two defaultable loans.
  – Hedging portfolios based on standard underlying default swaps
  – Uncertain hedge ratios if:
    ➢ simultaneous default events
    ➢ Jumps of credit spreads at default times
• Simultaneous default events:
  – If counterparties default altogether, holding the complete set of default swaps is a conservative (and thus expensive) hedge.
  – In the extreme case where default always occur altogether, we only need a single default swap on the loan with largest nominal.
  – In other cases, holding a fraction of underlying default swaps does not hedge default risk (if only one counterparty defaults).
Real world hedging and risk-management issues
Case study: hedge ratios for first to default swaps

• What occurs if there is a *jump in the credit spread* of the second counterparty after *default* of the first?
  – default of first counterparty means *bad news* for the second.

• If hedging with short-term default swaps, *no capital gain* at default.
  – Since PV of short-term default swaps is not *sensitive* to credit spreads.

• This is not the case if hedging with long term default swaps.
  – If credit spreads *jump*, PV of long-term default swaps *jumps*.

• Then, the amount of hedging default swaps can be *reduced*.
  – This reduction is *model-dependent*. 
Firm-value structural default models:

− Stock prices follow a diffusion processes (no jumps).
− Default occurs at first time the stock value hits a barrier

In this modelling, default credit derivatives can be completely hedged by trading the stocks:

− “Complete” pricing and hedging model:

Unrealistic features for hedging basket default swaps:

− Because default times are predictable, hedge ratios are close to zero except for the counterparty with the smallest “distance to default”.
On the edge of completeness?

**hazard rate based models**

- In **hazard rate** based models:
  - default is a sudden, *non predictable* event,
  - that causes a sharp *jump* in defaultable bond prices.
  - Most credit contingent contracts and basket default derivatives have payoffs that are *linear* in the prices of defaultable bonds.
  - Thus, good news: default risk can be *hedged*.
  - Credit spread risk can be *substantially reduced* but not completely eliminated.
  - More *realistic* approach to default.
  - *Hedge ratios* are *robust* with respect to default risk.
On the edge of completeness

Conclusion

• Looking for a better understanding of credit derivatives
  – payments in case of default,
  – volatility of credit spreads.

• Bridge between risk-neutral valuation and the cost of the hedge approach.

• dynamic hedging strategy based on standard default swaps.
  – hedge ratios in order to get protection at default time.
  – hedging default risk is model-independent.
  – importance of quantitative models for a better management of the P&L and the residual premiums.