Recent Issues in the Pricing of Collateralized Derivatives Contracts

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Recent Issues in the Pricing of Collateralized Derivatives Contracts

- LVA and FVA: where do we stand?
  - *Asymmetries between discounting receivables and payables ?*
    - Different lending and borrowing rates
    - Own default risk treatment
  - *A discount curve for uncollateralized trades: which market?*
  - *FVA connected to a cash-synthetic basis?*
- Trade contributions when pricing rule is not linear
    - BSDE, Euler's and marginal price contribution rules
- Consistency issues for pricing collateralized trades
    - Additive and recursive valuation rules.
- Bilateral initial margins
  - *Hedging recognition*
  - *Netted IM and multilateral default resolution*

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LVA and FVA: where do we stand?

- Different lending and borrowing default-free rates
  - $r(t)$ *(pure theoretical) default-free short rate*
    - Unobserved. Use of EONIA or Fed fund rate as a proxy is questionable
  - *Accounting for different lending and borrowing default-free rates*
    - $r(t), R(t), r(t) \leq R(t)$
    - $R(t) - r(t)$ pure funding liquidity premium or "liquidity basis"
    - Does not include a own credit risk component
    - Morini and Prampoloni (2010), Pallavicini et al. (2012), Castagna (2013)
  - *Bergman (1995): savings and borrowing accounts*
    - $\beta(t) = \exp \int_0^t r(s)ds, L(t) = \exp \int_0^t R(s)ds$
    - To preclude arbitrage opportunities, it is not possible to borrow at r and lend at R

Different lending and borrowing default-free rates

- Martingale measure?
  - *Korn (1992), Cvitanić and Karatzas (1993), thanks to Girsanov theorem, construct a Q^β – measure such that prices of primary (hedging) assets discounted by r are Q^β – martingales*
  - *In such a framework, derivatives can be replicated*
    - Consider such a derivative with terminal payoff $X(T)$
    - To cancel out such payoff, we need to replicate $-X(T)$
    - We define the PV of $X(T)$ as the opposite of the replication price of $-X(T)$
  - *PV is obtained as the unique solution of the BSDE*
  - $p_U(t) = E_t^{Q^\beta}\left[X(T)\exp\left(-\int_t^T \left(R(s)1_{\{p_U(s)>0\}} + r(s)1_{\{p_U(s)\leq 0\}}\right)ds\right)\right]$
    - Due to the difference between lending and borrowing rates, $\exp\left(-\int_0^t r(s)ds\right) \times p_U(t)$ is not a Q^β – martingale

Different lending and borrowing default-free rates

- PV computations
  - $p_U(t) = E_t^{Q^\beta}\left[X(T)\exp\left(-\int_t^T \left(R(s)1_{\{p_U(s)>0\}} + r(s)1_{\{p_U(s)\leq 0\}}\right)ds\right)\right]$
    - Non linear effects: discount rate $R(s)1_{\{p_U(s)>0\}} + r(s)1_{\{p_U(s)\leq 0\}}$ depends upon the PV
    - PV of portfolio is not equal to the sum of standalone PVs
    - Trade contributions discussed further
  - *Take a derivative receivable of* 1 *paid at T*
    - Under the previous approach, PV is equal to $E_t^{Q^\beta}\left[\exp\left(-\int_t^T R(s)ds\right)\right]$
    - This is if the case if the derivative receivable is stuck in the balance sheet (no securitization or repo funding of derivative receivable).
    - If the same cash-flow is paid through a bond, its price would be $E_t^{Q^\beta}\left[\exp\left(-\int_t^T r(s)ds\right)\right]$. Shorting this bond and buy the derivative receivable is not permitted (to preclude arbitrage opportunities).

Different lending and borrowing default-free rates

- PV computations (cont.)
  - *The previous pricing approach assumes that when the pricing entity borrows cash, it will pay the high rate $R(s)$*
  - *And when pricing entity lends cash, it pays the small rate $r(s)$*
    - Pricing entity then acts as a price taker (or liquidity taker)
    - If it is price maker in the money market, substitute $R(s)$ and $r(s)$
    - Positive externality of hedging derivatives
    - bid-ask spread $R(t) - r(t)$ can be viewed as a cost or a benefit
    - Taking a mid-point view leads to a symmetric LVA treatment for receivables and payables
  - *Systemic implications*
    - In the two dealers in a derivatives transaction are price takers, then the net PV of the two entities is negative
    - Even though the two entities only exchange cash-flows

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Different lending and borrowing default-free rates

- Pricing books of swaps: Model based approaches
  - *The funding spread conundrum*
  - *In the default-free setting of Piterbarg (2010, 2012), funding/lending rates essentially acts as usual short-term rate r*
    - If no repo and no collateral, discount a default-free receivable at funding rate
    - Note that the pure funding spread $R(t) - r(t) = 0$
  - *… In non linear approaches*
    - Castagna (2013), Crépey (2012) Pallavicini et al. (2012), etc.
    - $R(t) - r(t)$: So-called liquidity premium or liquidity basis
    - Short-term funding rate: $r + \lambda(1 - \delta) + (R - r)$
    - Only the sum is known, it is difficult to derive $R - r$ …
    - And isolate a funding adjustment (leaving aside non additivity issues)

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Own credit risk impact on valuations?

- Pricing books of swaps: Model based approaches
  - *Burgard and Kjaer (2011) framework*
    - The premise is different: specific treatment of own default risk
    - For a default-free receivable, the discount rate is $r + \lambda(1 - \delta)$ (see equation (2.1) p. 78).
    - Thus higher than r, even though the applicable discount rate is also equal to the funding rate
    - There is is no pure funding spread in the funding rate
    - Apart from CVA treatment, quite close to Piterbarg (2010).
  - *Piterbarg (2010), Burgard and Kjaer (2011) lead to lower the PV of receivables*
    - Discount at funding rate compared with discount at risk-free rate
  - *Other approaches require knowledge of liquidity premium*

Theoretical pricing framework

- Martingale measure? Complete markets?
  - *Replication?*
    - When considering interest rate derivatives, usually much more hedging assets (continuous tenors) than dimension of risk (number of Brownian motions), HJM setting
    - No specific underlying asset
  - *(Semi-)replication in the context of own default risk*
    - Defaultable bonds and possibly defaultable savings account are required to hedge default risk of entity
    - Practical difficulties in implementing the hedge
  - *Classical pricing approach*
    - Implies a consistent approach for derivatives and primitive assets
    - Same discount rate for a payment received through a bond or a derivative contract

Which inputs? Perfect collateralization scheme

- Theoretical pricing framework: Collateralized contracts
  - *Simplest case: perfect collateralization, cash-collateral*
    - Perfect collateralization: no slippage risk, no price impact of IM
  - $p_{Collat}(t) = E_t^{Q^\beta}\left[X(T)\exp\left(-\int_t^T c^\$(s)ds\right)\right]$
  - $c^\$$*, fed fund rate, OIS discounting under* Q^β
  - ***Pricing involves market observable*** $c^\$$
  - *Price is not related to default characteristics of the parties*
    - Not entity specific: easier to transfer the trade
  - *Derivatives assets can be seen as primitive assets.*
    - Settlement prices $h(t)$ for vanilla products and be observed and lead a model-free calibration of collateralized discount factors $E_t^{Q^\beta}\left[\exp\left(-\int_t^T c^\$(s)ds\right)\right]$

Which inputs? Perfect collateralization scheme

- Pricing books of swaps: Model based approaches
  - *In the case of fully collateralized contracts*
      - With no slippage risk at default
    - Discount rates are tied to the (expected) rate of return of posted collateral
      - Say EONIA or Fed funds rates in the most common cases
    - Calibration can be done on market observables with little adaptation and thus little model risk
      - Collateralized OIS and Libor swaps, possibly futures' rates
  - *This contrasts the case of uncollateralized contracts*
    - Modern math finance contributors (see references) use a funding spread but are short when it comes to figures
    - We miss out-of the money swap prices to calibrate discount factors

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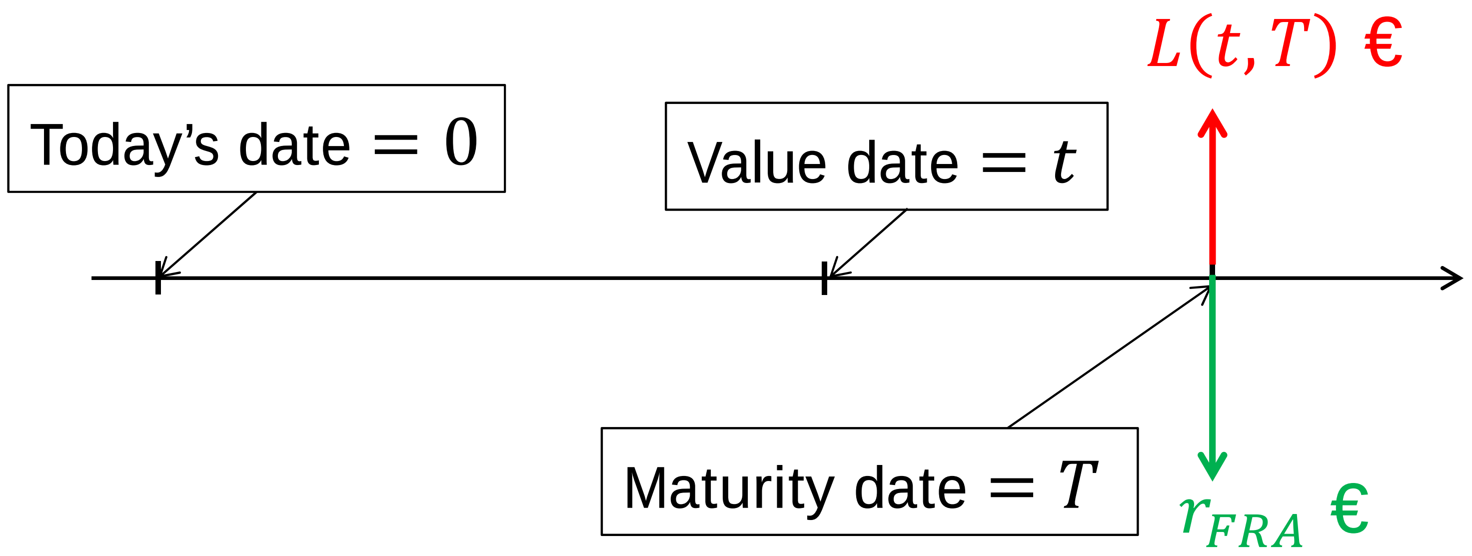
A discount curve for uncollateralized trades: which market?

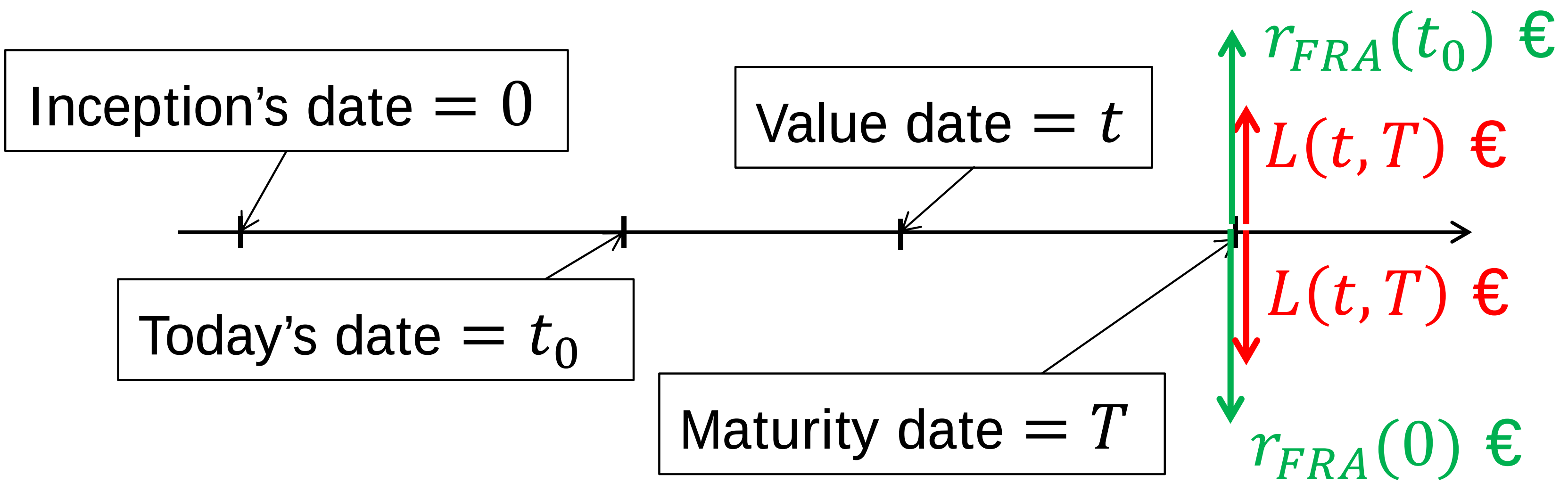
- Pricing books of uncollateralized swaps: the puzzle
  - For simplicity, leave aside CVA/DVA and focus on FVA/LVA



- ***r_{FRA} is the forward price of unknown Libor as seen from today's date.***
  - Mercurio (2009)

A discount curve for uncollateralized trades: which market?

- Pricing books of uncollateralized swaps: the puzzle
  - *Consider a legacy FRA with given fixed rate* r_{FRA}
  - *Enter an at the money FRA with opposite direction at* t_0



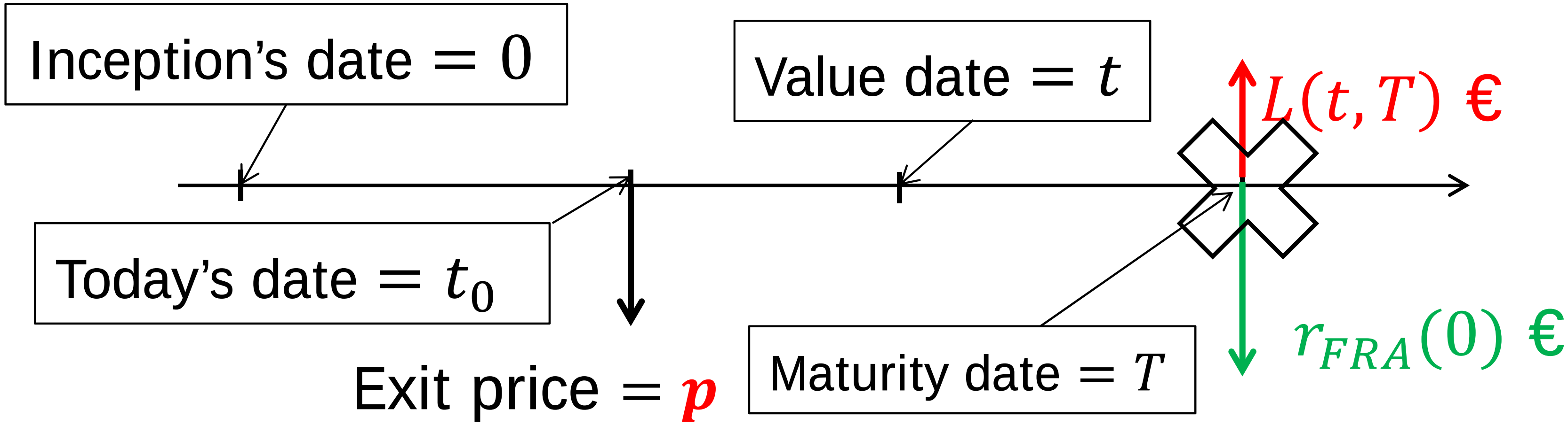
  - *Cancels out floating rate payments, only left with a fixed cash-flow of* $r_{FRA}(t_0) - r_{FRA}(0)$ *paid at* T
  - ***No funding need at any point in time*** *(only forward contracts)*

A discount curve for uncollateralized trades: which market?

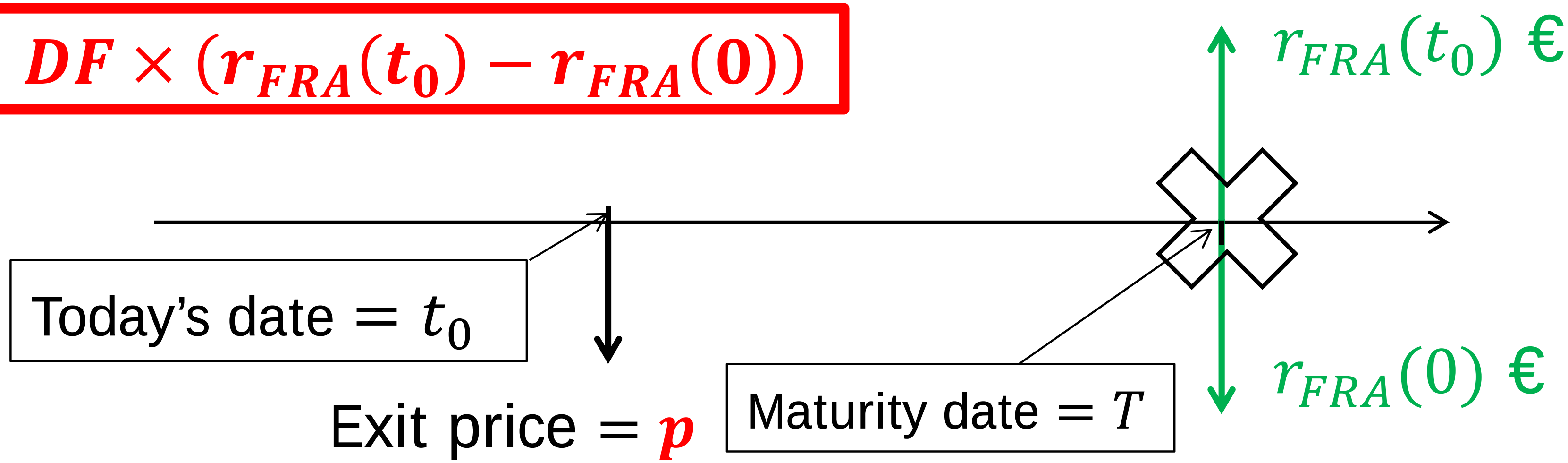
- Pricing books of uncollateralized swaps: the puzzle
  - *Which discount rate to be used is the question*
  - *Market based approach based on the concept of exiting the legacy trade against some cash at exit date*
  - *The cash paid to exit the trade is the price of the FRA*
    - Discount factors are inferred from such market prices
  - *Exiting the FRA is implemented through a novation trade*
    - Lack of novation trades?
  - *Related concept is the trading of out of / in the money FRA with upfront premiums*
    - Or to the securitization of derivative receivables
    - Or to financing such cash-flows in a repo market

A discount curve for uncollateralized trades: which market?

- Using novation trades to compute the fair value of a FRA
  - And discount factors ***DF*** for derivative receivables



$$\boxed{\boldsymbol{p = DF \times (r_{FRA}(t_0) - r_{FRA}(0))}}$$



FVA connected to a cash-synthetic basis?

- Let us go back to practical issues
  - *"It Cost JPMorgan \$1.5 Billion to Value Its Derivatives Right"*
    - http://www.bloomberg.com/news/2014-01-15/it-cost-jpmorgan-1-5-billion-to-value-its-derivatives-right.html
  - *"JP Morgan takes \$1.5 billion FVA loss"*
    - http://www.risk.net/risk-magazine/news/2322843/jp-morgan-takes-usd15-billion-fva-loss
    - "If you start with derivative receivables (…) of approximately \$50 billion,
    - Apply an average duration of approximately five years and a spread of approximately 50 basis points,
    - That accounts for about \$1 billion plus or minus the adjustment".
      - Marianne Lake, JP Morgan CFO

FVA connected to a cash-synthetic basis?

- From JP Morgan Fourth Quarter 2013 Financial Results

- The Firm implemented a Funding Valuation Adjustments ("FVA") framework this quarter for its OTC derivatives and structured notes, reflecting an industry migration towards incorporating the cost or benefit of unsecured funding into valuations
  - For the first time this quarter, we were able to clearly observe the existence of funding costs in market clearing levels
  - As a result, the Firm recorded a $1.5B loss this quarter
- FVA – which represents a spread over LIBOR – has the effect of "present valuing" market funding costs into the value of derivatives today, rather than accruing the cost over the life of the derivatives
  - Does not change the expected or actual cash flows
- FVA is dependent on the size and duration of underlying exposures, as well as market funding rates
- The adjustment this quarter is largely related to uncollateralized derivatives receivables, as
  - Collateralized derivatives already reflect the cost or benefit of collateral posted in valuations
  - Existing DVA for liabilities already reflects credit spreads, which are a significant component of funding spreads that drive FVA
- Current quarter reflects a one-time adjustment to the current portfolio
  - The P&L volatility of the combined FVA/DVA going forward is expected to be lower than in the past
- Refinements to the valuation approach will be made as appropriate, based on market evidence

http://files.shareholder.com/downloads/ONE/2956498186x0x718041/2a52855e-8269-4cfb-9ab9-d226e5d43844/4Q13presentation.pdf

FVA connected to a cash-synthetic basis?

FVA for Uncollateralized Trades

J.P.Morgan

- For uncollateralized trades, any future positive cash flow is equivalent to investors are purchasing a bond issued by the counterparty, hence its value should simply be given by $TV = Z^{+}e^{-(r+s_c)T}$
- For uncollateralized trades, any future negative cash flow is equivalent to investors are issuing a bond to the counterparty, hence its value should simply be given by $TV = Z^{-}e^{-(r+s_u)T}$
- When netting is allowed, then

$$
\begin{aligned}
TV &= Z^{+}e^{-(r+s_c)T} - Z^{-}e^{-(r+s_u)T} \\
&= Ze^{-rT} - Z^{+}e^{-rT}\left(1-e^{-s_cT}\right) + Z^{-}e^{-rT}\left(1-e^{-s_uT}\right) \\
&= RV - CVA + DVA - FVA + Residual
\end{aligned}
$$

where

$$
\begin{aligned}
RV &= Ze^{-rT} \\
CVA &= Z^{+}e^{-rT}\left(1-e^{-c_cT}\right) \\
DVA &= Z^{-}e^{-rT}\left(1-e^{-c_uT}\right) \\
FVA &= Ze^{-rT}\left(1-e^{-bT}\right)
\end{aligned}
$$

and *b* is cash-synthetic basis (assumed to be same for both counterparty and investor)

- In general, FVA can be approximated through

$$CVA = \int EEPV(t)P_c(t)\tilde{c}_c(t)dt \qquad DVA = \int RevEEPV(t)P_u(t)\tilde{c}_u(t)dt \qquad FVA = \int MEPV(t)\tilde{b}(t)dt.$$

CVA, FVA and Counterparty Credit Risk, Liu, JP Morgan, August 2013

FVA connected to a cash-synthetic basis?

- First item of previous slide suggests to use the same discount rate for a receivable payment on a derivative and for a bond of the same counterparty
  - *Consistency across bond and derivatives valuations*
  - *If CVA is market implied (i.e. using CDS quotes)*
  - *And a (collat.) swap curve is used as a base curve*
  - *Then, for global consistency, one needs to introduce a bond – CDS (or cash-synthetic) basis*
    - As above (with same basis for pricing entity and counterparty).
  - *And define this as a "funding valuation adjustment"*
    - Even if the connection with funding is loose
    - There are many components in the cash-synthetic basis, not only funding risk

Deutsche Bank

FVA connected to a cash-synthetic basis?

- Negative bond cds basis could imply positive fva effect?
  - *Deutsche Bank Corporate Banking & Securities 4Q2013*
  - *Fourth quarter results were also affected by a EUR 110 million charge for Debt Valuation Adjustment (DVA) and a EUR 149 million charge for Credit Valuation Adjustment (CVA)*
  - *Which offset a **gain** of EUR 83 million for Funding Valuation Adjustment (FVA).*
  - *FVA is an adjustment being implemented in 4Q2013 that reflects the implicit funding **costs** borne by Deutsche Bank for uncollateralized derivative positions.*
- **Volatile FVA would eventually lead to a capital charge**
  - *As for CVA …*
  - *Need to embed these in AVA charges?*

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LVA and FVA methodologies: some comments

- Limits of swaps / bonds analogy regarding funding
  - *If you start with derivative receivables (…) of $50 billion …"*
  - *Vanilla IR swaps do not involve upfront premium*
    - Therefore, no need of Treasury at inception
    - Treasury involved in fixed and floating leg accrued payments
  - *Receivables mainly result from accumulated margins*
    - Bid – offer on market making activities
    - Cash in directional trades
  - *Above $50 billion might not be funded on bond/money markets*
    - Do not interfere with prudential liquidity ratios
  - *What about different lending and borrowing rates?*
    - (See next slide)

LVA and FVA methodologies: some comments

- Different discount rates for (default-free) receivables and payables?
  - *Use of pure funding liquidity premium* $R(t) - r(t)$
    - Above quantity is difficult to calibrate
  - *Discounting receivables at R for <u>prudent valuations</u>?*
    - Limits of “cash-extraction” detrimental to bondholders
  - *Impact of own credit risk on discounting receivables?*
    - Drawbacks are already well documented
- Lack of novation trades
    - Calibration of uncollat. discount factors on market observables?
- FVA connected to a cash-synthetic basis
    - Money market rates: short maturities, bond rates: longer maturities…
    - Deal with basis volatility, term-structure, entity specific effects
    - FVA terminology is a bit misleading

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Trade contributions when pricing rule is not linear

- Trade contributions when pricing rule is not linear (asymmetric CSAs)
  - ***Marginal price*** *of Z within portfolio X :* $\frac{\boldsymbol{P(X+\varepsilon Z)-P(X)}}{\boldsymbol{\varepsilon}}$
  - ***Euler's price contribution rule***
  - *If* $P(\lambda \times X) = \lambda \times P(X)$
  - *Compute* $\boldsymbol{E}[\boldsymbol{P'(X)Z}]$
  - $P'(X)$*: Stochastic discount factor at the portfolio and CSA level*
    - Is related to a CSA change of measure, see Laurent et al. (2012)
    - Simplifies numerical pricing of new deals (use of Monte Carlo)
  - *Adapting El Karoui et al (1997), it can be proved that the two approaches lead to the same price contribution of trade Z within portfolio X*

Consistency between internal pricing models

- Consistency within and among pricing models
  - *For simplicity, let us restrict to cash collateral at rate* c
  - *And no difference between lending and borrowing rates*
    - r : default-free short rate
    - No default risk: concentrate on PV impact of variation margins
  - *Settlement price for collateralized contracts can be written as the sum of the uncollat. PV + the PV of collateral flows*
    - Additive approach
    - If we denote by V the collateral amount, the additive term to switch from uncollateralized to collateralized is
    - $E_t^{Q^\beta}\left[\int_t^T (r(s)-c(s))V(s)e^{-\int_t^s r(u)du}\,ds\right]$

Consistency between internal pricing models

- Consistent collateralized prices
  - *If collateral amount V is based on the collateralized price (settlement price) h only, we are led to recursive pricing formulas*
    - Possibly with non linear effects
  - *In some cases, for theoretical or practical reasons, the margin calls can be based on some proxy for h*
    - Use of Eurodollar futures instead of collateralized OIS contracts in the short end of yield curve (LCH at some point in time)
    - Use of Libor discounting in an asymmetric CSA
  - *Then* $V \neq h$ *(possibly by inadvertence) and recursive formula*
    - $E_t^{Q^\beta}\left[X(T)\exp\left(-\int_t^T c(s)ds\right)\right]$ (OIS discounting)
  - *Is not valid*

Consistency between internal pricing models

- Consistent collateralized prices
  - *Let us assume that V is derived from contractual payoff $X(T)$ through discounting at r_V (see previous slide)*
    - Thus $V(t) = E_t^{Q^\beta}\left[X(T)\exp\left(-\int_t^T r_V(s)ds\right)\right]$
  - *Accounting for actual collateralization scheme involves an additive adjustment term to OIS discounting*
  - *Settlement price:*
    - Sum of $h_C(t) = E_t^{Q^\beta}\left[X(T)\exp\left(-\int_t^T c(s)ds\right)\right]$
    - And of the adjustment term, which be written as:
    - $E_t^{Q^\beta}\left[\int_t^T (r(s) - c(s)\ ))(V(s) - h_C(s))e^{-\int_t^s r(u)du}\, ds\right]$

Non mandatory cleared swap contracts

- Scope of Dodd-Frank EMIR MiFID for mandatory clearing
  - *Many regulators involved (CFTC, SEC, ESMA, EBA) …*
  - *Status of compression trades, hedging trades?*
- Which model for bilateral IM?
  - *ISDA SIMM Initiative (Standard Initial Margin Model)*
    - ISDA, December 2013
- Hedging recognition for IM computations
  - *CFTC ruling?*
- Multilateral default resolution
  - *Tri-optima tri-reduce*
    - http://www.trioptima.com/services/triReduce/triReduce-rates.html
  - *Multilateral vs bilateral IM*
    - Sub-additivity of risk measure based initial margins.

Non mandatory cleared swap contracts

- Based on (too ?) rough computations, the need for bilateral IM might blow up to 1 trillion$
    - Applicable to new trades: room for adaptation and increased netting
  - *Still, collateral shortage issue cannot wiped out.*
    - New QIS? Monitoring working group? EBA schedule?
- Apart from liquidity and pricing issues, major concerns about systemic counterparty risk
  - *Collateral held in a third party custodian bank*
    - Which becomes highly systemic (wrong way risk)
    - Increased interconnectedness within the banking sector …
  - *IM cannot be seized by senior unsecured debt holders*
    - Lowers guarantees to claimants of collateral posting company
    - Moral hazard issues …

Non mandatory cleared swap contracts

- Hedging recognition for IM computations
  - *From Bank of England Quarterly Bulletin, Q1 2013*
    - *"Portfolios with certain counterparties comprise clearable products as hedges against other products which are not currently clearable".*
    - *"If those portfolios remained entirely bilateral, the clearable and non-clearable trades would be able to offset each other."*
  - *Let us consider an exotic swap sold by a dealer*
    - Swap cannot be centrally cleared
    - Ruled by a bilateral CSA (with small Independent Amount)
    - Due to Variation Margins, counterparty risk reduces to slippage risk
  - *Let us now consider a DV01 hedging swap*
    - If hedging swap is in the same bilateral netting set, slippage risk reduces to second order risks (gamma, vega, correlation risks …)
    - Zero DV01 of exotic swap + hedge at inception

Non mandatory cleared swap contracts

- Hedging recognition for IM computations
  - *Note that the two parties involved in the exotic swap have to agree about the DV01*
    - In order to agree with the hedging swap
    - Note that ISDA SIMM will be quite useful
    - Advocates the use of pre-computed DV01 for 2yr, 5yr, 10yr and 30 yr tenors.
    - Resolution of disputes on bilateral IM should lead to convergence of DV01 for exotic trades among parties
  - *Use of a bundle (exotic + hedge) as in FX options market*
  - *Or treat the hedging swap with a separate ID (for Swap Data Repositories)*
    - Question is whether hedging swap is out of the scope of mandatory clearing or needs some exemption (see next slide)

Non mandatory cleared swap contracts

- Hedging recognition for IM computations
  - *The hedging swap usually has a non standard amortization scheme and is not ready to clear*
  - *However, it could be disentangled into clearable components*
    - CFTC, Federal Register / Vol. 77, No. 240 / Thursday, December 13, 2012 / Rules and Regulations / Disentangling Complex Swaps
    - "*Adherence to the clearing requirement does not require market participants to structure their swaps in a particular manner or disentangle swaps that serve legitimate business purposes.*"
  - *Keeping the hedging swap in the bilateral netting set would result in a more efficient counterparty risk management*
    - Reduction of CCR (slippage risk) should be considered as a legitimate business purpose.
    - To be confirmed by regulators: the above statement applies to TriOptima rebalancing and compression exercises.

Non mandatory cleared swap contracts

- Multilateral default resolution
  - *Case of one (or more) major dealer defaulting*
  - *In a disordered default process, each surviving party would use collected bilateral IM to wipe out open positions with defaulted party*
  - $\Rightarrow$ *turmoil in the underlying market*
    - Tri-reduce algorithm from TriOptima is a pre-default compression process
  - *Idea is to make the compression process contingent to default (through a series of contingent CDS)*
  - *To minimize non-defaulted counterparty exposures*
  - *Efficient use of collateral* $\sum_i IM(X_i) \rightarrow IM(\sum_i X_i)$ *fully protects the netting set of non-defaulted counterparties as is the case with central clearing.*

Non mandatory cleared swap contracts

- Multilateral default resolution implementation
  - *As a starting point, let us go back to SIMM model and a given asset class, say rates*
  - *This provides daily equivalent exposures on a specified set of tenors (say 2 yr, 5 yr, 10 yr, 30 yr).*
  - *For all bilateral exposures within the netting set of swap dealers (and possibly other major swap participants)*
  - *A counterparty exposure can be seen as a vector X with coordinates equal to nominal amounts in 2 yr, 5 yr, 10 yr, 30 yr vanilla interest swaps*
  - *(SI)IM is then a risk measure mapping the previous vector into a cash amount.*
    - We will further assume that $IM(X+Y) \leq IM(X) + IM(Y)$ (sub-additivity) holds for considered portfolios.

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Non mandatory cleared swap contracts

- Multilateral default resolution implementation
  - *Let us denote by X the* ***aggregate net exposure of defaulted party***
  - *Which can be subdivided as $X = \sum X_i$ where X_i is the bilateral exposure to counterparty i*
    - Netted IM (as with central clearing) is $IM(X)$
    - With bilateral initial margining, posted IM is $\sum IM(X_i)$
  - *Step 1 (regression): $X_i = \beta_i X + \varepsilon_i$*
    - $E[\varepsilon_i|X] = 0$
    - By construction, $\sum \beta_i = 1, \sum \varepsilon_i = 0$
    - $\beta_i X$ fraction of aggregate risk exposure allocated to counterparty i
    - For simplicity, we will assume that $\beta_i \geq 0$
    - ε_i : residual risk, can be cancelled among the netting set of non defaulted counterparties.
    - Thus does not require IM

Non mandatory cleared swap contracts

- Multilateral default resolution implementation
  - *Step 2: cancellation of residual exposures* $\varepsilon_i = \left(N_i^{2Y}, N_i^{5Y}, N_i^{10Y}, N_i^{30Y}\right)$
  - *Since* $\sum \varepsilon_i = 0,\ \sum N_i^{2Y} = 0$ *and so on with other tenors.*
  - *Numerical example (3 non – defaulted parties)*
    - $N_1^{2Y} = 100, N_2^{2Y} = -70, N_2^{2Y} = -30$
    - Replace exposure 100 over defaulted party of counterparty 1 by two exposures of 70 and 30 over counterparty 2 and 3.
    - Rebalancing could be done at mid-prices out of the market (SEF) in order to minimize volatility and price impacts
    - Only involves non-defaulted parties
    - Need to account for heterogeneous credit quality of survived parties

Non mandatory cleared swap contracts

- Multilateral default resolution implementation
  - *Step 2: cancellation of residual exposures (legal issues)*
    - SEF exemptions (as with today's TriOptima trades)
    - Pre-commitment within the netting set?
    - Update of ISDA master agreements for multilateral IM CSA?
    - Use of contingent CDS: at counterparty default, the netting interest rate swaps are implemented.
  - *Step 3: managing aggregate net exposure*
    - Each non-defaulted party shares a fraction β_i of aggregate net exposure of defaulted party
    - Since $\beta_i X$ are comonotonic with X, $IM(X) = \sum IM(\beta_i X)$
    - For comonotonic-additive risk-based IM
    - As a consequence, netted IM can be split among non defaulted parties

Non mandatory cleared swap contracts

- Multilateral default resolution implementation
  - *Efficient use of collateral* $\sum_i IM(X_i) \rightarrow IM(\sum_i X_i)$ *fully protects the netting set of non-defaulted counterparties as is the case with central clearing.*
    - Allows to deal with swap contracts that cannot be centrally cleared in a an efficient manner.
    - Robust to multiple defaults
  - $IM(\beta_i X) \leq IM(X_i)$
    - Under technical conditions (Bäuerle and Müller (2006))
    - Counterparty risk on custodian banks is reduced
    - Netted IM could be posted to a single custodian bank and split at default
  - *Orderly default: non-defaulted parties need to cancel out a fraction of the same aggregate risk X*
    - Need of a common IM model among participants

Non mandatory cleared swap contracts

- Multilateral default resolution implementation
  - *Many legal and regulatory issues need to be solved*
  - *"ESMA considered that portfolio compression was a risk-reducing exercise and proposed that counterparties (…) had procedures to regularly (..) analyse the possibility to conduct a portfolio compression exercise."*
    - ESMA Draft technical standards under the Regulation (EU) No 648/2012 of the European Parliament and of the Council of 4 July 2012 on OTC Derivatives, CCPs and Trade Repositories
  - *Compression reduces interconnectedness and is usually viewed as a way to reduce systemic counterparty risk*
  - *The proposed scheme is a step in that direction*
    - While mitigated costs (collateral shortage, etc.)
    - And dealing with specificities of exotic swaps

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