Latest techniques in hedging credit derivatives

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Joint work with Jon Gregory, BNP Paribas

Latest techniques in hedging credit derivatives

- Default, credit spread and correlation hedges
- Analytical computations vs importance sampling techniques
- Dealing with multiple defaults
- Choice of copula and hedging strategies

Latest techniques in hedging credit derivatives

- Hedging of basket default swaps and CDO tranches
  - *With plain CDS*
  - *Hedging of quanto default swaps, options on CDO tranches not addressed.*
- Related papers:
  - *"I will survive", RISK, June 2003*
  - *"Basket Default Swaps, CDO's and Factor copulas", www.defaultrisk.com*
  - *"In the Core of Correlation", http://laurent.jeanpaul.free.fr*

Latest techniques in hedging credit derivatives

- Survey
  - *Payoff definitions:*
    - CDS, k^{th} to default swaps, CDO tranches
  - *Standard modelling framework*
    - Factor copulas and semi-analytical approach vs importance sampling
    - One factor Gaussian copula,
    - Gaussian copulas, Clayton, Student *t*, Shock models
  - *Default hedges*
    - Multiple default issues
  - *Credit Spread hedges*
  - *Correlation hedges*

Basket default swaps and CDO tranches

- $i = 1, \ldots, n$ names.
- $\tau_1, \ldots, \tau_n$ default times.
- N_i nominal of credit *i*,
- δ_i recovery rate (between 0 and 1)
  - $N_i(1 - \delta_i)$ loss given default (of name i)
  - if $N_i(1 - \delta_i)$ does not depend on *i*: homogeneous case
  - otherwise, heterogeneous case.

Basket default swaps and CDO tranches

- Credit default swap (CDS) on name *i*:
- Default leg:
  - *payment of* $N_i(1-\delta_i)$ *at* τ_i *if* $\tau_i \leqslant T$
  - *where T is the maturity of the CDS*
- Premium leg:
  - *constant periodic premium paid until* $\min(\tau_i, T)$

Basket default swaps and CDO tranches

- k^{th} to default swaps
- $\tau^1, \ldots, \tau^n$ ordered default times
- Default leg:
  - Payment of $N_i(1-\delta_i)$ at τ^k
  - where *i* is the name in default,
  - If $\tau^k \leqslant T$ maturity of *k*-th to default swap
- Premium leg:
  - *constant periodic premium until* $\min(\tau^k, T)$

Basket default swaps and CDO tranches

- Payments are based on the accumulated losses on the pool of credits
- Accumulated loss at *t*:

$$L(t) = \sum_{1 \leq i \leq n} N_i(1-\delta_i)N_i(t)$$

  - where $N_i(t) = 1_{\tau_i \leq t}$, $N_i(1-\delta_i)$ loss given default.
- Tranches with thresholds $0 \leq A \leq B \leq \sum N_j$
  - *Mezzanine: losses are between A and B*

Basket default swaps and CDO tranches

- Cumulated payments at time *t on mezzanine tranche*

$$M(t) = (L(t) - A))\,1_{[A,B]}(L(t)) + (B - A)1_{]B,\infty[}(L(t))$$

- *Payments on default leg:*
  - $\Delta M(t) = M(t) - M(t^-)$ at time $t \leq T$
- *Payments on premium leg:*
  - periodic premium,
  - proportional to outstanding nominal $B - A - M(t)$

Modelling framework for default times

- *Copula approach*
- *Conditional independence*
- *One factor Gaussian copula*
- *Gaussian copula with sector correlations*
- *Clayton and Student t copulas*
- *Shock models*

Modelling framework

- Joint survival function:

$$S(t_1, \ldots, t_n) = Q(\tau_1 > t_1, \ldots, \tau_n > t_n)$$

  - *Needs to be specified given marginal distributions.*
  - $S_i(t) = Q(\tau_i > t)$ *given from CDS quotes.*

- (Survival) Copula of default times:

$$C(S_1(t_1), \ldots, S_n(t_n)) = S(t_1, \ldots, t_n)$$

  - *C characterizes the dependence between default times.*

Modelling framework

- Factor approaches to joint distributions:
  - *V: low dimensional factor, not observed « latent factor ».*
  - *Conditionally on V, default times are independent.*
  - *Conditional default probabilities:*

$$p_t^{i|V} = Q\left(\tau_i \le t \mid V\right), \quad q_t^{i|V} = Q\left(\tau_i > t \mid V\right).$$

  - *Conditional joint distribution:*

$$Q(\tau_1 \le t_1, \ldots, \tau_n \le t_n \mid V) = \prod_{1 \le i \le n} p_{t_i}^{i|V}$$

  - *Joint survival function (implies integration wrt V):*

$$Q(\tau_1 > t_1, \ldots, \tau_n > t_n) = E\left[\prod_{i=1}^{n} q_{t_i}^{i|V}\right]$$

Modelling framework

- One factor Gaussian copula:
  - $V, \bar{V}_i,\ i = 1, \ldots, n$ *independent Gaussian,*

$$V_i = \rho_i V + \sqrt{1-\rho_i^2}\bar{V}_i$$

  - *Default times:* $\tau_i = F_i^{-1}(\Phi(V_i))$
  - *Conditional default probabilities:* $p_t^{i|V} = \Phi\left(\frac{-\rho_i V + \Phi^{-1}(F_i(t))}{\sqrt{1-\rho_i^2}}\right)$
- Joint survival function:

$$S(t_1, \ldots, t_n) = \int \left(\prod_{i=1}^{n} \Phi\left(\frac{\rho_i v - \Phi^{-1}(F_i(t_i))}{\sqrt{1-\rho_i^2}}\right)\right) \varphi(v) dv$$

- Can be extended to Student *t* copulas (two factors).

Modelling framework

- Why factor models ?
  - *Standard approach in finance and statistics*
  - *Tackle with large dimensions*
- Need tractable dependence between defaults:
  - *Parsimonious modelling*
    - One factor Gaussian copula: n parameters
    - But constraints on dependence structure
  - *Semi-explicit computations for portfolio credit derivatives*
    - Premiums, Greeks
    - Much quicker than plain Monte-Carlo
  - *No need of product specific importance sampling schemes*

Modelling framework

- Gaussian copula with sector correlations

$$\begin{pmatrix}
1 & \beta_1 & \beta_1 & & & & & & & \\
\beta_1 & 1 & \beta_1 & & & & & & \gamma & \\
\beta_1 & \beta_1 & 1 & & & & & & & \\
 & & & 1 & & & & & & \\
 & & & & \cdot & & & & & \\
 & & & & & \cdot & & & & \\
 & & & & & & 1 & & & \\
 & & & & & & & 1 & \beta_m & \beta_m \\
 & \gamma & & & & & & \beta_m & 1 & \beta_m \\
 & & & & & & & \beta_m & \beta_m & 1
\end{pmatrix}$$

- *Analytical approach still applicable*

Modelling framework

- Clayton copula:
  - *Archimedean copula*
  - *lower tail dependence:* $\lambda_L = 2^{-1/\theta}$
    - no upper tail dependence
  - *Kendall tau* $\rho_K = \frac{\theta}{\theta + 2}$
    - Spearman rho has to be computed numerically
  - C_θ *increasing with* θ
  - $\theta = 0$ *independence case*
  - $\theta = +\infty$ *comonotonic case*

Modelling framework

- Shock models
  - Duffie & Singleton, Wong
- *Default dates:* $\tau_i = \min(\bar{\tau}_i, \tau)$
- *Simultaneous defaults:* $Q(\tau_i = \tau_j) \geq Q\left(\tau \leq \min(\bar{\tau}_i, \bar{\tau}_j)\right) > 0$
- *Conditional default probabilities:*

$$p_t^{i|\tau} = 1_{\tau > t} Q(\bar{\tau}_i \leq t) + 1_{\tau \leq t}$$

- $\tau, \bar{\tau}_i$ *exponential distributions with parameters* $\lambda, \bar{\lambda}_i$
- *Symmetric case:* λ_i *does not depend on name*
  - $\lambda = 0$ independence case, $\lambda_i = 0$ comonotonic case
  - Copula increasing with λ
- *Tail dependence*

Model dependence

- Example: first to default swap
  - *Default leg* $$\int_0^T \sum_{i=1}^{n} M_i B(t) E\left[\prod_{j\neq i}\left(1-p_t^{j|V}\right)\frac{dp_t^{i|V}}{dt}\right]dt$$
  - *One factor Gaussian* $$p_t^{i|V} = \Phi\left(\frac{-\rho_i V + \Phi^{-1}(F_i(t))}{\sqrt{1-\rho_i^2}}\right)$$
  - *Clayton* $$p_t^{i|V} = \exp\left(V\left(1-F_i(t)^{-\theta}\right)\right)$$
  - *Shock model* $$p_t^{i|\tau} = 1_{\tau>t} Q(\bar{\tau}_i \leq t) + 1_{\tau\leq t}$$
- Semi-explicit computations

Model dependence

- From first to last to default swap premiums
  - *10 names, unit nominal*
  - *Spreads of names uniformly distributed between 60 and 150 bp*
  - *Recovery rate = 40%*
  - *Maturity = 5 years*
  - *Gaussian correlation: 30%*
- Same FTD premiums imply consistent prices for protection at all ranks
- Model with simultaneous defaults provides very different results

Rank	Clayton	Gaussian	MO
1	723	723	723
2	277	274	160
3	122	123	53
4	55	56	37
5	24	25	36
6	10	11	36
7	3.6	4.3	36
8	1.2	1.5	36
9	0.28	0.39	36
10	0.04	0.06	36
kendall	9%	19%	NS

Model dependence

- CDO margins (bp pa)
  - *Credit spreads uniformly distributed between 80bp and 120bp*
  - *100 names*
  - *Gaussian correlation = 30%*
  - *Parameters of Clayton and shock models are set for matching of equity tranches.*
- For the pricing of CDO tranches, Clayton and Gaussian copulas are close.
- Very different results with shock models

	Gaussian	Clayton	Shock
Equity	2158	2158	2158
Mezzanine	669	663	139
Senior	22	21	52

Default Hedges

- Default hedge (no losses in case of default)
  - CDS hedging instrument
- *Example: First to default swap*
  - If using *short term* credit default swaps
  - Assume no simultaneous defaults can occur
  - Default hedge implies 100% in all names
  - When using *long term* credit default swaps
    - Default of one name means bad news (positive dependence)
    - Jumps in credit spreads at (first to) default time
    - The amount of hedging CDS can be <u>reduced</u> (model dependent)
  - Default hedge may be not feasible in case of simultaneous defaults
- *CDO tranches*
  - <u>Recovery risk</u> may not be hedged

Credit Spread Hedges

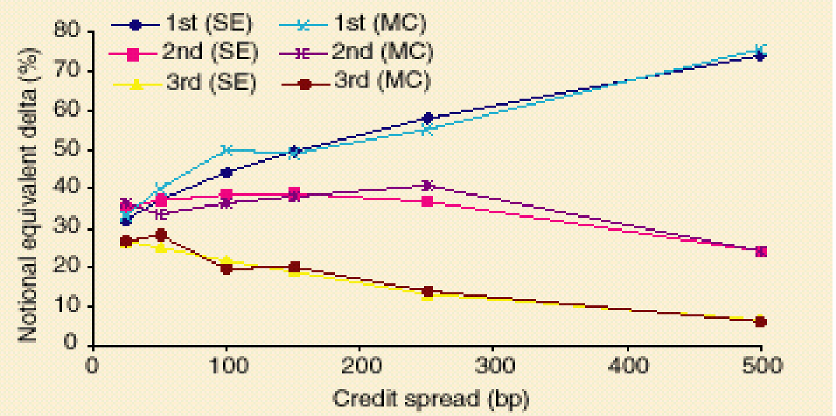
- Amount of CDS to hedge a shift in credit spreads
- Example: six names portfolio
- Changes in credit curves of individual names
- Semi-analytical more accurate than 10^5 Monte Carlo simulations.
- Much quicker: about 25 Monte Carlo simulations.

A. Comparison of the semi-explicit formulas with Monte Carlo simulations

	First to default		Second to default		Third to default	
	SE	MC	SE	MC	SE	MC
0%	1,075.1	1,075.9	214.8	214.7	28.2	27.7
20%	927.0	925.9	247.2	247.5	61.4	61.8
30%	859.9	857.9	256.8	257.6	77.6	78.0
40%	796.6	795.2	263.3	264.2	92.7	93.0
60%	679.6	678.0	268.8	268.9	119.5	119.8
80%	573.1	571.7	266.2	266.1	141.0	140.9
100%	500.0	500.0	250.0	250.0	150.0	150.0

Premiums in basis points per annum as a function of correlation for a five-year maturity basket with credit spreads of 25, 50, 100, 150, 250 and 500bp and equal recovery rates of 40%

1. Deltas calculated using semi-explicit formulas and Monte Carlo approaches

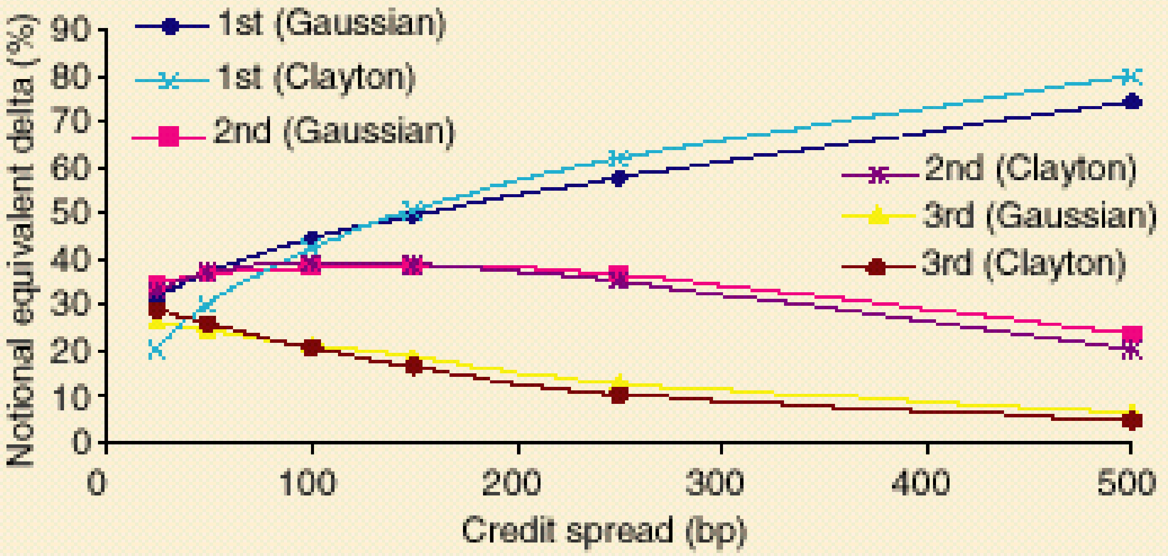


Comparison of deltas calculated using the analytical formulas and 105 Monte Carlo simulations for the example given in table A. The Monte Carlo deltas are calculated by applying a 10bp parallel shift to each curve

Credit Spread Hedges

- Changes in credit curves of individual names
  - *Dependence upon the choice of copula for defaults*

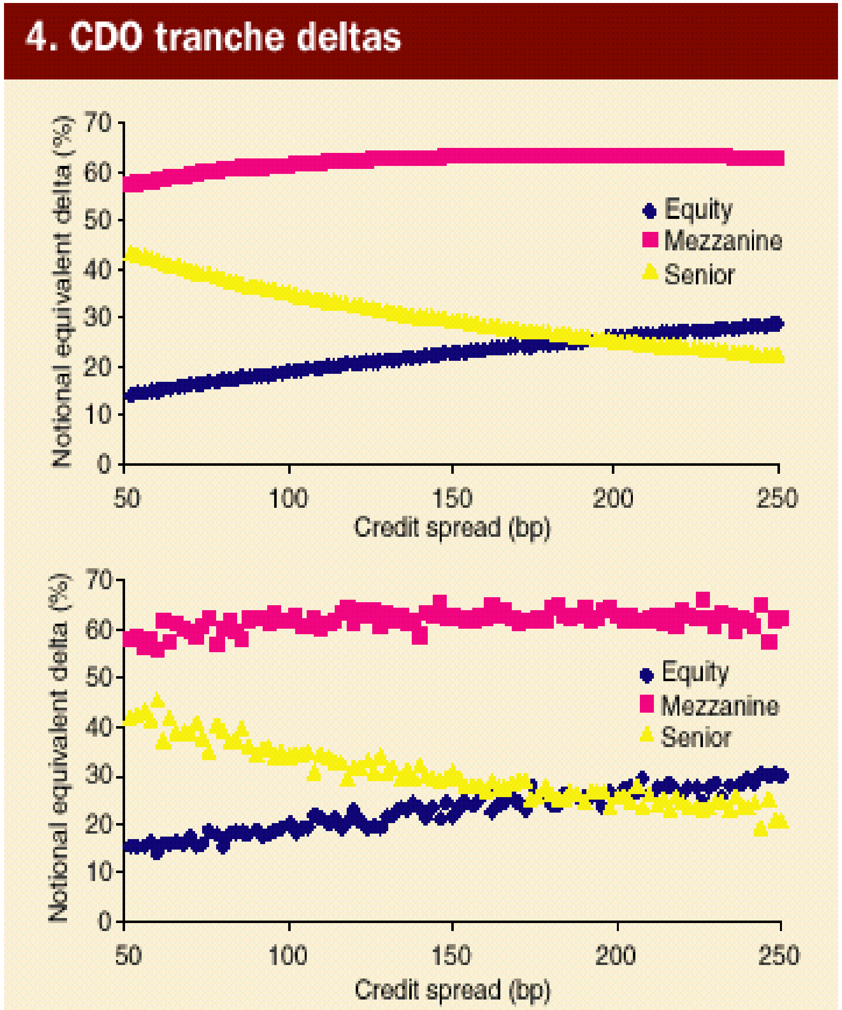
2. Deltas using Gaussian and Clayton copula



Comparison of deltas calculated using Gaussian (30% correlation) and Clayton copulas (θ = 0.27)

Credit Spread Hedges

- Hedging of CDO tranches with respect to credit curves of individual names
- Amount of individual CDS to hedge the CDO tranche
- Semi-analytic : some seconds
- Monte Carlo more than one hour and still shaky
  - *Importance sampling improves convergence but is deal specific*



CDO tranche deltas using the analytical method (top) and Monte Carlo (bottom) for a correlation of 50%

Correlation Hedges

- CDO premiums (bp pa)
  - *with respect to correlation*
  - *Gaussian copula*
  - *Attachment points: 3%, 10%*
  - *100 names, unit nominal*
  - *5 years maturity, recovery rate 40%*
  - *Credit spreads uniformly distributed between 60 and 150 bp*
- Equity tranche premiums decrease with correlation
- Senior tranche premiums increase with correlation
- Small correlation sensitivity of mezzanine tranche

ρ	equity	mezzanine	senior
0 %	6176	694	0.05
10 %	4046	758	5.8
30 %	2303	698	23
50 %	1489	583	40
70 %	933	470	56

Correlation Hedges

- TRAC-X Europe
  - *Names grouped in 5 sectors*
  - *Intersector correlation: 20%*
  - *Intrasector correlation varying from 20% to 80%*
  - *Tranche premiums (bp pa)*
- Increase in intrasector correlation
  - *Less diversification*
  - *Increase in senior tranche premiums*
  - *Decrease in equity tranche premiums*

$$\begin{pmatrix}
1 & 60\% & 60\% & & & & & & & \\
60\% & 1 & 60\% & & & & & & 20\% & \\
60\% & 60\% & 1 & & & & & & & \\
 & & & 1 & & & & & & \\
 & & & & \cdot & & & & & \\
 & & & & & \cdot & & & & \\
 & & & & & & 1 & & & \\
 & & & & & & & 1 & 60\% & 60\% \\
 & 20\% & & & & & & 60\% & 1 & 60\% \\
 & & & & & & & 60\% & 60\% & 1
\end{pmatrix}$$

	0-3%	3-6%	6-9%	9-12%	12-22%
20%	1273.9	287.5	93.4	33.3	6.0
30%	1226.6	294.4	102.7	39.9	7.9
40%	1168.9	303.5	114.0	47.3	10.3
50%	1100.5	314.2	127.6	56.3	13.3
60%	**1020.9**	**325.8**	**143.8**	**67.2**	**17.0**
70%	929.1	337.5	163.6	80.8	21.6
80%	821.9	349.3	188.0	98.8	27.2

Correlation Hedges

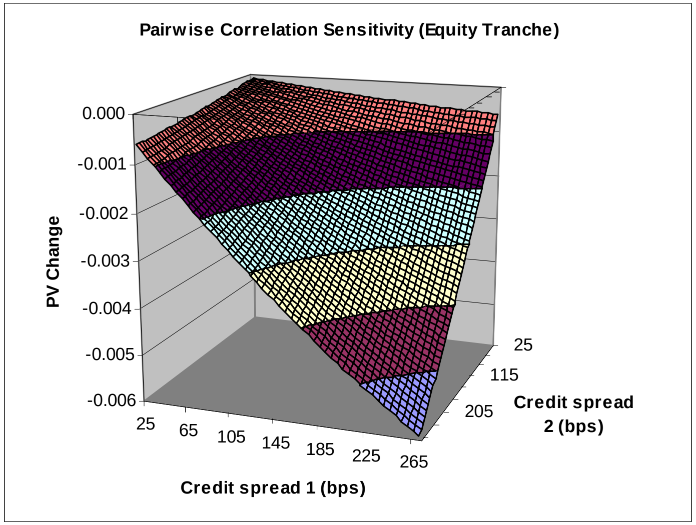
- Implied flat correlation
  - *With respect to intrasector correlation*
- * premium cannot be matched with flat correlation
  - *Due to small correlation sensitivities of mezzanine tranches*
- Negative corrrelation smile

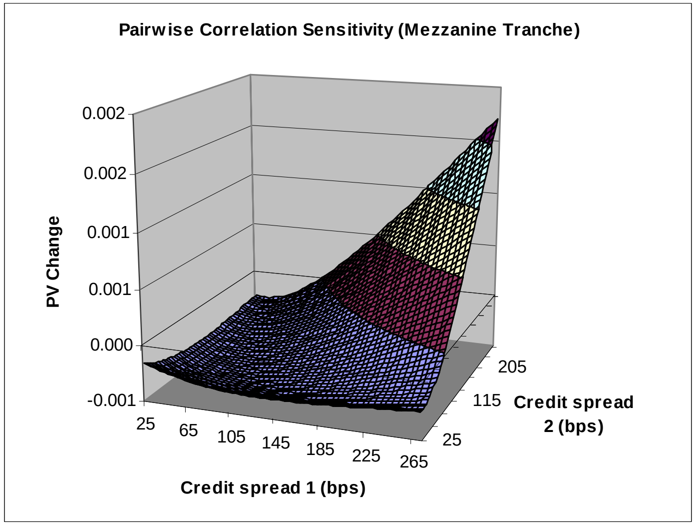
$$\begin{pmatrix} 1 & 60\% & 60\% & & & & & & & \\ 60\% & 1 & 60\% & & & & & & 20\% & \\ 60\% & 60\% & 1 & & & & & & & \\ & & & 1 & & & & & & \\ & & & & \cdot & & & & & \\ & & & & & \cdot & & & & \\ & & & & & & 1 & & & \\ & & & & & & & 1 & 60\% & 60\% \\ & 20\% & & & & & & 60\% & 1 & 60\% \\ & & & & & & & 60\% & 60\% & 1 \end{pmatrix}$$

	0-3%	3-6%	6-9%	9-12%	12-22%
20%	20.0%	20.0%	20.0%	20.0%	20.0%
30%	22.2%	22.6%	22.1%	22.2%	22.0%
40%	25.0%	27.6%	25.2%	24.6%	24.2%
50%	28.5%	*	29.7%	27.3%	26.8%
60%	32.8%	*	40.5%	30.6%	29.8%
70%	44.9%	*	*	34.8%	33.1%
80%	44.8%	*	*	41.3%	37.1%

Correlation Hedges

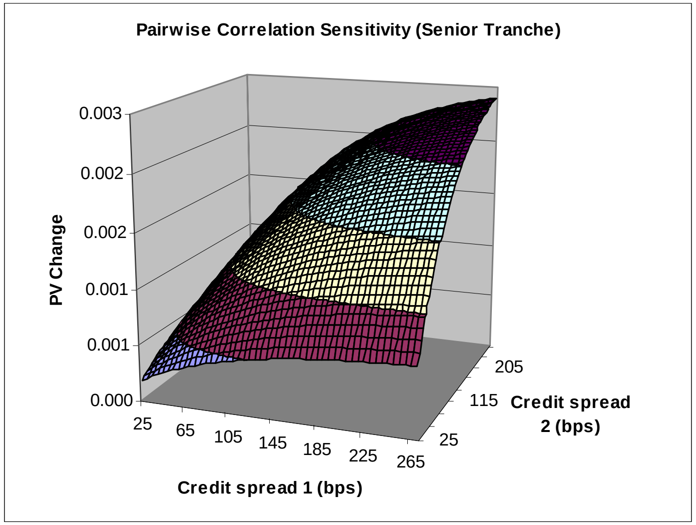
- <u>Correlation sensitivities</u>
  - *Protection buyer*
- 50 names
  - *spreads 25, 30,…, 270 bp*
- Three tranches:
  - *attachment points: 4%, 15%*
- Base correlation: 25%
- Shift of pair-wise correlation to 35%
- Correlation sensitivities wrt the names being perturbed
- equity (top), mezzanine (bottom)
  - *Negative equity tranche correlation sensitivities*
  - *Bigger effect for names with high spreads*





Correlation Hedges

- Senior tranche correlation sensitivities
  - *Positive sensitivities*
  - *Protection buyer is long a call on the aggregated loss*
    - Positive vega
  - *Increasing correlation*
    - Implies less diversification
    - Higher volatility of the losses
- Names with high spreads have bigger correlation sensitivities



Conclusion

- Factor models of default times:
  - *Deal easily with a large range of names and dependence structures*
  - *Simple computation of basket credit derivatives and CDO's*
    - Prices and risk parameters
- Gaussian and Clayton copulas provide similar patterns
- Shock models quite different