Pricing of sovereign defaultable bonds and stripping issues

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Presentation slides connected to the paper Sovereign recovery schemes: discounting and risk management issues Updated version to be available on <u>www.defaultrisk.com</u> Slides and paper also available on <u>http://laurent.jeanpaul.free.fr/</u> Pricing of sovereign defaultable bonds and stripping issues

- Aim of the presentation
  - Pricing of sovereign bonds with respect to coupon and maturity
  - Plain vanilla coupon bonds, principal and coupon strips
  - Accounting for specificities of default event
    - Change of currency
    - Debt restructuring through a bond swap
  - And CAC features, such as the new euro model CAC applicable from January 2013
  - Challenge current bond methodologies based on hypothetical default-free curves and Z – spreads
    - Discuss sovereign CDS stripping issues



• Choice of recovery rate, base curve, independent defaults and rates





F + C

- Defaultable level coupon bond
  - <u>Contractual</u> cash-flow schedule
  - *Face value F, coupon C*
  - *Payment dates* : 1,2, ..., *t*, ..., *T*



- Street method for pricing bonds
  - Postulates issuer specific risky discount factors B\*(i)



Bond price  $P_T^*$  given by  $P_T^* = C \times (\sum_{i=1}^T B^*(i)) + F \times B^*(T)$ 

## Street method

- Street method does not account explicitly for the payment to bondholders in case of default
  - Other methods are indeed being used for emerging markets
- As will be further shown, existence of risky discount factors is only valid for certain default schemes
  - Such as change of currency
  - And under the absence of arbitrage opportunities
- $P_T^* = C \times \left(\sum_{i=1}^T B^*(i)\right) + F \times B^*(T)$ 
  - Underlies BVAL and fair value market curves (Bloomberg)
    - Lee (2007), Ward (2010, 2011)
  - $\sum_{i=1}^{T} B^{*}(i)$  risky annuity, PV01, risky level





Street method



- $P_T^* = C \times \left(\sum_{i=1}^T B^*(i)\right) + F \times B^*(T)$  is not innocuous
  - $\sum_{i=1}^{T} B^{*}(i)$  risky annuity or PV01
  - Provides the dependence of bond price  $P_T^*$  wrt to coupon C
- Greek bonds, 23<sup>rd</sup> of November 2011
  - *Coupon rate* 3.7%, *maturing on* 20/07/2015
  - *Coupon rate* 6.1% *maturing on* 20/08/2015
- Both having the same clean price of 29% of face value
  - Discounting rule hardly consistent with market prices for distressed bonds
    - If sovereign financial distress cannot be expelled
    - By backward induction, discrepancies should translate to normal situations



## Street method

- One step beyond: Z spreads (OAS)
  - Given set of hypothetical default-free discount factors B(i)
  - B(i) can be derived by stripping swap curves
    - Which swap curve in today's multicurve setting?
    - OIS & Libor swaps, Futures for short maturities, collat. & uncollat IRS
  - B(i) can be derived from a base treasury curve
    - US treasuries, German bunds
    - Or bund rates minus German CDS?
  - Define risky discount factors  $B^*(i) = B(i)\exp(-z(i) \times i)$
  - z(i): Z-spread
    - Can occasionally become negative
    - Challenges the B(i) being default-free



## Getting the risky discount factors (bond street approach)

## Central banks provide Constant Maturity Treasury (CMT)

Par yield curves

Date	1 mo	3 mo	6 mo	1 yr	2 yr	3 yr	5 yr	7 yr	10 yr	20 yr	30 yr
02/01/2013	0.02	0.06	0.11	0.15	0.27	0.4	0.88	1.4	2.04	2.83	3.21

- <u>http://www.treasury.gov/resource-center/data-chart-center/interes</u> <u>rates/Pages/TextView.aspx?data=yield</u>
  Daily Treasury Yield Curve
- US Treasuries
- On the right, estimated par rates
- Computed from « on the run » securities
- And interpolation methods
  - "Treasury does not provide the computer formulation of our quasi-cubic Hermite spline yield curve derivation program"





## Getting the risky discount factors (bond street approach)

• Greek bond prices and yields: which data should we interpolate?

Trade.MaturityDate	Chae	Live	Basis	Chg	Yield
20/03/2012	41.00	35.25	-14,268	-5.75	1,724.48
18/05/2012	32.00	30.00	-3,803	-2.00	809.17
20/08/2012	28.00	26.00	-1,906	-2.00	497.77
20/05/2013	26.50	24.50	-532	2.00	205.52
20/08/2013	25.00	24.00	-616	-1.00	162.80
11/01/2014	26.00	24.00	-488	-2.00	122.42
20/05/2014	23.00	22.00	-421	-1.00	105.41
20/08/2014	23.00	22.00	-393	-1.00	95.51
20/07/2015	24.00	22.00	-237	-2.00	61.78
20/08/2015	23.00	22.00	-261	-1.00	69.03
20/07/2016	23.00	22.00	-194	-1.00	49.23
20/04/2017	23.00	22.00	-147	-1.00	49.18
20/07/2017	23.00	22.00	-151	-1.00	42.73
20/07/2018	23.00	22.00	-127	-1.00	38.10
19/07/2019	23.00	22.00	-113	-1.00	38.38
22/10/2019	23.00	22.00	-129	-1.00	39.13
19/06/2020	23.75	24.00	-88	0.25	35.75
22/10/2022	22.50	22.50	-101	0.00	31.98
20/03/2024	22.00	22.00	-76	0.00	26.97
25/07/2025	22.00	21.00	-67	-1.00	19.81
20/03/2026	22.00	22.00	-67	0.00	27.36
25/07/2030	21.25	21.00	-54	-0.25	15.21
20/09/2037	22.00	22.00	-48	0.00	20.93
20/09/2040	22.00	22.00	-45	0.00	21.12



## Getting the risky discount factors (bond approach)

- Once the (theoretical, untraded) par rates are derived for 1Y, 2Y, ... maturities
- One can derive discount factors by bootstrapping
  - Given 1 and 2 year maturity coupon bonds
  - Short-sell 1 year maturity bond to get rid of first coupon on a 2 year maturity bond
- Provides a 2 year discount bond
  - Standard textbook approach
- Does the financial engineering stripping approach hold if default occurs (prior to one year)?
  - Requires a further investigation of cash-flows in default



What are the default features of synthetic strips?

## Pre-default cash-flows

- Further investigation of actual bond cash-flows required
- Pre-default cash-flows are contractual cash-flows paid until default time τ



• On dates t = 1, ..., T - 1, payment of  $C \times 1_{\tau > t}$ 



• On date t = T, payment of  $(F + C) \times 1_{\tau > T}$ 

Pre-default cash-flows

• Pre-default cash-flows are contractual cash-flows paid until default time  $\tau$ 



- At default time  $\tau$ , if  $\tau \leq T$ , a default payment is made
- Payment will depend on the recovery scheme
  - Recovery of face value
  - Exit of eurozone (change of currency)







Forced conversion  $\delta < 1$  $\delta = 1, \delta > 1$ ? Ostmark parity, German reunification

- Default date  $\tau$  corresponds to exiting the eurozone
- Euro payments are converted to « new euros », i.e. old drachmas
- After default, cash-flows are scaled down by a factor  $\delta$ 
  - $\delta < 1$  acts as an exchange rate
- Same scaling factor applied to coupon and principal payments



- For a contractual payment of 1 with scheduled maturity *t*, defaultable bond pays
  - 1 at maturity t if  $\tau > t$
  - $\delta$  at same maturity t if  $\tau \leq t$  (recovery at scheduled maturity)
- Actual payment of  $1_{\tau > t} + \delta 1_{\tau \le t}$  at date *t* 
  - Defaultable discount bonds of Jarrow and Turnbull (1995)
    - One could either compute the present value at time  $\tau$  of  $\delta$ 
      - Using a discount rate reflecting post-default credit risk of issuer
      - "cash-settlement" instead of bond settlement"
    - Or consider the date t exchange rate  $\delta_t$  and receive  $\delta_t$  at t

Same building blocks for coupon and principal payments

 $B^*(t)$  today's price of above defaultable discount bond

- Defaultable coupon bond with price *P* 
  - Linear combination of defaultable discount bonds of Jarrow and Turnbull type

• 
$$P = C \times \left(\sum_{t=1}^{T} B^*(t)\right) + F \times B^*(T)$$

- B\*(t) : defaultable discount factor
- $\sum_{t=1}^{T} B^{*}(t) : PV \text{ of defaultable annuity}$
- By definition, par rate *y* fulfills:

• 
$$F = yF \times \left(\sum_{t=1}^{T} B^*(t)\right) + F \times B^*(T)$$
  
•  $P = F + (C - yF) \times \left(\sum_{t=1}^{T} B^*(t)\right)$ 



- Exit of eurozone scenario complies with Elton and Green (1998) statement
  - "Cash-flows of non-callable treasury securities are fixed and certain, simplifying the pricing of these assets to a present value calculation".
- Why so?
  - Discount bond payments are stochastic if expressed in  $\in$
  - But there are not if expressed in drachmas
    - Prior to euro, drachma with floating exchange rate to euro
    - During the single currency episode, fixed exchange rate
    - Exit of eurozone, back to floating exchange rates
  - Correct numéraire is then drachma
    - Default risk disappears and we can rightfully apply textbook methods for default-free bonds
    - Usefulness of capital charges on sovereign bonds in local currency?



- Exit of eurozone becomes less likely
  - Even for major issuers
  - ECB, ESM, CAC (easier private sector involvement)
  - Debt restructuring is increasingly being considered as a debt management tool
  - Greek bond swap
  - Emerging market techniques apply to emerged issuers too
- Recovery of face value important kind of credit event
  - With some adaptation for sovereigns
  - Local law bonds, with and without CAC
  - Possibility of selective default: bonds of short or long maturities may be excluded from bond forced exchange



- Recovery of Face Value (RFV)
- Standard recovery mechanism for corporates
  - At default time  $\tau$ , a fraction  $\delta$  of the face value F
  - Standard assumption for corporate bonds
  - Principal acceleration
    - The principal payment can be claimed immediately
  - Loss of any right on any further coupon payment
    - Zero recovery on coupon payments
  - Different treatment for coupon and principal payments
- At default, all bonds have the same value, irrespective of maturity and coupon rate



Distressed bonds trade on price and not on yields

### Greek default consistent with recovery of Face Value

Trade.MaturityDate	Close	Live	Basis	Chg	Yield
20/03/2012	41.00	35.25	-14,268	-5.75	1,724.48
18/05/2012	32.00	30.00	-3,803	-2.00	809.17
20/08/2012	28.00	26.00	-1,906	-2.00	497.77
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20/09/2040	22.00	22.00	-45	0.00	21.12

## Sovereign recovery schemes

## Heterogeneity of sovereign debt

- Different default dates
- Local law and foreign law
  - Selective (S&P) or restrictive defaults (Fitch)
- Payment at default may depend upon the bond
  - Coupon, maturity, bond holder identity
  - Individuals, Hedge fund, ECB (implied seniority), local banks
- Different recovery rates
  - Strips and level coupon bonds ...





## Sovereign recovery schemes

G10 recommendation (Other relevant features in italics)	UK	Italy	Mexico	Uruguay	Brazil	Belize	South Africa	Turkey	Poland	Korea
Governing law	English	NY	NY	NY	NY	NY	NY	NY	NY	NY
Permanent bondholders' representative										
Bondholders' negotiating representative elected by 2/3 of bondholders										
Bondholders' meeting on request of 10% of bondholders										
Majority action provisions for amendments to reserved matters with 75% vote										
List of reserved matters	*	*	*	*	*	*	*	*	*	*
Majority action on non-reserved matters with 66 2/3% vote				*						
Non-material amendments may be made without the bondholders' consent										
Acceleration requires support of 25% of bondholders	*			*						
Rescission of acceleration by 66 2/3 % of bondholders	*		*				*		*	*
Litigation to be instituted solely by the permanent representative										
Majority action on continuation and outcome of litigation	*									
Pro rata distribution of proceeds										
Disenfranchisement provision		*		*						
Information provision – to be included on a case by case basis				*						
Events of default										

#### Collective action clauses in some recent sovereign bonds - comparison with G10 recommendations



Same as	Different	Mixed	*	Some
G10 in	from		1995.C	minor
substance	G10			variation

- CDS triggering event
- ISDA two-step auction to determine settlement price
- Auction settlement price =  $\delta$  (recovery rate)

#### Final Results of the Hellenic Republic CDS Auction, 19 March 2012

Dealer	Bid	Offer	Dealer
Bank of America N.A.	21.625	23.625	Bank of America N.A.
Barclays Bank PLC	21.0	23.0	Barclays Bank PLC
BNP Paribas	20.75	22.75	BNP Paribas
Citigroup Global Markets Limited	20.5	22.5	Citigroup Global Markets Limited
Credit Suisse International	20.25	22.25	Credit Suisse International
Deutsche Bank AG	20.25	22.25	Deutsche Bank AG
Goldman Sachs International	21.125	23.125	Goldman Sachs International
HSBC Bank PLC	20.25	22.25	HSBC Bank PLC
JPMorgan Chase Bank N.A.	21.25	23.25	JPMorgan Chase Bank N.A.
Morgan Stanley & Co. International PLC	21.0	23.0	Morgan Stanley & Co. International PLC
Nomura International PLC	20.0	22.0	Nomura International PLC
Société Générale	21.0	23.0	Société Générale
The Royal Bank of Scotland PLC	22.0	24.0	The Royal Bank of Scotland PLC
UBS AG	20.5	22.5	UBS AG



#### Net Open Interest

EUR 291.6 million to sel







 Merrick (2001), Andritzky (2005), Vrugt (2011) in the context of sovereign bond pricing (emerging markets)

# • Principal payment $F \text{ if } \tau > T$ • Principal payment T

- Recovery of Face Value scheme
- Payment of 1 at maturity T if  $\tau > T$ 
  - Pre-default payment
- Payment of  $\delta$  at  $\tau$  if  $\tau \leq T$ 
  - Default payment, principal acceleration
- Principal payment in Recovery of Face Value Scheme
  - zero-coupon with uncertain maturity and payoff



• Payoff depends on unknown default date  $\tau$  and recovery rate  $\delta$ 

- Stripping coupon payments for defaultable bonds
  - Payments of C on coupon payment dates until default date τ or maturity T
  - Stream of payments  $C \times 1_{\tau > t}$ , paid at dates t = 1, ..., T
  - No claim on coupons after default
  - Zero-recovery defaultable annuity
- Coupon payments defaultable annuity with zero recovery
  - Can be further stripped into defaultable discount bonds with zero-recovery





## Defaultable coupon bond with price *P*

Coupon bond = linear combination of coupon and principal payments

• 
$$P = C \times \left(\sum_{t=1}^{T} B^{C}(t)\right) + F \times B^{P}(T)$$

- $\sum_{t=1}^{T} B^{C}(t)$  : PV of defaultable annuity with zero-recovery
- *B<sup>C</sup>(t)* price of defaultable discount bond with maturity *t* and zero-recovery
- $B^{P}(T) > B^{C}(T)$ : Extra-payment of  $\delta F$  if  $\tau \leq T$
- <u>Two discount curves</u>: coupons and principal payments



This is not consistent with applying a same defaultable discount factor to all bond payments with maturity t

Financial engineering: coupon rate sensitivity

Bond pricing formulas : recovery of face value

• 
$$P = C \times \sum_{t=1}^{T} B^{C}(t) + F \times B^{P}(T)$$
  
•  $F = yF \times \sum_{t=1}^{T} B^{C}(t) + FB^{P}(T)$ 

By definition of par rate y

- $\Rightarrow P = F + (C yF) \times \left(\sum_{t=1}^{T} B^{C}(t)\right)_{\mathsf{r}}$
- Remind bond price with exit of eurozone
- $P = F + (C yF) \times \left(\sum_{t=1}^{T} B^*(t)\right) \checkmark$ 
  - $\sum_{t=1}^{T} B^{C}(t) < \sum_{t=1}^{T} B^{*}(t)$
  - Assessment of coupon rate sensitivity

Different price sensitivities to coupon rate Bond origination



- Consider a set of defaultable coupon bonds
  - With different coupon rates and maturities
  - Assume recovery of face value mechanism (say)
- Assume no arbitrage opportunities within bonds
  - Then, there exists some <u>positive</u> defaultable discount factors B<sup>C</sup>(t), B<sup>P</sup>(t)
  - Such that price P of bond with coupon C, face value F and maturity T fulfills discounting rule:

• 
$$P = \sum_{t=1}^{T} C \times B^{C}(t) + F \times B^{P}(T)$$

- Thanks to a suitable application of Farkas' lemma
  - The reverse proposition is obvious



- Hypothetical derivation of principal strips from coupon bonds
  - Level coupon bonds with same maturity, different coupon rates need to be traded
- Use two level coupon bonds
  - Same maturity T, face value 1
  - Coupon rates  $c, c^* > c$
- Buy  $c^*/(c^* c)$  level coupon bond with coupon rate c
- Sell  $c/(c^* c)$  level coupon bond wit coupon rate  $c^*$
- This replicates the maturity *T P* −Strip



• Model-free computation of  $B^P(T)$ ,  $B^C(t)$ 

- Computation of discount factors for principal and coupon payments
  - Two (almost) independent curves  $t \rightarrow B^{P}(t)$ ,  $B^{C}(t)$  need to be calibrated
- CDS approach applied to bonds
  - *Extra layer of modelling* in the RFV approach
  - B(t): default-free discount factors given
  - *Default time τ independent of default-free rates*
  - $S(t) = Q(\tau > t)$  survival probabilities
  - $B^{C}(t) = B(t) \times S(t)$
  - Constant recovery parameter δ



•  $B^P(t) = B^C(t) - \delta \times F \int_0^t B(s) dS(s)$ 

Based on Italian government bond prices, Feb. 2013



CDS stripping approach to bonds Recovery parameter = 40% Blue curve: discount rates for coupon payments **Red curve: discount rates for principal** payments Intermediate curve: same discounting rate for coupon and principal payment Intermediate curve corresponds to zero recovery rate huge impact of recovery rate on strips Intermediate curve also corresponds to exit of eurozone scenario

- Different features in markets for strips
  - Principal and coupon strips may or may not be fungible
- If *P* and *C* strips are fungible
  - They have the same price by necessity
    - Same ISIN/CUSIP number
- If *P* and *C* strips are not fungible
  - They should have the same price if change of currency is the privileged default scenario
  - Quite different prices (previous slide) if bond restructuring (RFV) is the market default scenario
    - Unless recovery rate equals zero
  - Fungibility can be introduced after issuance
    - Legal uncertainty about rights of strip holders



- Even when *P* and *C* strips are not fungible, differences in prices are far below one could expect under a <u>bond swap</u>
  - Debt restructuring with RFV as in Greek case example
  - And most emerging markets debt restructurings

Discrepancies between principal and coupon strips – US Treasuries Maturing on 15 February 2031, differences in yields are capped by 10 bps





New CAC in the eurozone provides further insights

- CAC specifies voting rights for debt holders
  - Strips and level coupon bonds
- Voting rights likely to be connected to recovery payments
- <u>Different computations</u> for level coupon bonds and strips
- For level coupon bonds, voting rights are based on nominal value, irrespective of coupon rate
- Voting rights of strips are based on discounted value of contractual payments
  - As in change of currency case, recovery of "cash-flows"
- Discount rate based on coupon rate structure at default date



• Uncertainty on the applicable discount rate

• New CAC in the eurozone provides further insights (cont.)

- As a consequence, voting rights of coupon and principal strips will have be equal
- Leading to same recovery and same prices even without fungibility
- Fungibility also leads to same recovery on principal and coupon strips
  - By necessity
- Potential inconsistencies in prices of coupon bonds and strips
  - Coupon bond: recovery of face value (usual bond swap)
  - Strips: recovery of cash-flows as in change of currency



• Next slide provides an example based on two French OATs

Same payment dates and maturity, two different coupons

- Prices of two bonds need to be <u>equal</u> at maturity under RFV
- Same bonds reconstituted from strips
  - High coupon bond price will be <u>higher</u> than low coupon bond price

Bloomberg Name	FRTR 8.5 10/19	FRTR 3.75 25/10/19		
Bond Type Name	France Govt	France Govt		
Coupon	8.5	3.75		
Coupon Length	12	12		
Issue Date	25-Jan-89	07 <b>-</b> Jul-09		
First Settlement Date	25-Jan-89	07-Jul-09		
First Coupon Date	25-Oct-89	25-Oct-09		
Maturity	25-Oct-19	25-Oct-19		
Issue Price	95.8	99.37		
Callable	FALSE	FALSE		
Outstanding Amount	8844392893	28078000000		
ISIN	FR0000570921	FR0010776161		
Par Value	100	100		
Issuer Location	PAR	PAR		
ECB Eligibility	TRUE	TRUE		
DB Currency	EUR	EUR		



Same payment dates and maturity, two different coupons

• Actually under the new CAC, three different recovery basis

- 1 for the traded level coupon bond
- $\sum_{i=1}^{n} C_2 \times B_N(i) + B_N(n)$  for bond with coupon  $C_2 = 8.5\%$
- $\sum_{i=1}^{n} C_1 \times B_N(i) + B_N(n)$  for bond with coupon  $C_1 = 3.75\%$

• Illustrative example since new CAC only apply to new issues

- Arbitrage opportunities are far from being granted
  - Unlikely that stripping / reconstitution would be allowed around a debt restructuring
- We have left aside connections between strips and CDS markets



 Yet, close inspection of stripping methodologies, bond and CDS pricers is likely to be required

## Conclusion

Simple pricing formulas for defaultable bonds

- As a function of coupon rate and maturity
- Consistent with quoted prices of traded bonds
- Model-free with respect to distribution of default-date
- No need of default-free bonds
- Building blocks depend on recovery scheme
  - Recovery of face value, Greek bond swap
    - Different discount rates for principal and coupon payments
  - *Exit of Eurozone scenario* 
    - Jarrow & Turnbull (Recovery of Treasury) defaultable discount factors
- $\neq$  bond prices w.r.t. coupon and maturity



Consistency issues with strip and CDS markets