Alternative Risk Measures for Alternative Investments

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Outline

- **Optimizing under VaR constraints**
  - **Estimation techniques**
  - **VaR analytics and efficient portfolios comparison**
- **Optimizing under alternative risk constraints**
  - **Expected Shortfall, Downside Risk measure,…**
  - **Risk measures analytics and efficient portfolios comparison**

Data set

16 individual Hedge Funds

Fund	Style	Mean	Std	Skewness	Kurtosis	Granger VaR	ES	Correl / underlying index
AXA Rosenberg	Equity Market Neutral	5,61%	8,01%	0,82	13,65	3,72%	5,59%	-28,36%
Discovery MasterFund Ltd	Equity Market Neutral	6,24%	14,91%	-0,27	0,25	6,78%	8,98%	3,27%
Aetos Corp	Event Driven	12,52%	8,13%	-1,69	7,78	2,73%	5,17%	34,05%
Bennett Restructuring	Event Driven	16,02%	7,48%	-0,74	7,37	1,79%	3,67%	64,15%
Calamos Convertible	Convertible Arbitrage	10,72%	8,09%	0,71	2,59	3,14%	4,24%	32,75%
Sage Capital	Convertible Arbitrage	9,81%	2,45%	-3,19	3,00	0,60%	1,05%	52,30%
Genesis Emerging Markets	Emerging Markets	10,54%	20,03%	-3,34	6,40	8,44%	13,15%	88,06%
RXR Secured Note	Fixed Income Arbitrage	12,29%	6,45%	2,33	4,84	1,84%	2,84%	1,14%
Arrowsmith Fund	Funds of Funds	26,91%	27,08%	14,51	60,70	6,67%	12,84%	
Blue Rock Capital	Funds of Funds	8,65%	3,47%	1,66	7,51	0,76%	1,40%	
Dean Witter Cornerstone	Global Macro	13,95%	23,19%	7,42	9,17	7,55%	8,78%	31,62%
GAMut Investments	Global Macro	24,73%	14,43%	3,38	4,61	4,45%	6,27%	57,58%
Aquila International	Long Short Equity	9,86%	16,88%	-1,22	2,32	7,99%	10,98%	72,07%
Bay Capital Management	Long Short Equity	10,12%	19,31%	1,94	0,70	7,31%	9,68%	27,85%
Blenheim Investments LP	Managed Futures	16,51%	29,59%	3,07	10,25	11,80%	17,47%	22,77%
Red Oak Commodity	Managed Futures	19,80%	29,08%	1,94	3,52	11,33%	16,00%	21,60%

Hedge funds summary statistics

Data structure

- monthly data
- 139 observations

Non Gaussian features (confirmed by Jarque Bera statistics)

Wide range of correlation with the CSFB tremont indexes

Data set (2)

- **Rank correlation**

	Skewness	Kurtosis	Std	Semi-variance	Granger VaR	ES
Skewness	100%	38%	40%	32%	23%	25%
Kurtosis	38%	100%	15%	15%	-3%	6%
Std	40%	15%	100%	99%	93%	95%
Semi-variance	32%	15%	99%	100%	95%	98%
Granger VaR	23%	-3%	93%	95%	100%	96%
ES	25%	6%	95%	98%	96%	100%

- **Risk measured with respect to kurtosis and VaR are almost unrelated**
- **Std, semi-variance, VaR and ES are almost perfect substitutes for the risk rankings of hedge funds**

Data set (3)

- **Correlations**

100	14	-3	0	-20	-13	-33	-4	-15	5	7	8	-24	-7	-2	-28
14	100	-1	-13	-5	-2	-5	6	-10	28	11	-2	3	2	-5	7
-3	-1	100	35	35	21	31	12	16	11	1	-13	30	-5	0	12
0	-13	35	100	25	31	40	-3	16	-4	3	-8	31	9	-4	11
-20	-5	35	25	100	40	50	21	18	30	-15	5	49	12	1	16
-13	-2	21	31	40	100	40	8	14	7	2	1	34	-5	-2	2
-33	-5	31	40	50	40	100	15	22	10	-8	-9	64	14	16	31
-4	6	12	-3	21	8	15	100	8	25	37	41	25	10	11	31
-15	-10	16	16	18	14	22	8	100	1	-9	-1	17	7	-1	11
5	28	11	-4	30	7	10	25	1	100	8	1	21	13	-5	13
7	11	1	3	-15	2	-8	37	-9	8	100	37	-3	7	6	17
8	-2	-13	-8	5	1	-9	41	-1	1	37	100	-2	10	22	29
-24	3	30	31	49	34	64	25	17	21	-3	-2	100	14	6	24
-7	2	-5	9	12	-5	14	10	7	13	7	10	14	100	10	9
-2	-5	0	-4	1	-2	16	11	-1	-5	6	22	6	10	100	50
-28	7	12	11	16	2	31	31	11	13	17	29	24	9	50	100

Correlation matrix

- **Wide range of correlations**
  - Some of them negative

Data set (4)

- **Betas with respect to the S&P 500 index**

Funds	Beta	t
AXA Rosenberg	-0,14	3,08
Discovery MasterFund Ltd	0,02	0,17
Aetos Corp	0,25	5,17
Bennett Restructuring	0,16	3,33
Calamos Convertible	0,37	9,22
Sage Capital	0,07	3,55
Genesis Emerging Markets	0,78	7,79
RXR Secured Note	0,21	5,21
Arrowsmith Fund	0,37	2,28
Blue Rock Capital	0,09	3,78
Dean Witter Cornerstone	-0,03	0,22
GAMut Investments	0,06	0,67
Aquila International	0,69	8,42
Bay Capital Management	0,24	2,10
Blenheim Investments LP	0,10	0,56
Red Oak Commodity	0,70	4,23

- **12 funds have a significant positive exposure to market risk, but usually with small betas.**

- **Results of a Principal Component Analysis with the correlation matrix**

  - **8 factors explain 90% of variance**
  - **13 factors explain 99% of variance**
    - ➔ **high potential of diversification**
    - ➔ **some assets are not in the optimal portfolios but may be good substitutes**

  - **Factor-loadings lead to a portfolio which is high correlated with the S&P 500 (60%)**

Value at Risk estimation techniques

- **Empirical quantile**
  - Quantile of the empirical distribution
- **“L-estimator” (Granger & Silvapulle (2001))**
  - Weighted average of empirical quantiles
- **Kernel smoothing: (Gourieroux, Laurent & Scaillet (2000))**
  - Quantile of a kernel based estimated distribution
- **Gaussian VaR**
  - Computed under the assumption of a Gaussian distribution

VaR estimators analysis (1)

- **We denote by $(a'r)_{1:n} \leq \ldots \leq (a'r)_{n:n}$ the rank statistics of the portfolio allocation *a***
- **VaR estimators depend only on the rank statistics**
- **VaR estimators are differentiable and positively homogeneous of degree one (with respect to the rank statistics)**

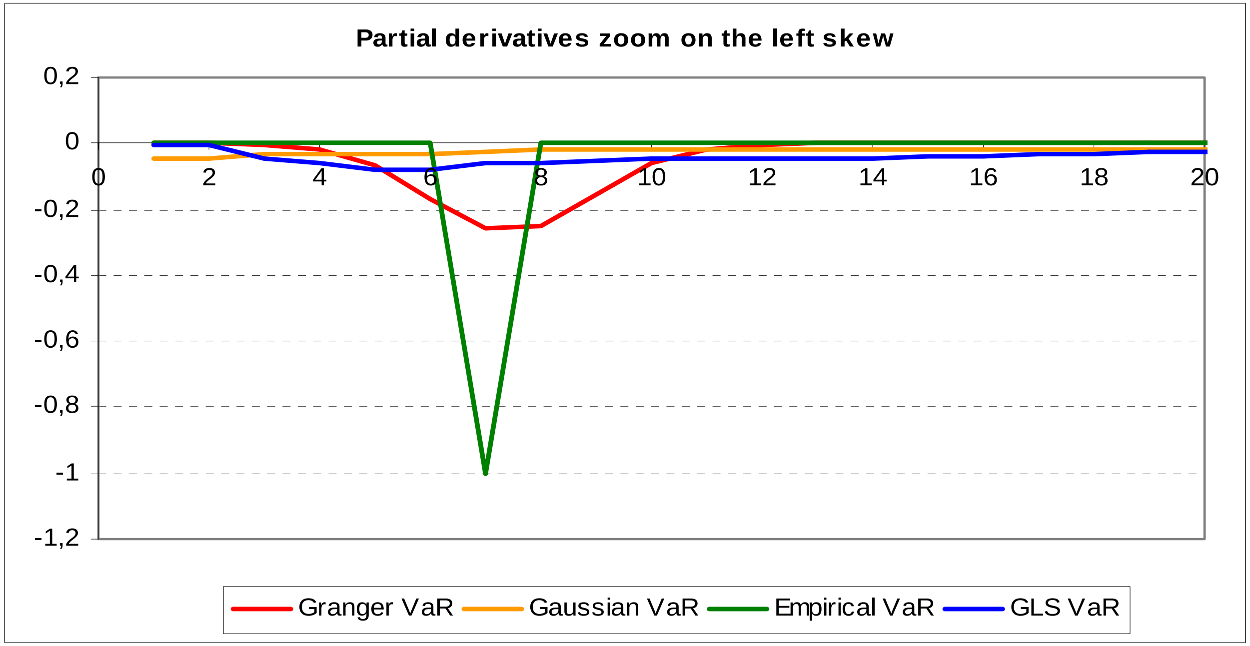
Thus, we can decompose VaR using Euler 's equality :

$$VaR(a'R) = \sum_{i=1}^{n} \frac{\partial VaR(a'R)}{\partial (a'r)_{i:n}} (a'r)_{i:n}$$

see J-P. Laurent [2003]

VaR estimators analysis (2)

- Weights associated with the rank statistics for the different VaR estimators

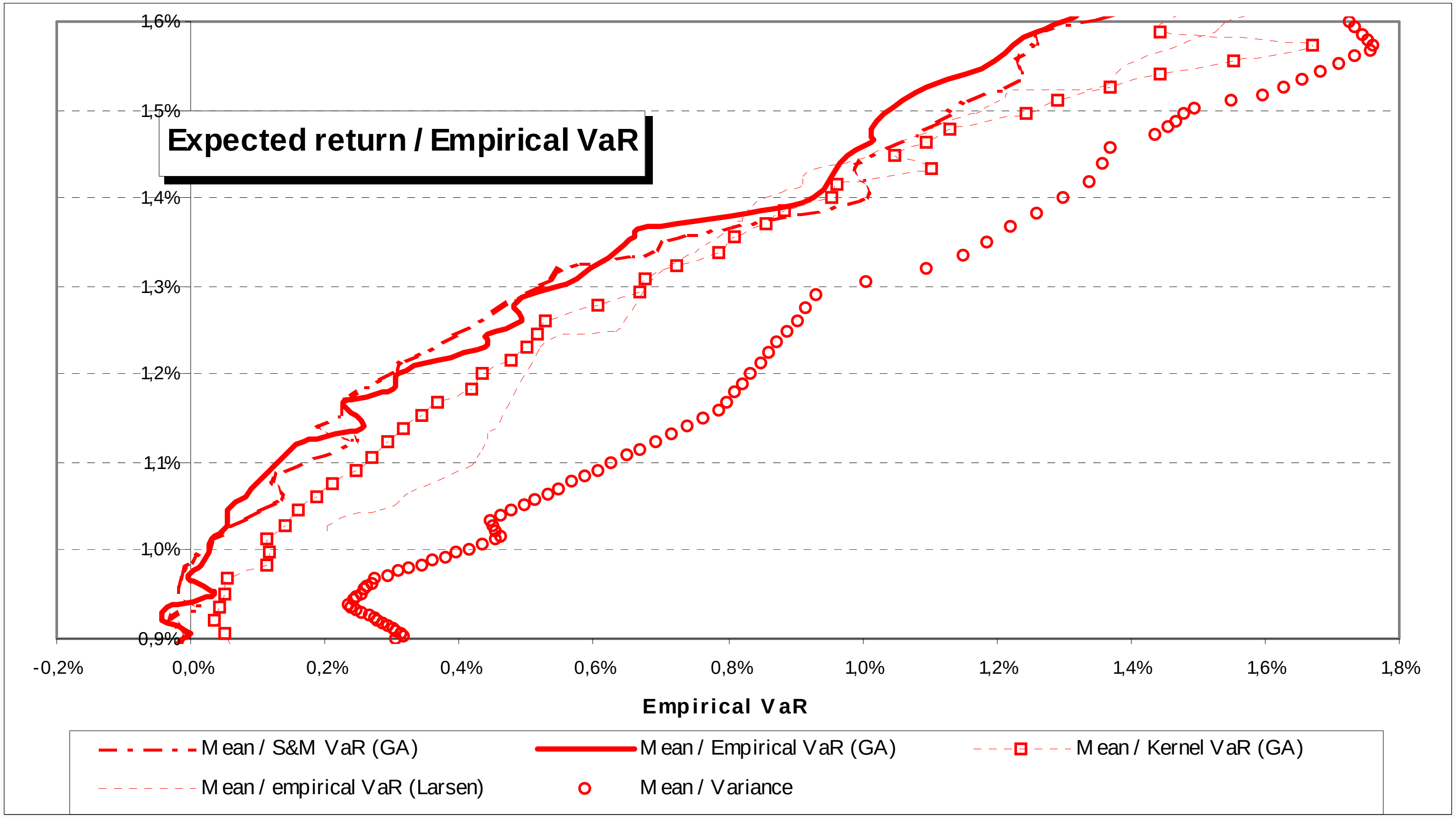


- Empirical VaR is concentrated on a single point
- Granger VaR is distributed around empirical VaR
- GLS VaR : smoother weighting scheme
- Gaussian VaR involves an even smoother pattern

Mean VaR optimization

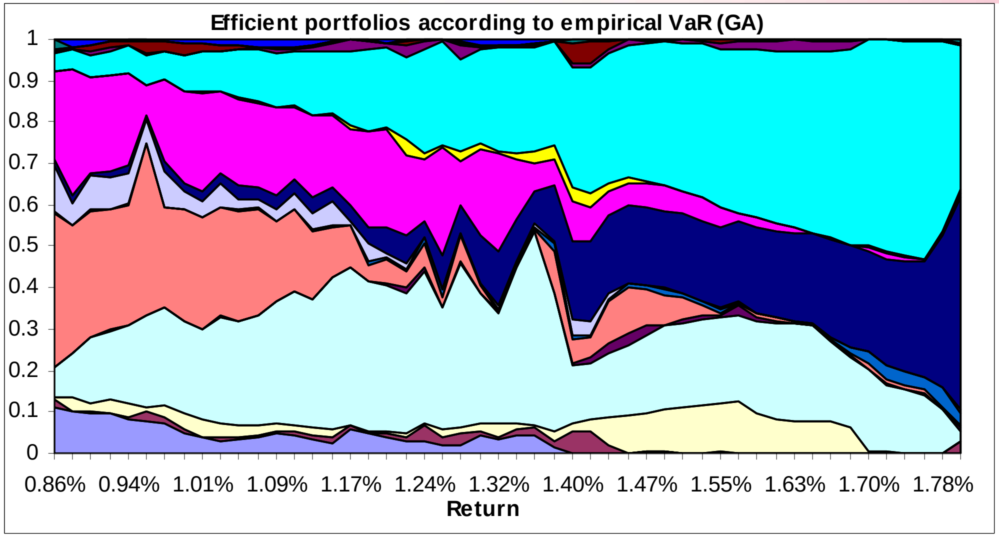
- **A non-standard optimization program**
  - **VaR is not a convex function with respect to allocation**
  - **VaR is not differentiable**
  - **Local minima are often encountered**
- **Genetic algorithms**
  - **(see Barès & al [2002])**
  - **Time consuming: <u>slow convergence</u>**
  - **1 week per efficient frontier**
- **Approximating algorithm Larsen & al [2001]**
  - **Based on Expected Shortfall optimization program**
  - **We get a <u>sub-optimal solution</u>**

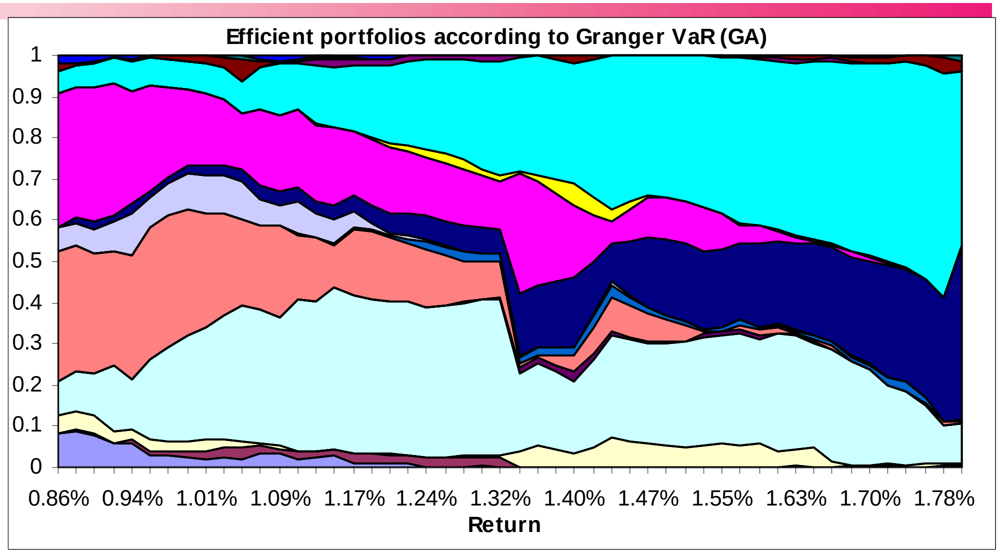
Mean VaR efficient frontier

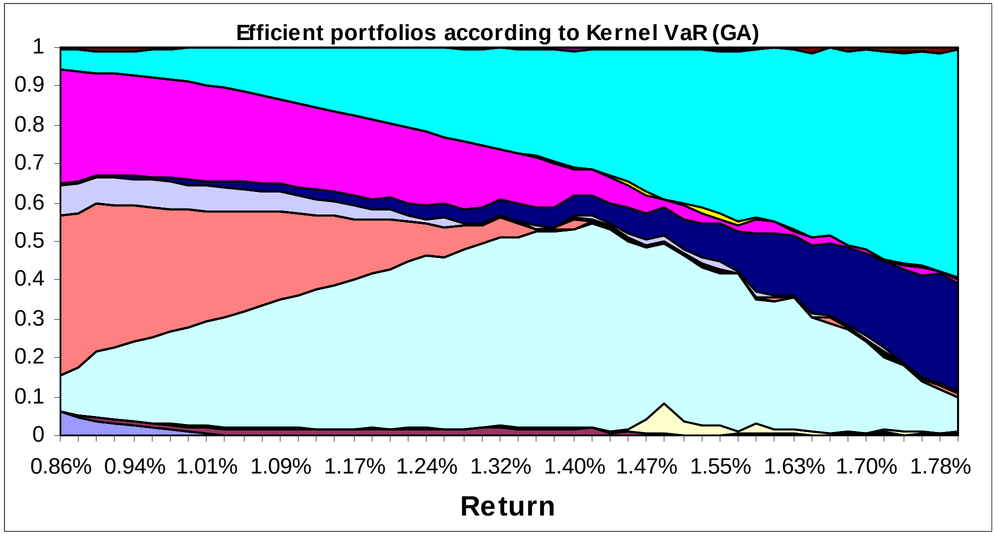


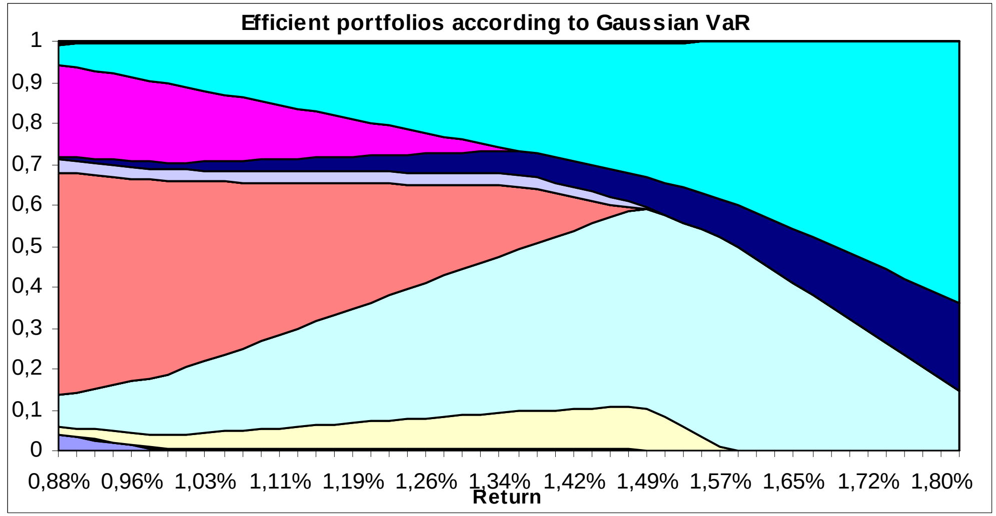
- **VaR efficient frontiers are close**
- **Far from the mean-Gaussian VaR efficient frontier**
- **Larsen & al. approximating algorithm performs poorly**

Mean VaR efficient portfolios (1)









- AXA Rosenberg Market Neutral Strategy LP
- Bennett Restructuring Fund LP
- Genesis Emerging Markets Fund Ltd
- Blue Rock Capital Fund LP
- Aquila International Fund Ltd
- Red Oak Commodity Advisors Inc
- Discovery MasterFund Ltd
- Calamos Convertible Hedge Fund LP
- RXR Secured Participating Note
- Dean Witter Cornerstone Fund IV LP
- Bay Capital Management
- Aetos Corporation
- Sage Capital Limited Partnership
- Arrowsmith Fund Ltd
- GAMut Investments Inc
- Blenheim Investments LP (Composite)

Mean VaR optimal portfolios (2)

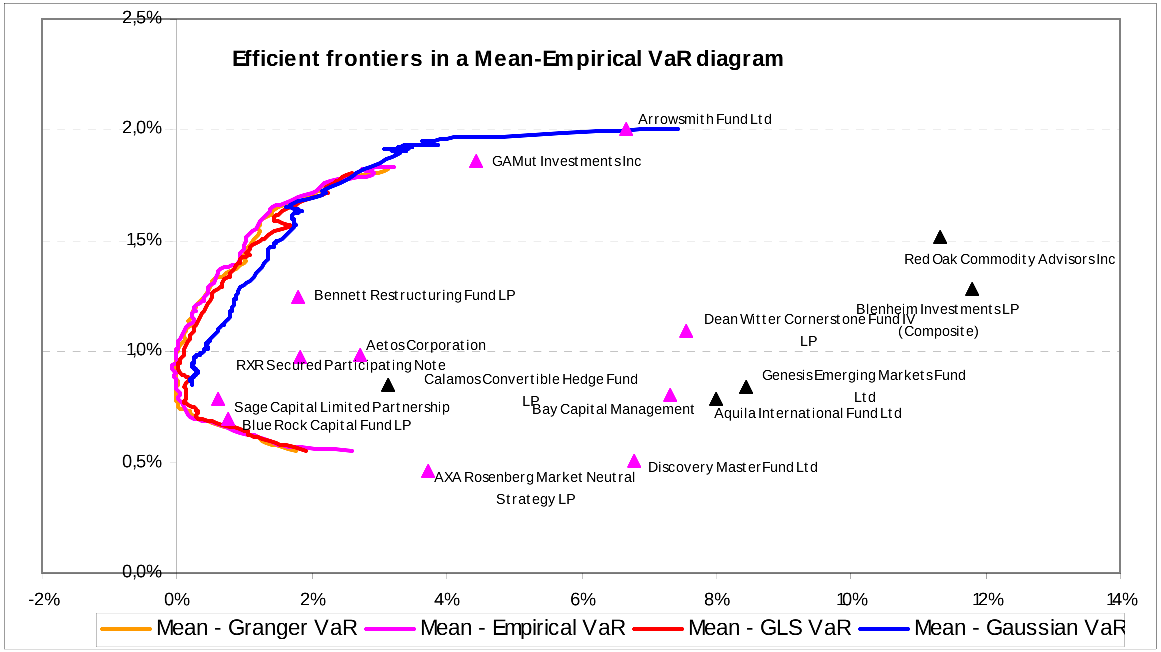
- **Optimal allocations with respect to the expected mean**
  - **Empirical VaR leads to portfolio allocations that change quickly with the return objectives**
  - **GLS VaR leads to smoother changes in the efficient allocations**
  - **Gaussian VaR implies even smoother allocation**

Optimal allocations

- **Almost the same assets whatever the VaR estimator**

Funds	Gaussian VaR	Empirical VaR	Kerner VaR	Granger VaR
AXA Rosenberg	0,0%	4,0%	0,0%	0,9%
Discovery MasterFund Ltd	0,5%	1,1%	1,7%	2,2%
Aetos Corp	6,9%	0,2%	0,1%	0,1%
Bennett Restructuring	30,5%	35,2%	41,2%	37,1%
Calamos Convertible	0,0%	0,4%	0,0%	0,0%
Sage Capital	27,2%	5,9%	12,5%	15,5%
Genesis Emerging Markets	0,0%	0,5%	0,0%	0,7%
RXR Secured Note	2,9%	1,1%	2,5%	0,4%
Arrowsmith Fund	4,1%	6,2%	3,0%	4,8%
Blue Rock Capital	7,2%	23,5%	19,2%	16,1%
Dean Witter Cornerstone	0,0%	0,7%	0,0%	0,8%
GAMut Investments	20,1%	19,4%	19,6%	19,1%
Aquila International	0,0%	0,8%	0,0%	1,4%
Bay Capital Management	0,0%	0,1%	0,0%	0,0%
Blenheim Investments LP	0,5%	0,0%	0,0%	0,0%
Red Oak Commodity	0,0%	0,9%	0,0%	0,8%

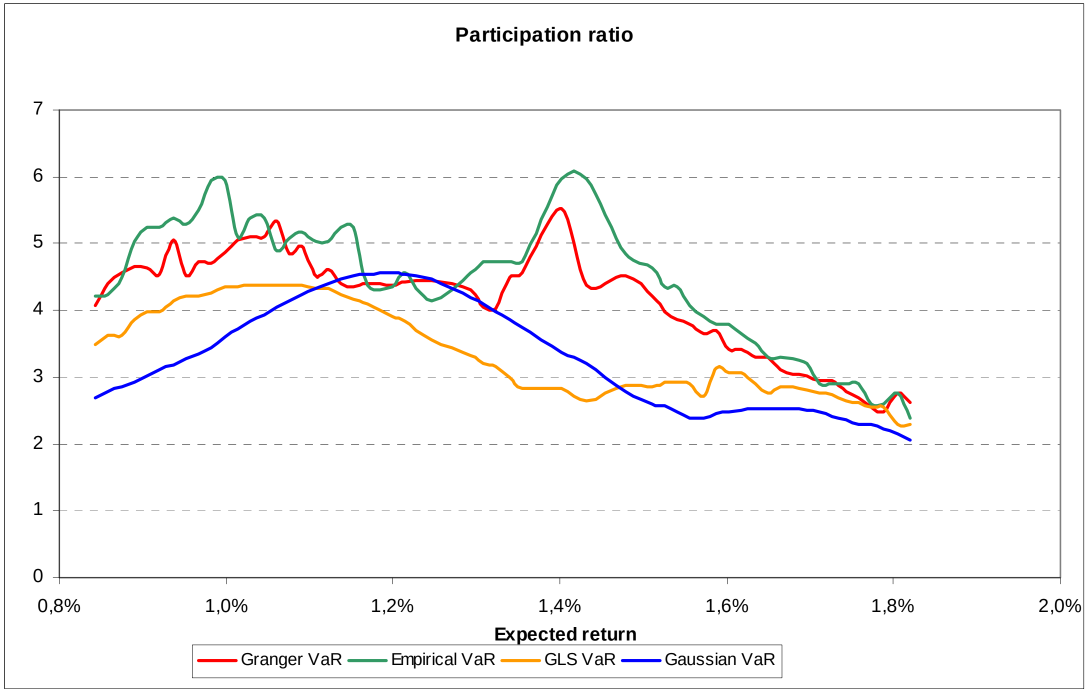
Efficient portfolios for a 1.2% level of expected return



- **Since the rankings with respect to the four risk measures are quite similar, the same hedge funds are close to the different efficient frontiers**
- **VaR is not sub-additive but…we find a surprisingly strong diversification effect**
- **Malevergne & Sornette [2004], Geman & Kharoubi [2003] find less diversification…but work with hedge funds indexes**

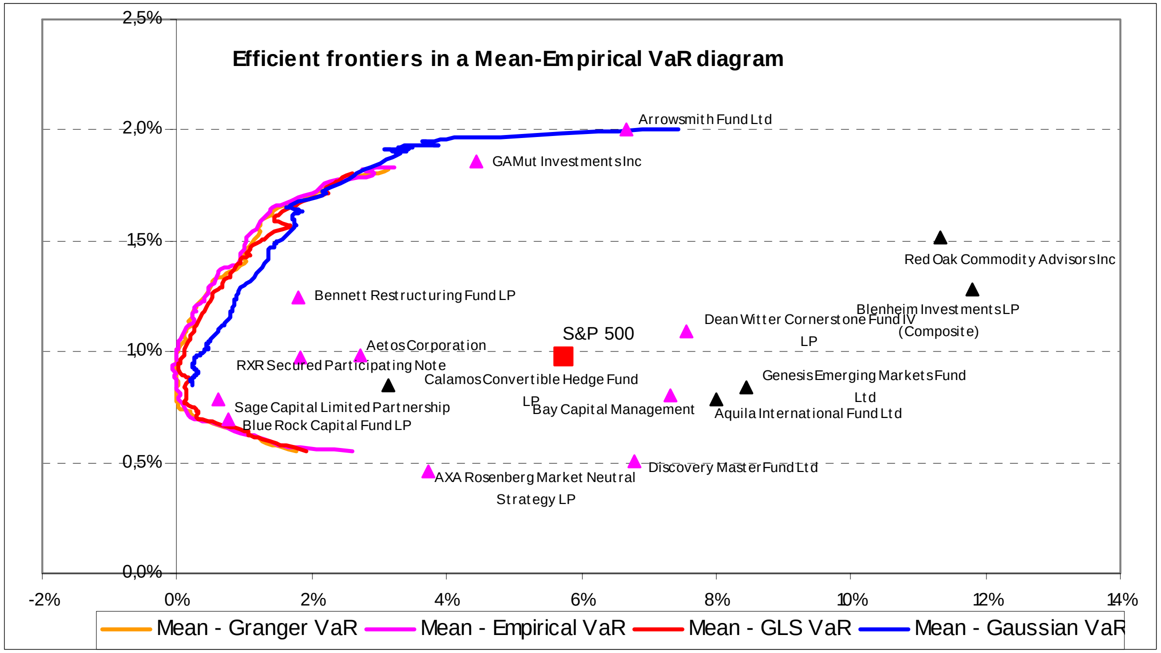
Diversification

- **Analysis of the diversification effect using :** $Participation\,ratio = \frac{1}{\sum_{i=1}^{n} a_i^2}$



- **Gaussian VaR leads to less diversified efficient portfolios**
- **Against « common knowledge » : non subadditivity of VaR implies risk concentration increases**

- **Analysis including S&P 500…**



- **…no change in the efficient frontiers**

Alternative Risk Measures

Alternative risk measures

- **Recent works about risk measures properties**
  - **Artzner & al [1999], Tasche [2002], Acerbi [2002], Föllmer & Schied [2002]**
  - **Widens the risk measure choice range**
- **Some choice criteria**
  - **Coherence properties**
  - **Numerical tractability**
- **Properties of optimal portfolios analysis**
  - **Comparison of different optimal portfolios**

Expected shortfall

- Definition: mean of “losses “ beyond the Value at Risk
- Properties
  - Coherent measure of risk
  - Spectral representation

$$ES_\alpha(X) = \frac{1}{\alpha}\int_0^\alpha VaR_u(X)du$$

➔ optimal portfolio may be very sensitive to extreme events if α is very low

- Algorithm
  - Linear optimization algorithms (see Rockafellar & Uryasev [2000])

➔ may be based on the simplex optimization program

  - Quick computations

Downside risk

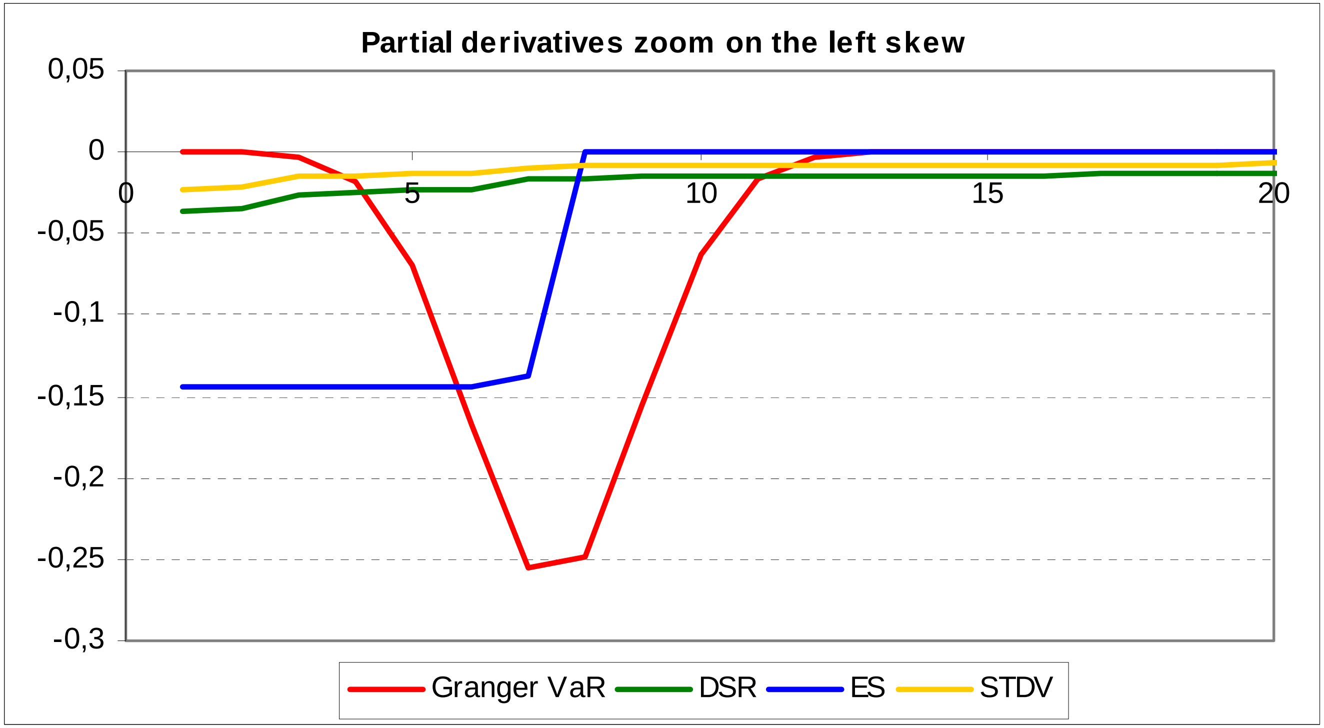
- **Definitions**
  - Let $x_1, x_2, \ldots x_n$ be the values of a portfolio (historical or simulated)
  - The downside risk is defined as follows

$$SV(X) = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left[\left(\bar{x} - x_i\right)^+\right]^2} - \bar{x}$$

- **Properties**
  - Coherent measure of risk
    - ➔ See Fischer [2001]
  - No spectral representation
    - ➔ fails to be comonotonic additive
  - Could be a good candidate to take into account the investors positive return preference
- **Algorithms**
  - Athayde's recursive algorithm ([2001])
    - ➔ Derived from the mean - variance optimization
  - Konno et al ([2002])
    - ➔ Use of auxiliary variables

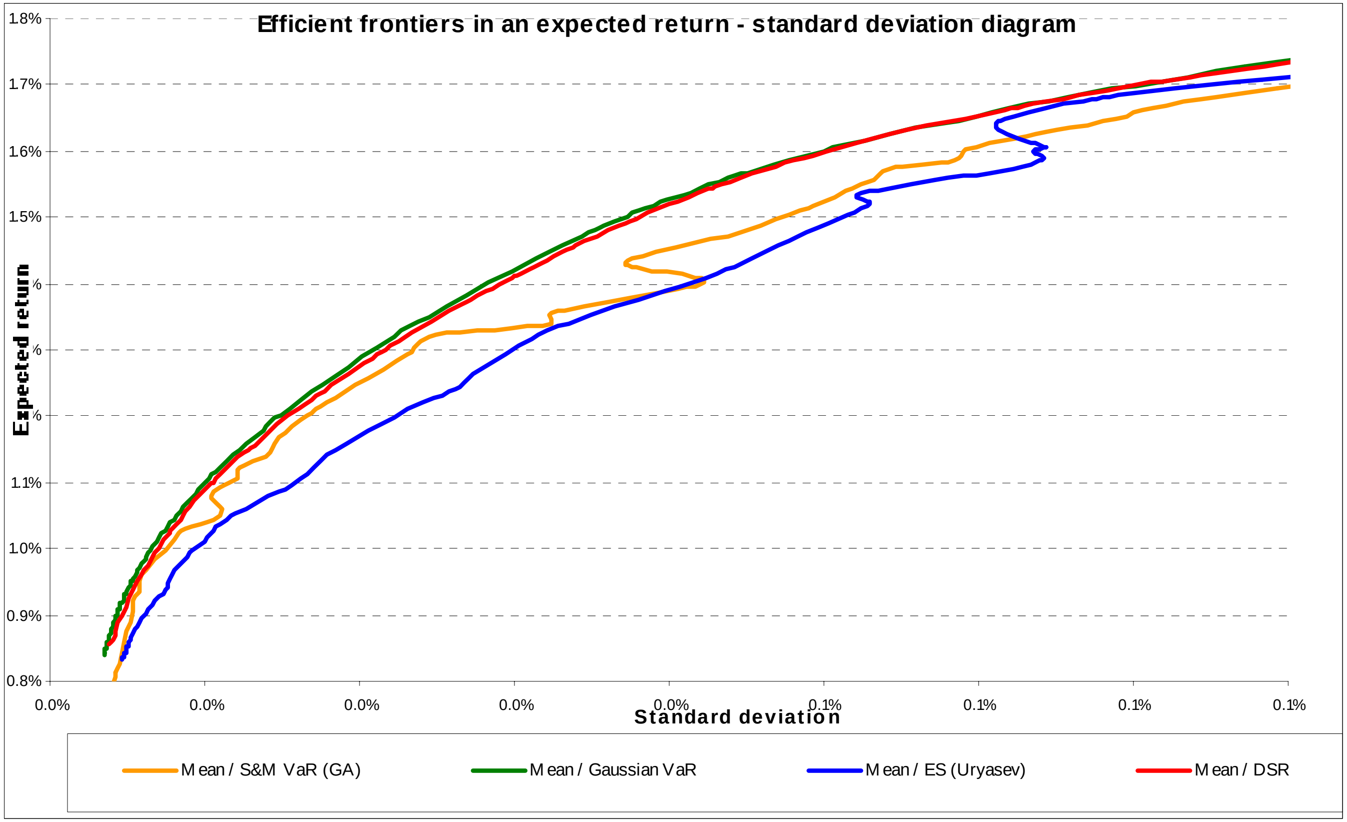
Contribution of rank statistics

- **Decomposition of the risk measures as for the VaR case**



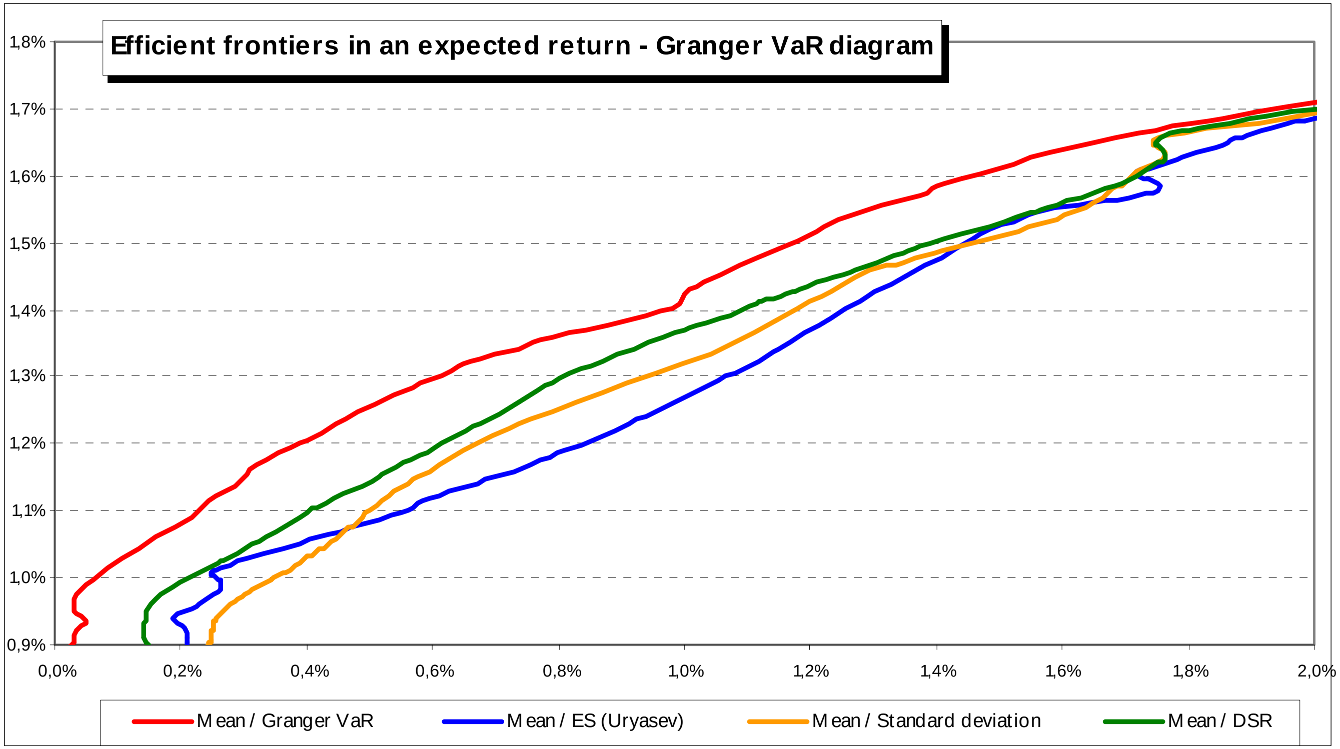
- **VaR and ES weights are concentrated on extreme rank statistics**
- **Variance and Downside risk weights exhibit a smoother weighting scheme**

Efficient frontiers: the Variance point of view



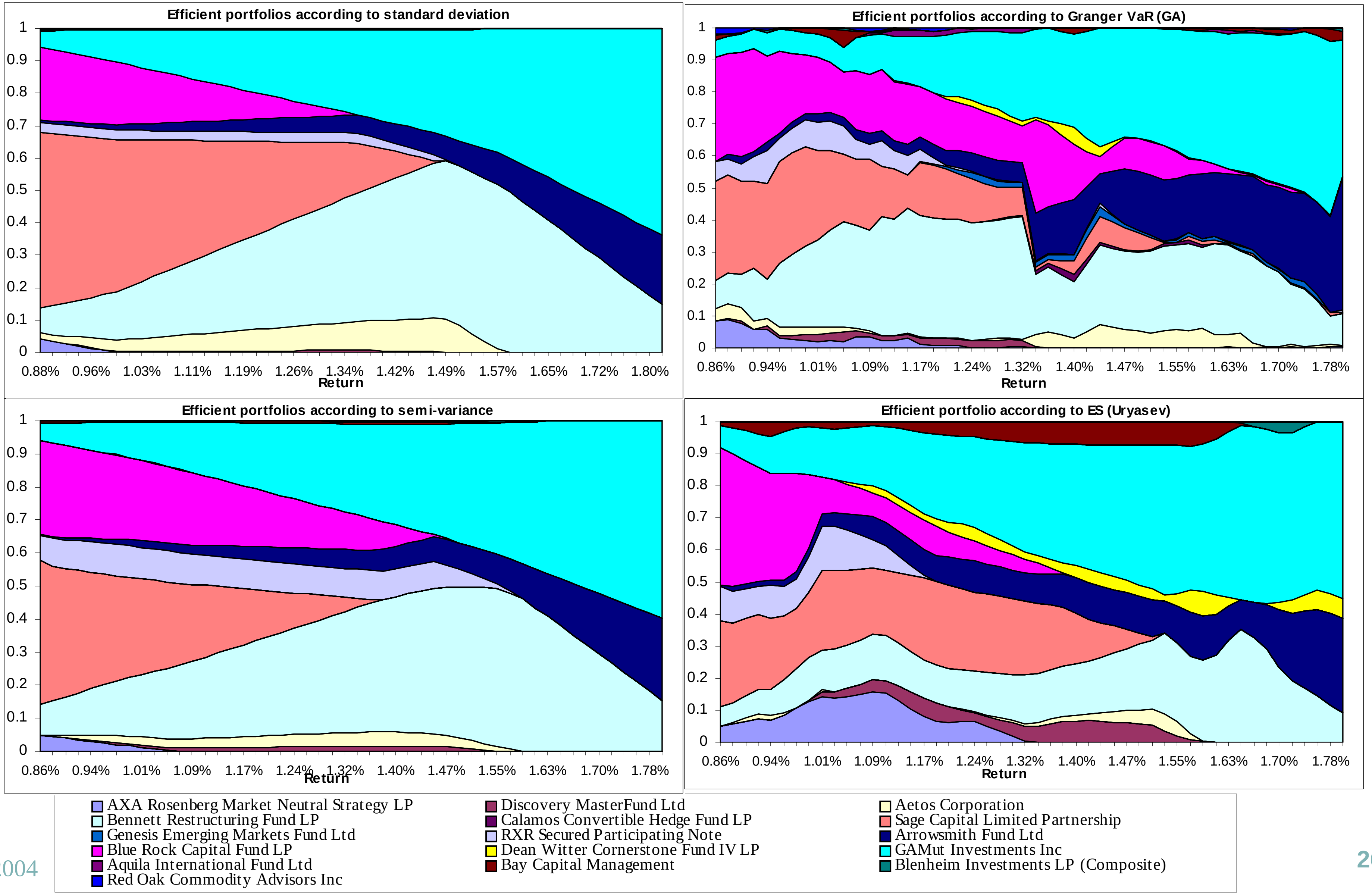
- **Variance and downside risk are very close**
- **Contrasts created by the opposition of**
  - **Small events based measure: variance and downside risk**
  - **Large events based measure: VaR and Expected Shortfall**

The VaR point of view



- **VaR efficient frontier is far from the others (even from Expected Shortfall)**
  - **VaR estimation involve a few rank statistics than the other risk measures**
- **No differences between downside risk, Variance, Expected shortfall in the VaR view**

Optimal portfolios



Optimal portfolios

- Almost the same assets whatever the risk measure

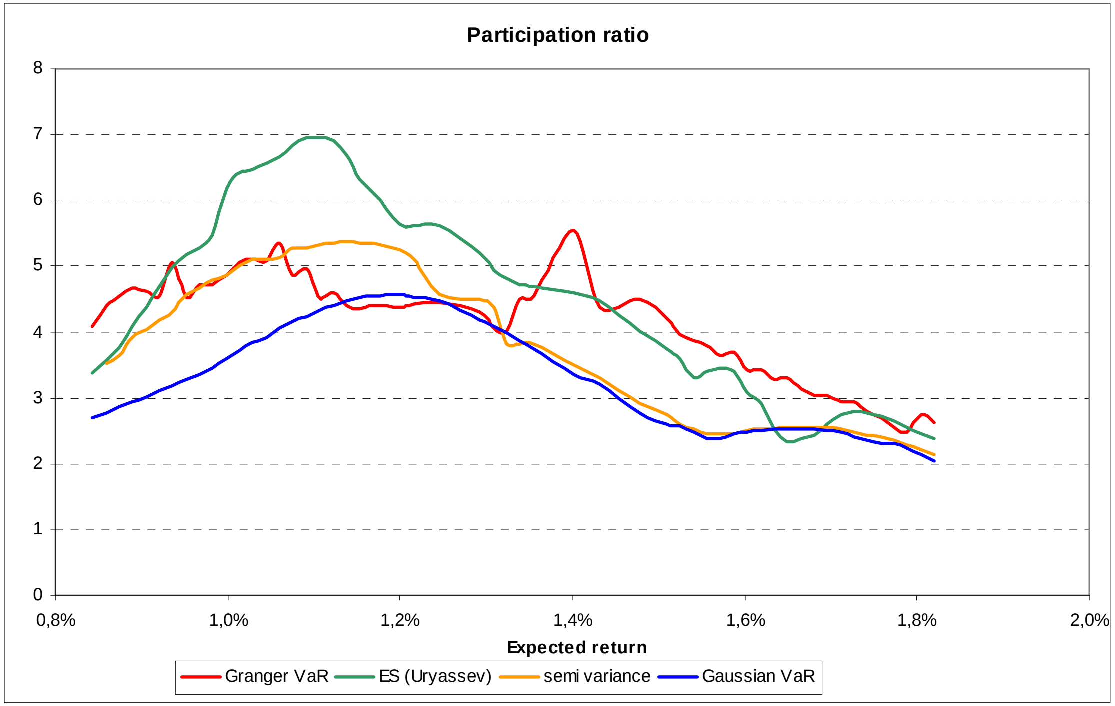
Funds	ES	VaR	Semi-variance	Std
AXA Rosenberg	6,3%	0,9%	0,0%	0,0%
Discovery MasterFund Ltd	4,7%	2,2%	1,3%	0,5%
Aetos Corp	0,0%	0,1%	3,5%	6,9%
Bennett Restructuring	12,0%	37,1%	29,9%	30,5%
Calamos Convertible	0,0%	0,0%	0,0%	0,0%
Sage Capital	26,0%	15,5%	14,0%	27,2%
Genesis Emerging Markets	0,0%	0,7%	0,0%	0,0%
RXR Secured Note	0,0%	0,4%	8,9%	2,9%
Arrowsmith Fund	8,7%	4,8%	4,3%	4,1%
Blue Rock Capital	8,0%	16,1%	16,5%	7,2%
Dean Witter Cornerstone	3,0%	0,8%	0,0%	0,0%
GAMut Investments	27,0%	19,1%	20,9%	20,1%
Aquila International	0,0%	1,4%	0,0%	0,0%
Bay Capital Management	4,3%	0,0%	0,7%	0,0%
Blenheim Investments LP	0,0%	0,0%	0,1%	0,5%
Red Oak Commodity	0,0%	0,8%	0,0%	0,0%

Efficient portfolios for a 1.2% level of expected return

- Some assets are not in the optimal portfolios but may be good substitutes
- As for the VaR, risk measures with smoother weights leads to more stable efficient portfolios.

Diversification

- **Analysis of the diversification effect**



- **Expected Shortfall leads to greater diversification than other risk measures**
- **Gaussian VaR leads to less diversified efficient portfolios**

- **Rank correlation analysis between risk levels and optimal portfolio weights**

	Optimal ptf sc semi-v.	Optimal ptf sc VaR	Optimal ptf sc ES	Semi-variance	Granger VaR	ES
Optimal ptf sc semi-v.	100%	38%	40%	37%	53%	38%
Optimal ptf sc VaR	38%	100%	60%	39%	43%	35%
Optimal ptf sc ES	40%	60%	100%	15%	28%	16%
Semi-variance	37%	39%	15%	100%	95%	98%
Granger VaR	53%	43%	28%	95%	100%	96%
ES	38%	35%	16%	98%	96%	100%

Rank correlation

- **No direct relation**

Conclusion

- The same assets appear in the efficient portfolios, but allocations are different
  - The way VaR is computed is quite important
  - Expected shortfall leads to greater diversification
- No direct relation between individual amount of risk and weight in optimal portfolios:

  Large individual risk $\not\Rightarrow$ low weight in optimal portfolios

  Small individual risk $\not\Rightarrow$ large weight in optimal portfolios
  - Importance of the dependence between risks in the tails
- The risk decomposition (can be compared to spectral representation) allows to understand the structure of optimal portfolios
- Open question:
  - Relation between risk measures and investors' preferences