Mortality Fluctuations Modelling with a Shared Frailty Approach

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Abstract

We propose a Shared Frailty model for the representation of lifetimes stochastic dependence. The estimation of the parameters is computed using two models: the first model is a standard Gompertz model, the second one is a Gompertz model with a shared frailty term. Non-linear pricing measures, such as percentile premiums and reinsurance premiums, are computed with the two models and their respective estimated parameters. We analyse how these measures are changed by the introduction of a frailty. As a result, we observe that even a small amount of dependence can dramatically increase these risk measures.

Key Words: Lifetimes Dependence, Frailty, Gompertz model, EM Algorithm, Term Life Insurance, Life Annuities, Excess of Loss Reinsurance Arrangement, Percentile Principle

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1 Introduction

Over the past few years, we have observed a significant development of Life Insurance risk transfer activities. The issued notional of **Life Insurance risk securitizations** has grown exponentially from 1.8Bn USD in 2001 to 15Bn USD in 2007. With capital markets investors looking for risk diversification, activities like Life Settlements have litterally exploded. The **Life Reinsurance** market has significantly increased between 1990 and 2000. According to Clark [2003], only 15% of US life insurance business was reinsured in 1993. In 2000, this percentage grew up to 64%. In 1995, 17 life reinsurers represented 90% of the life reinsurance market. These same 90% were only represented by 10 companies in 2002.

The risk management in these risk transfer activities strongly relies upon **non-linear risk measures**. For instance, Stop-Loss premiums and Percentile measures are common practice in Life Reinsurance. On the other hand, capital markets are looking to trade Synthetic Tranches of life insurance portfolios. Such derivative product offers an enhanced yield to the investors, in exchange of the payment of death benefits when the portfolio aggregate losses is located between two thresholds.

Unlike their linear counterparts used by traditional insurance companies, non-linear risk measure are **very sensitive to the modelling of the mortality fluctuations**, in particular the **stochastic dependence of the lifetimes** of the individuals in the pool. Intuitively, we give value to very out-of-the-money options by increasing the tail of the underlying risk distribution. In the case of a multiple risk portfolio, the tail of the aggregate risk distribution is controlled by the correlation between the single risks.

Although it might not be intuitive in Life Insurance, there are situations where lifetimes can be correlated. We observe dependent lifetimes in married couples, or twins. Events like epidemics or wars generate also correlation in lifetimes. The medical improvement also correlates lifetimes because it is not observable and common to a large number of individuals (Hougaard [2000]). In general, dependence of lifetimes in a greater population is not a cause to effect phenomenon, but is rather generated by the common effect of some unobservable risk factors on the death intensity of a group of individuals.

The aim of this article is to propose a simple framework to model dependent lifetimes, in order to analyse the impact of this dependence in some of the non-linear risk measures. We will compare two models: one traditional Gompertz model and the same model enhanced with a **shared frailty** component whose aim is to create some dependence between lifetimes. We will calibrate the models to observed mortality data, and use the calibrated parameters to calculate some risk measures, aggregate loss distribution and life reinsurance premium.

We will start with a review of the most common approaches to mortality deviations modelling (section 2). In the third section, we will introduce the shared frailty model in more detail, and make the specification of its underlying functions and distributions. Results of the estimation of a model without frailty and the shared frailty model are presented briefly. We use two data sets with low and high dependence respectively. Finally, in section 4, we propose the application of the frailty model to a life reinsurance pricing example. We analyse the impact of the dependence on the tail of the aggregate loss distribution and then we look at the impact on non-linear life reinsurance premiums.

2 Mortality models

2.1 Intensity models

These models assume a dynamic for the mortality rate, and the individual lifetimes are assumed to be independent. For a given individual with age x at year t, a general formulation of the mortality rate $\mu_x(t)$ can be expressed as the solution of the stochastic equation:

$$d\mu_x(t) = \nu(x, t, \mu_x(t))\, dt + \sigma(x, t, \mu_x(t))\, dW_t$$

where $\{W_t\}$ is a Brownian motion. We will now give some examples from this class of models, amongst the best known in actuarial science and demographics.

The **Gompertz model** is traditionally used in the construction of mortality tables by actuaries. We find this model when we take $\nu(x, t, \mu_x(t)) = p\mu_x(t)$ with $p = cte$ and $\sigma(x, t, \mu_x(t)) = 0$, resulting in the following expression:

$$\mu_x(t) = \gamma \exp[p(t + x)]$$

where γ is a strictly positive constant. In the remainder of the article, we will use the Gompertz model, as a "base case" actuarial model.

Lee-Carter method is considered as a reference by many actuaries and demographers, hence we found useful to mention it, even if we won't use it in the rest of the paper. This model can be obtained by taking:

$$\begin{aligned}\nu\left(x,t,\mu_x\left(t\right)\right) &= \left(\alpha\left(x\right)+\beta\left(x\right)\kappa'\left(t\right)-\frac{1}{2}\sigma^2\right)\mu_x\left(t\right)\\ \sigma\left(x,t,\mu_x\left(t\right)\right) &= \sigma\mu_x\left(t\right)\end{aligned}$$

where $\alpha(x)$ and $\beta(x)$ are two functions of age x and $\kappa(t)$ is scalar function of time t. Applying Itô's Lemma to $\ln\mu_x(t)$ leads to the formulation of the model, separating the mortality rate into an age-specific component and a time-specific component:

$$\mu_x\left(t\right)=\exp\left(\alpha\left(x\right)+\beta\left(x\right)\kappa\left(t\right)+\varepsilon_{x,t}\right)$$

2.2 Models of dependent lifetimes

Comonotonic models represent a case of perfect dependence. A set of random variables $(Y_1,...,Y_n)$ is comonotonic if there exists a random variable Z and a set of non-decreasing functions $(g_1,...,g_n)$ of the real variable such that we have the equality in distribution:

$$(Y_1,...,Y_n)\sim\left(g_1\left(Z\right),...,g_n\left(Z\right)\right)$$

Mixture models belong to another class of lifetime dependence models. For instance, we can assume that the lifetimes $\tau_1,...,\tau_n$ follow a $Weibull\left(a,b\right)$ distribution, where the parameters a and b are random (Pitacco and Olivieri [2002]). Conditionally on a and b, the lifetimes $\tau_1,...,\tau_n$ are independent.

The **shared frailty model** has been suggested by Hougaard [1984] for the representation of lifetimes dependence within a group of individuals. In the model, each individual has a mortality rate, function of the observed risk factors such as age or sex. In addition, an unobservable risk factor, the frailty, has a common effect on the mortality of all the individuals. Such common risk dependence could be illustrated by pollutions (asbestos), epidemics (SRAS, Esbola), lifestyle (smoking, obesity), wars, catastrophes and medical improvement. In the remainder of the paper, we will use the Shared Frailty model to represent the dependence of lifetimes in a group of individuals.

3 Formulation of the shared frailty model

In this section, we explain the assumptions of the shared frailty model, its specification and how to estimate its parameters.

3.1 Assumptions and notations

For the sake of simplicity, we assume only one group of individuals sharing the same frailty. We use the following notations:

- Z is the frailty shared by all individuals of the group
- $\varphi(z)$ or $\varphi(z;\boldsymbol{\theta})$ is the common density probability of frailty Z, $\boldsymbol{\theta}$ being the vector of parameters of the frailty distribution.
- $\Psi(s)$ or $\Psi(s;\boldsymbol{\theta})$ is the Laplace transform of the distribution of Z, defined as $\mathbf{E}\left(e^{-sZ} \mid \boldsymbol{\theta}\right)$ for all $s \in \mathbb{R}_+$.
- $\mu_j(t \mid Z)$ is the mortality rate of individual j, conditional to Z
- $\mu_{0j}(t)$ is the baseline hazard rate of individual j and $M_{0j}(t)$ is the integrated baseline hazard rate

The assumptions of the shared frailty model are the following:

Assumption (A1) *conditional on the frailty Z, the random lifetimes of individuals within the group are independent*

Assumption (A2) *the frailty Z has a multiplicative effect on the mortality rate of the individuals:*

$$\mu_j(t \mid Z) = Z\mu_{0j}(t) \tag{1}$$

Additionally, we make the following distribution assumptions:

Assumption (A3) *the baseline mortality follows the Gompertz (p,γ) model*

Assumption (A4) *the shared frailty Z is Gamma (δ,δ) distributed*

In the *Gompertz* (p,γ) model, the integrated baseline hazard rate of an individual with age x_j is expressed:

$$M_{0j}(t) = \frac{\gamma}{p}\exp(px_j)(\exp(pt)-1)$$

and the Laplace transform of the *Gamma* (δ,δ) distribution is:

$$\Psi(s) = \left(\frac{\delta}{\delta+s}\right)^{\delta}$$

The lifetimes dependence is driven by the variance of the shared frailty.

3.2 Probabilistic functions

We will now describe the basic probabilistic functions of the shared frailty model. We focus on the individual and the multivariate survival functions, because they are used in the parameters estimation process. The individual survival function, conditional to frailty is:

$$\mathbf{P}\left(\tau_j > t \mid Z\right) = \exp\left(-ZM_{0j}\left(t\right)\right)$$

Using above expression, and conditional independence assumption (A1), one can write the conditional multivariate survival function of lifetimes $\tau_1, ..., \tau_n$ as:

$$S\left(t_1, ..., t_n \mid Z\right) = \exp\left[-Z\left\{\sum_{j=1}^{n} M_{0j}\left(t_j\right)\right\}\right] \tag{2}$$

We obtain the individual survival function after taking the expectation with respect to Z of the conditional individual survival function:

$$S_j\left(t\right) = \Psi\left(M_{0j}\left(t\right)\right) \tag{3}$$

Similarly, we calculate the multivariate survival function as a function of the Laplace transform Ψ and the individual integrated mortality rates:

$$S\left(t_1, ..., t_n\right) = \Psi\left(\sum_{j=1}^{n} M_{0j}\left(t_j\right)\right) \tag{4}$$

3.3 Estimation of the parameters

We will now provide some reasonable values of the dependence parameters and briefly describe the estimation procedure.

The **data** have been retrieved from the Human Mortality Database (HMD) website. We observe French populations from 1900 to 1997. The groups consist of males, aged from 30 to 80 years at the beginning of the study. Each group is observed for 5 years: 1900-1905 (Group 1), 1905-1910 (Group 2), ... , 1990-1995 (Group 19).

- **Data Set 1:** we have excluded the war periods. This data set represents a population with **low** level of fluctuations
- **Data Set 2:** we have included the war periods. This data set represents a population with **high** level of fluctuations

For the Shared Frailty model, the parameters have been estimated using the **EM algorithm**. This is an iterative Bayesian method, aiming at maximizing the sample likelihood in the case of incomplete data. Therefore, it is relevant to the Shared Frailty model, since the common risk factor Z is not directly observable. We refer to Klein and Moeschberger [1997] for a description of the EM algorithm in the frailty case and we refer to Dempster, Laird and Rubin [1977] for a theoretical presentation of the EM algorithm.

The estimated parameters are given in the two tables below, for data set 1 and 2 respectively:

Data Set 1	**Gompertz standard**	**Gamma-Gompertz Frailty**
$\widehat{\gamma}_1$	$\widehat{\gamma}_1^{Indep} = 0.000122$	$\widehat{\gamma}_1^{Frailty} = 0.000116$
$\widehat{p}_1$	$\widehat{p}_1^{Indep} = 0.081016$	$\widehat{p}_1^{Frailty} = 0.081826$
$\widehat{\delta}_1$	-	$\widehat{\delta}_1^{Frailty} = 227.6571$
$\widehat{\theta}_1 = \widehat{\delta}_1$	-	$\widehat{\theta}_1^{Frailty} = 227.6571$

Data Set 2	**Gompertz standard**	**Gamma-Gompertz Frailty**
$\widehat{\gamma}_2$	$\widehat{\gamma}_2^{Indep} = 0.000227$	$\widehat{\gamma}_2^{Frailty} = 0.000231$
$\widehat{p}_2$	$\widehat{p}_2^{Indep} = 0.077562$	$\widehat{p}_2^{Frailty} = 0.078801$
$\widehat{\delta}_2$	-	$\widehat{\delta}_2^{Frailty} = 10.06357$
$\widehat{\theta}_2 = \widehat{\delta}_2$	-	$\widehat{\theta}_2^{Frailty} = 10.06357$

The parameter δ is around 10 for a strong dependence and around 200 for a low dependence. Moreover, the Gompertz parameters γ and p are not significantly modified by the introduction of a frailty. In fact, they only adjust in order to conserve the value of the marginal mortality rate.

From such results, we can say that a high degree of dependence ($CV^2(Z) \geq 10\%$) corresponds to populations exposed to wars, mass epidemics or natural catastrophes. It could also correspond to specific groups of individuals exposed to the same risks *a priori*. For instance building workers have been exposed to asbestos before they knew its risks. The case of patients with Leukemia also illustrates high dependence. These patients lifetimes would all increase if a better treatment of the disease was discovered.

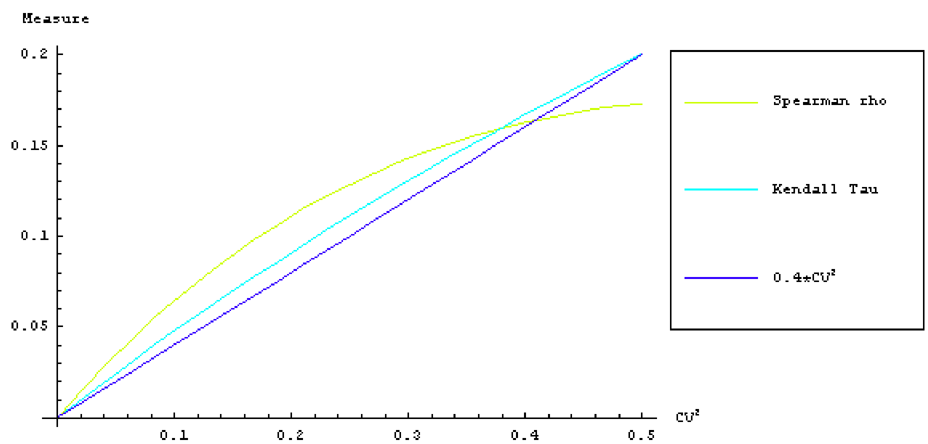

Figure 1: *Comparison of $CV^2(Z)$ with Kendall coefficient of concordance and Spearman correlation coefficient.*

3.4 Measure of the dependence obtained

In order to measure the degree of dependence generated by the model, we will use the coefficient of variability of the frailty (Hougaard [2000]), defined as:

$$CV^2(Z) = \frac{\mathbf{Var}(Z)}{\mathbf{E}(Z)}$$

In Figure 1, we plotted Kendall Tau $\kappa(\tau_i, \tau_j)$ and Spearman Rho $\rho_S(\tau_i, \tau_j)$ of two lifetimes τ_j and τ_j, against $CV^2(Z)$. The graphs seem quite close to each other for $CV^2(Z) < 0.5$, which corresponds to $\delta > 2$ in the Gamma-Gompertz case. We obtained the dependence levels $CV^2(Z) \approx 0.5\%$ for the data set 1 and $CV^2(Z) \approx 10\%$ for the data set 2.

4 Applications

4.1 Impact on the aggregate loss distribution for a portfolio of lives

We have derived the distribution of the total liability of an Insurance company (Figure 2). We assume a closed portfolio of $n = 1000$ temporary death

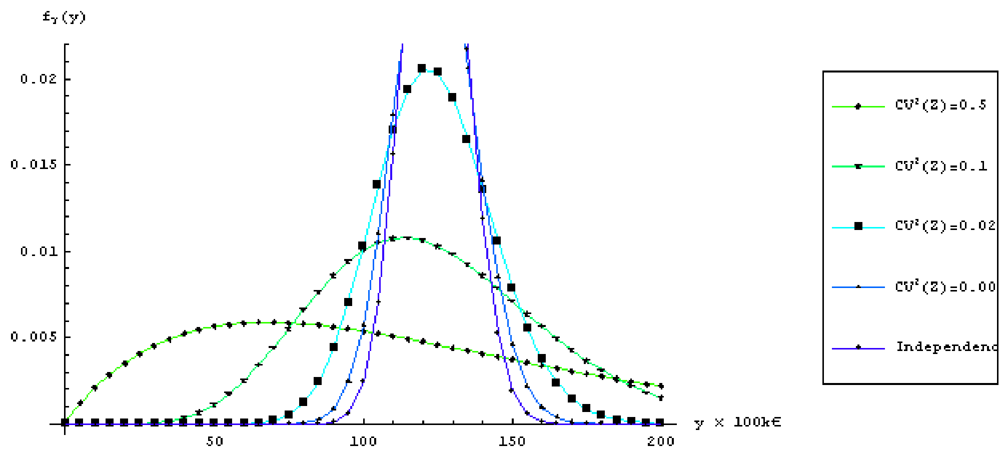

Figure 2: *Probability density function of Y for various levels of dependence, in the case of the death insurance portfolio (Age = 50 years)*

contracts of maturity $T = 8$ years. The premium is paid upfront and the death benefit is C = 100 000 €. The individuals have same age $x = 50$ years at underwriting date.

We have represented several degrees of dependence, including the ones implied by the two sets of parameters previously estimated. The main observation is an increase of both left and right tails with increased dependence.

4.2 Impact on aggregate risk measures

For the Life Insurance portfolio described previously, we have calculated the Value-At-Risk VaR_α, the Tail-Value-At-Risk CTE_α and the Wang Transform WT_α for a level of confidence $\alpha = 5\%$ and a time horizon of 8 years, corresponding to the maturity of the contracts. The results are in the table below, in k€:

	$\mathbf{CV}^2(\mathbf{Z})$	$VaR_{0.05}$	$CTE_{0.05}$	$WT_{0.05}$
$\boldsymbol{\delta = 2}$	**0.5**	272.54	322.72	281.24
$\boldsymbol{\delta = 10}$	**0.1**	191.35	211.98	194.23
$\boldsymbol{\delta = 50}$	**0.02**	158.29	161.59	158.81
$\boldsymbol{\delta = 200}$	**0.005**	147.91	150.35	147.62
Independence	**0.000**	141.64	145.30	141.58

We observe that the three measures are sensitive to dependence. They stay relatively close to each other with a moderate dependence level, and become significantly different with a higher dependence level ($\delta < 10$), with a dominance of the CTE. This is another illustration of the impact of dependence in the tail of the aggregate risk distribution. These results suggest the application of the shared frailty model to Economic Capital calculations.

4.3 Application to a Stop-Loss Life Reinsurance arrangement

We now evaluate the impact of the dependence on Stop-Loss Life Reinsurance arrangements similar to the ones described by Terrier [2000] or Olivieri [2002].

4.3.1 Notations and mechanism

The principle of the treaty is to cover negative Release of Surplus[1] of the Life Insurer on an annual basis. A time horizon is specified, with a start date h and an end date H. **The Reinsurer must make a payment if at date H, the value of the assets of the Insurer is lower than the value of his liabilities,** or, in other terms, **when the Insurer's Release of Surplus at H is negative.** The liability of the Reinsurer is limited to a share $1 - w$ of the negative Release of Surplus. The share of the Release of Surplus financed by the insurer is therefore w. The Stop-Loss coverage runs for the whole period $[0, \widetilde{T}]$. The mechanism of payments over a year of insurance is the following:

- At the beginning of the year h, the insurer creates a reserve for potential losses ${}_hV$
- At $h + 1$, the insurer must pay $\sum_{j=1}^{n} B_j \mathbf{1}_{\{\tau_j \in [h,h+1[\}}$, corresponding to the total death benefits between h and $h + 1$

[1] This quantity is also called Solvency Margin (Olivieri [2000])

- At $h+1$, the insurer must create a reserve ${}_{h+1}V$ for expected future liabilities

If $v(h,h+1)$ is the 1 year discount factor starting at date h, the Release of Surplus of the insurer at $h+1$ can be expressed:

$$\begin{aligned} M_{h+1} &= \frac{{}_{h}V}{v(h,h+1)} - \sum_{j=1}^{n} B_j \mathbf{1}_{\{\tau_j \in [h,h+1[\}} - {}_{h+1}V \\ {}_{h}V &= \sum_{j=1}^{n} \mathbf{1}_{\tau_j > h} \times {}_{h}V_j \end{aligned}$$

The reserves per individual ${}_{h}V_j$ and ${}_{h+1}V_j$ are calculated using the mortality assumptions of the insurer. The mortality risk assumption of the Reinsurer will be made for the random lifetimes τ_j.

4.3.2 Numerical results

We will now present the results of the pricing of the Stop-Loss contracts, using the Gompertz model without and with frailty. The insurer's portfolio is constituted of $n = 1000$ death insurance contracts, each paying 100k € upon death of the reference individual. We assume that this portfolio is closed[2] during the time of the reinsurance arrangement. The time horizon $\widetilde{T}$ for the reinsurance coverage and the term T of death insurance contract are equal to 8 years. We also assume that the share of participation of the insurer is $w = 10\%$. We assume that the insurer uses $Gompertz\left(\widehat{p}^{Indep}, \widehat{\gamma}^{Indep}\right)$ as a mortality assumption for pricing and reserving, whereas the Reinsurer will use alternatively independence model $Gompertz\left(\widehat{p}^{Indep}, \widehat{\gamma}^{Indep}\right)$ or dependence model $GammaGompertz\left(\widehat{p}^{Frailty}, \widehat{\gamma}^{Frailty}, \widehat{\delta}^{Frailty}\right)$.

We start with the analysis of simulated Release of Surplus paths. In Figure 3, we simulated 20 paths with the independence model (plain lines) and with the dependence model (dotted lines). The dotted lines are more dispersed than the plain lines, because of the heavier tails in the loss distribution of the Shared Frailty model.

Finally, we evaluate the Stop-Loss premiums. In Figure 4, using Monte-Carlo method with 1000 paths simulations. we have plotted the upfront

[2] The extension to non-closed portfolios can easily be made.

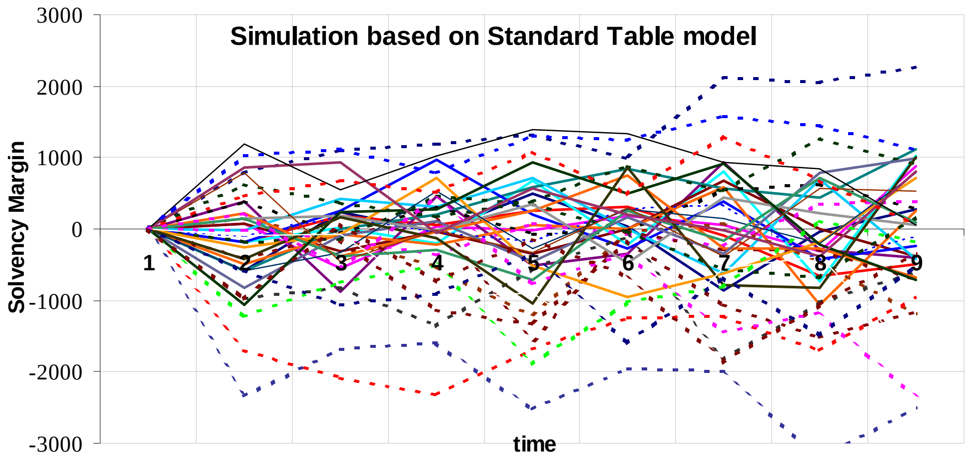

Figure 3: *Simulated Release of Surplus paths with time. Plain lines are simulations without dependence and dotted lines are simulations with dependence*

Stop-Loss Reinsurance premiums. We have represented the cash amount (k€) and percentage amount of the initial Insurer portfolio reserve ($\%\ {}_0V$) in order to give a better sense of the magnitude of the impact.

For data set 1, we observe an overall increase of the premium after introducing the dependence term in the model. The positive difference seem to increase with age and saturates after age 70. The premium is lower for ages 20 and 40, but we strongly believe that sampling error is the cause, as we have used only 1000 simulations. For data set 2, we observe a significant impact of introducing a dependence term. This impact seem to be reaching a maximum around age 70, corresponding to the mode of the Insurance portfolio loss distribution.

5 Conclusion

This article is an attempt to represent mortality fluctuations and apply it to the pricing and risk management of life insurance portfolios, using a Shared Frailty model. This model belongs to the class of dependent lifetimes models. We have considered the simplest form, with a stationnary risk factor, common to all invividuals in the population. This choice has been made in order to

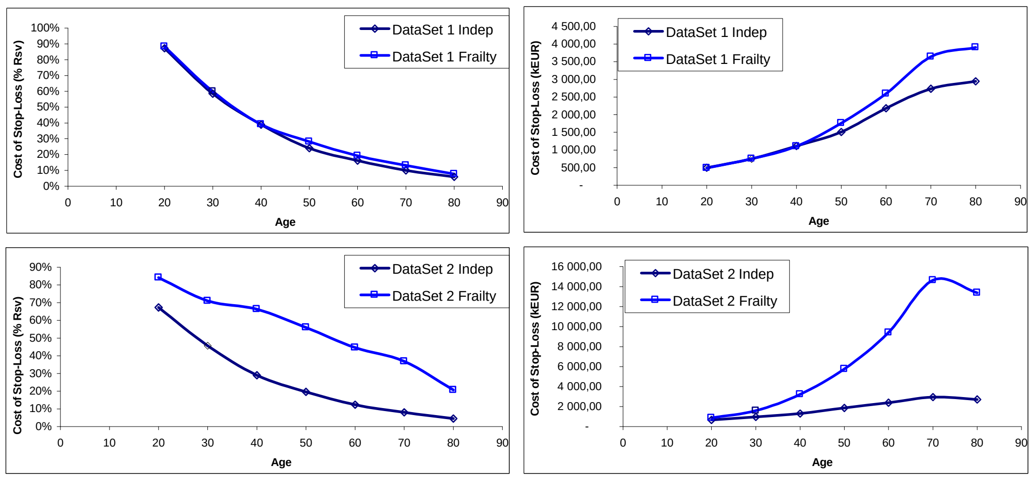

Figure 4: *Upfront Stop-Loss premium in kEUR and in % of* ${}_0V$*, for both dependence and independence cases and with both low and high fluctuations data sets.*

focus on the main effects of the model. We have used the EM algorithm for the parameter estimation, and used two data sets: one with a low level of mortality fluctuations, the other one with a large level of fluctuations.

The introduction of a frailty results in a modification of the portfolio aggregate loss distribution, mostly in the tails. Even a low level of dependence resulted in a significant increase. Aggregate risk measures increased in average by +4% for a low dependence level (0.5%) and by +39% for a large dependence level (10%). Stop-Loss premiums increased respectively by +15% and +200% for low and high dependence. These results suggest to use the Shared Frailty model in order to calculate Economic Capital, or any provision against tail risks.

Finally, we have found that the increase was reaching a saturation or a maximum at the age corresponding to the mode of the individual lifetime density function. This suggests to look at time consistent or more granular models, such as a non-stationary Shared Frailty, or a Shared Frailty by risk or age group. For further research, we could imagine combined approaches, using trend plus frailty or a dynamic frailty term, in order to represent longevity increase. We would have a complete modelling framework, representing both increase in longevity and shocks in mortality.

Bibliographie

ARTZNER P., F. DELBAEN, J. M. EBER & D. HEATH, 1999, *Coherent Measures of Risk, Mathématical Finance* 9(3), 203-228.

BROUHNS N. & M., J. DENUIT, 2001, *Risque de Longevité et Rentes Viagères, II. Tables de Mortalité Prospectives pour la Population Belge*, Belgian Actuarial Bulletin, vol. 2.

BROUHNS N. & M., J. DENUIT, 2001, *A Poisson Regression Approach to the Construction of Projected Lifetables for Life Annuitants.* Discussion paper, Institut de Statistique, Université Catholique de Louvain, Louvain-La-Neuve, Belgique.

CAIRNS A.J., D. BLAKE, K. DOWD, *A Two-Factor Model for Stochastic Mortality with Parameter Uncertainty: Theory and Calibration,* 2006, Journal of Risk and Insurance, 2006, Vol.73, No.4, 687-718.

C.E.I.O.P.S., 2006, *Quantitative Impact Study (QIS1) Summary Report*, available on www.ceiops.org

COX D. R., 1972, *Regression Models and Life Tables (with Discussion)*, Journal of the Royal Statistical Society, B 34, 187-220.

DEMPSTER A., N. LAIRD & D. RUBIN, 1977, *Maximum Likelihood from Incomplete Data via the EM Algorithm*, Journal of the Royal Statistical Society, Ser. B, 1-39.

DENUIT M., J. DHAENE, C. LE BAILLY DE TILLEGHEM & S. TEGHEM, 2001, *Measuring the Impact of the Dependence Among Insurance Lifelengths*, Belgian Actuarial Bulletin 1.

DEVOLDER P., 2005, *Affine Stochastic Models of Mortality and Longevity Risk*, Presentation held at Institut de Science Financière et d'Assurance, Lyon University.

DHAENE J. & M. DENUIT, 1999, *The Safest Dependence Structure among Risks*, Insurance: Mathematics and Economics, vol. 25, 11-21.

DHAENE J. & M. J. GOOVAERTS, 1997, *On the Dependency of Risks in the Individual Life Model*, Insurance, Mathematics and Economics, vol. 19, 243-253.

EMBRECHTS P., A. McNEIL & D. STRAUTMANN, 1998, Correlation and Dependency in Risk Management: Properties and Pitfalls, working paper, ETH Zürich.

FREES E. & E. VALDEZ, 1998, *Understanding Relationships Using Copulas*, North American Actuarial Journal, 2(1), 1-25.

FREES E. W., J. CARRIERE & E. VALDEZ, 1996, *Annuity Valuation with Dependent Mortality*, Journal of Risk and Insurance, vol. 63, 229-261.

HOUGAARD P., 1984, *Life Table Methods for Heterogeneous Populations: Distributions Describing the Heterogeneity*, Biometrika, vol. 71, 75-83.

HOUGAARD P., 2000, *Analysis of Multivariate Survival Data*, Springer.

HURLIMANN W., 2004, *Distorsion Risk Measures and Economic Capital*, North American Actuarial Journal, 8 (1), 86-95.

JOE H., 1997, *Multivariate Models and Dependence Concepts*, Monographs on Statistics and Applied Probability, volume 37, Chapman & Hall.

KLEIN J. P. & M. MOESCHBERGER, 1997, *Survival Analysis*, Springer Verlag, New-York.

KLEIN J. P., 1992, *Semiparametric Estimation of Random Effects Using the Cox Model Based on the EM Algorithm*, Biometrics 48, 795-806.

LAURENT J. P. & J. GREGORY, 2002, *Basket Default Swaps, CDO's and Factor Copulas*, Journal of Risk, Vol. 7, n°4, 103-122.

LEE R. D. & L. CARTER, 1992, *Modelling and Forecasting the Time Series of US Mortality*, Journal of the American Statistical Association, vol. 87, 659-671.

MARSHALL A. & L. OLKIN, 1967, *A Multivariate Exponential Distribution*, American Statistical Association Journal, vol. 62, 30-44.

OLIVIERI A. & E. PITACCO, 2002, *Inference About Mortality Improvements in Life Annuity Portfolios*, Transactions of the 27[TH] International Congress of Actuaries, Cancun, Mexico.

OLIVIERI A., 2001, *Uncertainty in Mortality Projections: an Actuarial Perspective*, Insurance: Mathematics and Economics, vol. 29, 231-245.

OLIVIERI A., 2002, *Risk and Reinsurance in Life Annuity Portfolios*, quaderni del dipartimento di mathematica applicata alle scienze economiche statistiche e attuarialli, Universita di Trieste.

PARKER G., 1997, *Stochastic Analysis of the Interaction between Investment and Insurance Risks*, North American Actuarial Journal 1, 55-84.

PETAUTON P., 2001, *Théorie et Pratique de l'Assurance Vie*, second edition, Dunod.

PITACCO E., 2003, *Survival Models in Actuarial Mathematics: from Halley to Longevity Risk*, Transactions of the 7th International Congress of Insurance: Mathematics and Economics.

RIBAS C., J. MARIN-SOLANO & A. ALEGRE, 2003, *On the Computation of the Aggregate Claims Distribution in the Individual Life Model with*

Bivariate Dependencies, Insurance: Mathematics and Economics, vol. 32(2), 201-215.

ROLSKI T., H. SCHMIDLI, V. SCHMIDT & J. TEUGELS, 1999, *Stochastic Processes for Insurance and Finance*, John Wiley & Sons, New York.

SOLVENCY WORKING PARTY, 2002, *Report of the Solvency Working Party*, Prepared for the IAA Insurance regulation Committee, available on www.soa.org

TOSETTI A., T. BEHAR, M. FROMENTEAU & S. MENART, 2000, *Assurance: Comptabilité, Règlementation et Actuariat*, Economica.

VALDEZ E. A., 1999, *The Distribution of Insurance and Annuity Benefits with Frailty*, working paper, Division of Actuarial Science and Insurance, Nanyang Technological University.

VAUPEL J.W. & J. OEPPEN, *Broken Limits to Life Expectancy*, Science, Vol 296, 1029-1031

WANG S. & J. DHAENE, 1998, *Comonotonicity, Correlation Order and Stop-Loss Premiums*, Insurance: Mathematics and Economics, vol. 22, 235-243.

WANG S. S. & R. L. BROWN, 1997, *A Frailty Model for Projection of Human Mortality Improvement*, Journal of Actuarial Practice, vol. 6, 221-241.

WANG S. S., 2000, *A Class of Distorsion Operators for Pricing Financial and Insurance Risks*, the Journal of Risks and Insurance, vol. 67(1), 15-36.

YASHIN A.I. & I. A. IACHINE, 1995, *Survival of related individuals: an extension of some fundamental results of heterogeneity analysis*, Mathematical Population Studies, vol. 5, 321-339.