

Meta-models and consistency issues

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Abstract

Rather than considering a “model” as a one-piece object, we can translate and adapt the concept of meta-models, commonly used in computer science, to the field of insurance management. We actually deal with a number of interconnected models. These models involve common concepts such as risk and value, assets and liabilities, reserves, management actions, etc. To avoid cacophonies (i.e. operational inefficiencies), every piece has to be placed in the right order. Depending on objectives and context, different levels of modelling will be required. Coherence in the modelling process does not mean uniformity. It is vital to understand correctly how models can effectively enhance business performance, yet not be blurred by undue complexity.

1) Life insurance models in a nutshell

Before going into the topic of this chapter, let us recall the two main modelling issues in the life insurance world:

- Computation of reserves since the scope of mark-to-model and best estimate approaches encompasses most of insurance liabilities.
- Computation of risk measures and the corresponding solvency requirements: pillar 1, such as standard SCR formula or internal model approaches, pillar 2 / ORSA approaches in Solvency II. Up to a certain extent, EIOPA dynamic scenario-based stress tests involve the same prerequisites as ORSA.

These two items may be combined to assess solvency over a prescribed time horizon, in most cases a single year in Basel II and III or Solvency 2 contexts, leading to the well-known issues of nested simulations². To be more specific, under standard modelling approaches, risk measure computation requires a great deal of reserve calculations, which may lead to process jamming. Clearly, when it comes to multi-period assessment of risk, as in the ORSA framework, computational and modelling issues blow up.

As discussed in the previous chapters of this book, a wide range of specialized modelling approaches exists, focusing on items such as economic scenario generation, management actions, loss quantile-based or standardized risk measures. To ease the exposition, we will pay extra attention to yield curve static or dynamic representations on one hand and on the dual view regarding risk models, i.e. internal models and standard formulas on the other. These two themes are well documented and allow for comparisons between insurance and finance fields. Even though the computation of risk-based capital charges is more challenging on theoretical grounds, proper accounting of best estimate insurance liabilities is by far the largest management issue for life insurance companies.

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² See Bauer et al. (2012), Broadie et al. (2011).

The term meta-model has different meanings, the most common refers to a model of a model, which can be understood as a simplified version, easier to grasp than the original one. For instance, a Vasicek model could be seen as a simplified version of a more general multifactor Gaussian HJM model³, leading to much simpler closed forms for interest rate option prices. This is related to the concept of nested models. As for Markov chains, the concept of embeddability discusses whether, or not, a discrete time Markov chain could be seen as a restriction to a discrete time scale of a continuous time Markov chain⁴. A similar issue is whether we can relate discrete and continuous time versions of a stochastic time or not. For instance, an Ornstein-Uhlenbeck standard mean reverting process or square root type is an AR(1) process when considered over a discrete time scale.

While the above is meaningful in our context, we will consider meta-modelling as the science or art that deals with model design and use. Our concern is not to determine the winner among a set of non-nested competing models, but to investigate the implications of using a number of models simultaneously. We are clearly faced with consistency issues and even though model choice is mostly driven by ease of implementation, model multiplicity might end up with undue overall complexity.

Eventually, meta-models refer to a highly specialized field of research. A model is viewed here as an engine providing some outputs. A meta-model or a surrogate model provides much easier to compute approximations to true model outputs. Response surface methodology, artificial neural networks, multivariate adaptive regression splines and radial basis function approximations are among the numerous techniques involved. As for pricing and risk measurement purposes in life insurance or finance businesses, kriging, a technique derived from geostatistics and the modelling of spatial data, is an appealing route. Unlike usual spline interpolation methods, it provides some confidence intervals on meta-model outputs. This is of high importance: when it comes to the assessment of reserves or the risks faced by a company, not being able to assess the magnitude of the approximation leaves a strong sense of discomfort. However, kriging approaches need to comply with some specific constraints when it comes to finance and insurance models. For instance, output approximate prices should not present arbitrage opportunities and the benefits of diversification should be acknowledged with it comes to approximating risk measures. These issues are currently on the academic research agenda, but still in limbo and will not be discussed thereafter⁵.

This chapter is organized as follows. In section 2, we will discuss the interplay between models and markets. This, of course, concerns the computation of best estimates, mark-to models calibrated up to some extent to market observables. As for risk models and especially standard formulas, we will investigate the extent to which they are properly connected to statistical features, i.e. actual changes in market risk factors. This could be broadly seen as consistency between models and data. In section 3, we will consider practical constraints that drive model building: pragmatism is associated with model multiplicity. Regulatory constraints on internal risk models and their departure from standard formulas (benchmarking, regulatory floors) also shape model construction. Section 4 investigates the practical implication of using models based on different premises, models need to be recalibrated to one another, making model governance trickier and leading to operational complexities. Section 5 considers the increased use of ad-hoc parametric approaches to approximate the output of internal pricing or risk models. Section 6 discusses the pros and cons of modelling shortcuts, i.e. replacing the

³ See Andersen and Piterbarg (2010) for a comprehensive review of interest rate models in theory and in practice.

⁴ This is related to the existence of an infinitesimal generator. See Kijima (1997) for mathematical details.

⁵ Applications to finance and insurance are not well developed yet. When dealing with quantile based risk measures, we can mention Chen et al. (2012), Chen et al. (2013), Liu (2010), Liu et al. (2010).

output of a questionable model by a parameter under management control. Section 7 reviews the standard approximation techniques, such as replicating portfolios, LSMC or the use of simplified closed-form expressions for prospective risk assessments such as ORSA. We end up with a brief reminder of practical challenges regarding model multiplicity in life insurance businesses.

2) Models and markets

The computation of reserves is likely to be the key challenge to life insurance companies. It is, therefore, worth recalling the theoretical context that underpins valuation procedures and how models come into play. The main risk drivers, or risk factors, are the changes in interest rates and the aggregate fluctuations in mortality rates or related biometric quantities. To ease the exposition, we will leave aside other possible risk factors that might be related to changes in currencies, stock or real estate prices.

Nowadays it is quite common for actuaries to think in terms of stochastic discount factors or "state-price deflators" or equivalently "pricing kernels" to follow the academic jargon. This allows to price contingent liabilities and in appropriate cases, to price them in a way that is consistent with observable and reliable market prices. In the mere case of fixed payables, the general approach collapses to standard discounting. Focusing on the simplest case is of first importance, since it is quite likely that the main driver of reserve uncertainty lies in the choice of the risk-free discounting curve:

- This starts from the choice of market data, either a set of collateralized or uncollateralized swap rates, index on Libor or overnight rates, yet it could also relate to rates inferred from currency-based bond prices. The way interbank default risk is taken into account, the crucial choice of an ultimate forward rate and how rates are interpolated and extrapolated up to that ultimate rate are obviously key points and are not technical involved. In Europe, EIOPA provides such a curve for the relevant currencies⁶. Clearly, the way market rates are translated to regulatory discount rates is of first importance for the computation of reserves and, here, more importantly when designing hedges, which requires calculation of risk sensitivities. When the market rate over a given time horizon moves by one basis point, the corresponding regulatory discount rate over the same time horizon may not change accordingly. Thus, both levels of discount rates and their changes are concerned at that stage.
- The second issue of importance is that, on top of this model-based regulatory discount curve, insurers are led to add various items, the so-called volatility adjustment and, under a number of restrictions, a matching adjustment so that the discount rate of liabilities becomes tied to the rate of return on the assets held to match liabilities. These items have led to numerous interactions between the insurance industry and the European regulators over the past years.

⁶ Colloquially, anyone investigating competing models has the impression of barging into a pet shop. The classical review paper by Hagan and West (2006) accounts for about 20 yield curve interpolation methods, leaving aside numerous parametric models and more statistical approaches (including the stochastic kriging briefly discussed below). They were not aware of the contribution of Smith and Wilson (2001) which underpins the official EIOPA interpolation methodology. It is conspicuous that actuarial yield curve modelling streams were apart from those followed by fixed income quants routinely involved in the pricing of trillions of swaps or billions of corporate or sovereign bonds.

One potential issue regarding long-term guarantee products⁷ is the reliance on the long part of the EIOPA curve, as this is largely model-based, due to the use of a normative ultimate forward rate (UFR) and the Smith-Wilson interpolation/ extrapolation scheme and leads to low-volatility of long maturity risk-free discount rates. Thus, even without accounting for shifts in credit spreads, it creates some discrepancies between assets and liabilities of similar maturities and cash-flow schedules (see Lageras (2014)).

As such, it comes of no surprise that this model driven and somehow ad-hoc shortening of liability duration needs to be compensated by a well-calibrated volatility adjustment. We should incidentally note that the volatility adjustment is not only required to balance credit spread volatility, but as an extra modelling layer to mitigate the unexpected consequences of a questionable yield curve stripping methodology. This is typical of constraint-based modelling (see below) whereas repair patches alleviate the perverse effects of previously set convenient shortcuts. Software and high-tech industries provide us with a number of textbook cases where such processes swerve off the road. To keep on track, an overall view of the issues at hand, a clear and sharp understanding of the technicalities and a perfectly driven implementation process are required.

Therefore, market driven input interest rates are filtered out by regulatory technical rules. This is to account for the specificities of the life insurance industry (long-term commitments, illiquidity of life insurance contracts) and the limited reliance on the longer part of the yield curve or on market implied credit spreads. Even though a number of theoretical arbitrage-free dynamic models of the yield curve predict constant ultimate forward rates, the current level of such UFR and the speed of convergence to this UFR in the Eurozone could be revisited. Hopefully, such discounting rules will be stabilized in the EU. Nevertheless, what drives risk is the possibility of changes of rules under changing economic environments and the degree of national discretion. It may be a difficult modelling task to lead an Own Risk and Solvency Assessment Exercise (ORSA). It is almost impossible to guess how key parameters would be recalibrated in stressed environments. Thus, forward looking risk exercises have to be understood as being set up under a fixed regulatory framework.

The second issue of importance is the range in which interest rates are likely to vary over the coming years. This range could be partly inferred from macroeconomic analyses; forward guidance by central banks is quite useful from this perspective. On the other hand, one-factor models, quite useful due to ease of implementation, could lead to implausible trajectories. AR(1) models will do a poor job when it comes to economics, mean-reversion parameters are prone to statistical noise and more involved state-space models are difficult to handle.

Going back to market observables such as interest rate options should then be considered when dealing with the pricing of life insurance liabilities. It also provides a direct route that bypasses the use of probabilistic models and stochastic discount factors. It will be later advocated in this chapter that typical life insurers are both short of out of the money caps and out of the money floors and thus would be better off if interest rates remain inside a reasonable corridor. Stated differently, extra rate volatility and pronounced volatility smiles are not the best context for looking for long-term solvency.

⁷ See https://eiopa.europa.eu/Publications/Reports/EIOPA_LTGA_Report_14_June_2013_01.pdf for some context.

To investigate the above issue, let us, for instance, consider a commitment to pay one euro at a five year horizon, we assume the payment date is certain, provided that the three months (Euro) Libor rate is above 4%. Provided that the interest rate derivatives, here the cap and floors market, is liquid enough to provide reliable prices, we will readily get the price of the above contingent (to the level of rates) five year zero coupon, as today's price of a five year so-called digital cap with a 4% strike. This is mere algebra. Please note that at this stage, we do not need to be bothered with a number of confusing concepts such as risk-neutral densities, historical (or statistical) probability measures and how these objects are related through risk premiums. We do not even need to call upon probability theory! As can be seen from this example, the higher the price of the digital cap, the higher will be the risk-neutral probability of rates being above that 4% strike. This is tautological. Then, this risk-neutral probability is the product of the actual probability of the reference Libor rate to be above the prescribed 4% threshold times a risk premium (to be investigated below).

As can be seen from the digital cap example, the higher the price of out of the money caps and floors the higher the priced dispersion of future Libor rates. A properly calibrated pricing kernel (or risk-neutral ESG) will account for such features. The intuition behind risk-neutral densities and pricing kernels may not be fully understood. It is then tempting to criticize the approach as too abstract. Let us put things differently. If one trades interest rate floors with zero strike at a positive premium, then the market puts a positive weight on negative rates. Rather than risk-neutral pricing, we should talk about market consistent pricing of liabilities. Unless we distrust market prices, say due to irrational exuberance or market illiquidity, an ESG should comply with such material information to provide best estimates (understood as market consistent) of insurance liabilities. It is sometimes advocated that models are performative, for instance interest rate option prices would be driven by disputable pricing models. Let us go the opposite way: If insurance companies were led to hedge against extreme changes in rates, it would raise the price of these hedges. Consequently, market-implied out of the money volatilities would develop, leading to a self-fulfilling fear of strongly diverging rates.

When we need to switch to the generation of meaningful interest rate scenarios, for the purpose of long-term solvency assessment (ORSA, Stress Tests) in the real world, we need to solve a puzzle. Up to now and under some restrictions, we advocated that markets prices could be meaningful inputs to assess insurance liabilities. Problem is that, unless some more or less arbitrary restrictions are put on risk premiums, one cannot infer actual probabilities from the risk-neutral ones. Stated slightly differently, such inference is highly subject to model risk. We will use a dual perspective to the construction of real world ESG together with risk-neutral ones.

- When we remain stuck to a standard Brownian framework, say a Vasicek type model to illustrate our purpose, we are faced with a huge restriction. As a consequence of the Girsanov theorem, volatility needs to be the same under risk-neutral and real worlds. This may lead to the kind of implausible explosive scenarios that are very much disliked by life insurers' executive committees.
- On the other hand, when we turn back to discretized versions of the original model, say random samples of rate trajectories, we have much more, too much, flexibility in computing risk premiums. We could reweight the probabilities of the sample paths quite arbitrarily, only being subject to weights that sum up to one. For instance, if monetary analysis states that rates are to remain for a while in a narrow corridor, we would put almost all probability mass on such scenarios and almost exclude explosive or negative rate scenarios in an ORSA exercise. This goes into the lines set by Avellaneda et al (2001) and weighted Monte Carlo approaches. The drawback of flexibility is that management risk appetite and discretion regarding risk premiums really makes it an own (subjective) risk assessment.

Another matter of concern regarding the coherence of the best estimate approach relates to the interplay between financial risks and biometric risks (mortality, longevity, dependence). As for the latter, it is often assumed that the law of large numbers applies at contract level. Let us try to be more specific regarding the previous assertion. Best estimates of insurance liabilities involve a market-consistent derivation as for financial risks and statistical approaches, say mortality tables for biometric risks. When it comes to the computation of life-contingent liabilities, an expectation is involved, thus everything looks fine with the best estimate approach. On the other hand, when looking at the more granular level of a single life insurance contract, there is some kind of discreteness in the payments, death is a zero-one event and an individual would not die gradually according to the certified relevant mortality table. Thus, when it comes to hedges, so that the best estimate does not remain a concept but could be translated in today's cash, things become blurred. One could think of using life reinsurance, balance guaranteed swaps or other forms of securitization and risk transfers, but actually such markets are still in limbo. Besides, it is the reason why insurance companies are asked for an extra risk-margin since the only credible transfer is some other insurance company taking over the commitments to the insured. We may also notice that such issues are at the core of the validity of matching adjustments, i.e. whether liability cash-flows can be well predicted.

Clearly, a proper assessment of the reliability of life tables is a legitimate concern when it comes to the computation of best estimates of insurance liabilities. Technical issues at hand are well-documented and will not be further discussed in this chapter.

One of the biggest concerns regarding reserves is the plurality of reference frames: prudential such as in Solvency II, financial when it comes to MCEV and accounting as with IFRS 4 Phase 2⁸. For instance, even though under IFRS, the cost of capital approach is a valid option for risk-margin computations, which should, in principle, be consistent with the Solvency II framework, principles to account for diversification or reinsurance benefits may differ. Consequently, models need to be rerun under different assumptions, obscuring the outputs. Regarding the key issue of discount rates, the scope of volatility adjustments might differ under the two metrics. As can be seen from this example, model multiplicity often arises from compliance constraints.

While the internal model approaches to solvency go along the previous lines, standard formulas are often remotely related to pricing and risk theory. We recall that, on the academic side, there is now a well-established theory of risk measures. As for the now classical arbitrage-free pricing theory, this theory relies heavily on probability tools. When it comes to applications, internal models can be implemented thanks to Monte Carlo (in most cases for insurance) or through filtered historical simulation (in most cases for market risks over short-time horizons). In the standard formula of Solvency 2, coupling risk measures for risk classes as if insurance risks could be modelled by multivariate Gaussian distributions with regulatory prescribed correlation parameters is a clear sign of that departure from standard statistical approaches that underlie internal models. A similar trend is at work in the banking world, either regarding the Basel III capital charge on securitizations⁹ or market and default risk treatments in the standard formulas of the FRTB (Fundamental Review of the Trading Book)¹⁰. In a number of cases, as securitization in the trading book, standard approaches will become compulsory and in other cases standard formulas will be involved in floors to the outputs of

⁸ Academic literature on evolving insurance accounting standards is scarce. We can mention Dal Moro et al. (2014), though they focus on reinsurance rather than the life insurance business. However, easy-to-access professional documentations and discussion papers by consultancies are widely published on the web.

⁹ <http://www.bis.org/bcbs/publ/d303.pdf>

¹⁰ <http://www.bis.org/publ/bcbs265.pdf>

internal models. Thus, the lack of risk sensitivity could become an issue. It may lead to improper capital allocations and misestimating diversification benefits.

Given the chosen horizon (1Y for Solvency II, default risk in Basel II and III) and the stated confidence level (99.5% in the case of Solvency II) standard formulas cannot be formally back-tested (see Chapter "Ex-ante model validation and backtesting" by Loisel and Nisipasu). On the other hand, standard formulas can be calibrated thanks to QIS so that they would still provide sensible results for actual lines of business. Standard formulas are meta-models of risk measures¹¹. As such, they need regular patches and updates when market environment changes: EIOPA issued a document entitled "The underlying assumptions in the standard formula for the Solvency Capital Requirement calculation" in July 2014, and some notes on the latest standard formula calibration in September 2014¹². These documents provide some important insights about the interest rate risk capital charge. It was calibrated on data prior to 2009, thus not accounting for the Eurozone 2011-2012 crisis or for the subsequent credit and quantitative easing. It was based on relative changes of market rates, including thirty years maturity interest rate swaps. The current version of the standard formula postulates a floor on the absolute positive changes in rates of 1%. Besides the trickiness of the approach, this calls for two remarks:

- There is a clear consistency issue between the assumption of a constant, non market based ultimate forward rate and the assumption that the 90-year might change in a +/- 80 bps range at a one-year horizon.
- Since regulators needed to accommodate to evolving market environments and close to zero or negative rates, some adjustments were deemed necessary. Consequently, it is wise to question the calibration of the standard formula in an ORSA framework. This is the typical recalibration issue when using meta-models. They may behave understandably at some point in time, once properly calibrated thanks to QIS exercises, but may fail to behave properly in a dynamic risk assessment framework. The major risk and modelling issue in the long term might actually be how the alpha term (driving the convergence to the ultimate forward rate) and the level of that ultimate forward rate are to be monitored...

In October 2013, the Basel Committee issued a discussion paper entitled "*The regulatory framework: balancing risk sensitivity, simplicity and comparability*"¹³. The motivation was largely due to increasing defiance on internal models, especially the divergence in RWA (Risk Weighted Assets) in banking and in the trading book. This could be equally of interest to insurers.

Standard formulas aim at being simple, though keeping a reasonable degree of risk sensitivity. Robustness, especially regarding correlation parameters or the frequency of extreme risks, is often a key issue. Since estimates of correlation parameters or default frequencies could be highly instable, there is a global trend towards the use of regulatory prescribed parameters. This is obviously the case with standard approaches.

¹¹ We refer to Devineau and Loisel (2009) for a discussion of the interplay between Solvency II standard formulas and internal models. Using the standard approach would clearly simplify computation of risk margins as compared with using internal risk models.

¹² https://eiopa.europa.eu/Publications/Standards/EIOPA-14-322_Underlying_Assumptions.pdf and <http://fr.slideshare.net/andrewcoffey1/notes-on-the-latest-standard-formula-calibration>

¹³ <http://www.bis.org/publ/bcbs258.pdf>

As for internal models, it is likely that modelling choices will be more and more constrained: An internal model that involves a large number of risk factors and deals with risk at a low level of granularity is likely to be more risk-sensitive but less robust, i.e. more prone to a number of arbitrary technical choices, and more difficult to run and manage (model governance). For that purpose, the Basel Committee compels a number of technical choices regarding internal models such as the maximum number of factors (two in the case of default risk within the trading book). Also, granularity and materiality of risk factors are closely monitored. Thus, on one hand, the estimation of correlation matrices (involved in diversification benefits) is constrained but, on the other, bank regulators are concerned about material risks under the radar, typically sovereign default and credit risks when it comes to European entities.

Standard formulas (should) give little room to interpretation and thus allow for greater comparability across regulated entities. In the banking world, disclosure of standard capital charges will become mandatory even for institutions using internal models. Moreover, too large departures between the two approaches will be more closely monitored and could lead to floors. Consequently, standard capital charges are part of the benchmarking of risk models (including RCAP, computation of RWA on hypothetical portfolios¹⁴ and benchmarking exercises conducted by EBA¹⁵).

Due to the above and since standard capital charges for market risks are to be considered as a credible alternative to internal models, the Basel Committee has come recently to much more granular market risk models (SBA, Sensitivity Based Approach¹⁶). As in Solvency II, shocks on risk factors and correlation parameters are given and aggregation across risk classes is carried out using a quadratic formula. This scenario-based approach is in the same vein as the CME margin model, SPAN¹⁷, and departs from the filtered historical simulation approaches (and to a much lesser extent to Monte Carlo) that prevail within financial institutions. The challenge is not to fall under undue complexity, for instance an excessive number of arbitrary inputs, when specifying standard formulas.

Overall, the juxtaposition of risk models built under different premises makes model governance trickier. In the insurance business, we are still at the dawn of this evolutionary process. Compared to the banking sector, regulatory solvency requirements are currently less binding than in the banking world where financial institutions need to build up huge amount of capital to meet the new regulatory constraints. On the other hand, the insurance industry is much more concerned by long-term solvency issues. Comparability and simplicity of risk models are thus clear issues: Currently EIOPA ORSA guidelines stay at a qualitative level and focus on organizational principles¹⁸. It might well be that modelling choices will eventually be restricted in the course of on-going supervisory processes¹⁹.

¹⁴ See <http://www.bis.org/publ/bcbs267.pdf> and <http://www.bis.org/publ/bcbs256.pdf>

¹⁵ <https://www.eba.europa.eu/-/eba-consults-on-technical-standards-on-supervisory-benchmarking-of-internal-approaches-for-calculating-capital-requirements>

¹⁶ See <http://www.bis.org/publ/bcbs265.pdf> , <http://www.bis.org/bcbs/publ/d305.pdf> and http://www.bis.org/bcbs/qis/biiiimplmoninstr_feb15.pdf

¹⁷ <http://www.cmegroup.com/clearing/span-methodology.html>

¹⁸ https://eiopa.europa.eu/Publications/Consultations/EIOPA-BoS-14-259_Final%20report_ORSA.pdf

¹⁹ To quote Eling et al. (2007), “We anticipate that the models with greatest predictive power will be highly complex, likely including some aspects of dynamic cash-flow. Complexity itself, however, does not guarantee a good model. Also, even if the model is reasonably successful at identifying financially weak companies, such

3) Constraint-driven modelling

We advocate that model multiplicity (and potential inconsistencies) arises from pragmatism on one hand and from diverging compliance constraints on the other.

Pragmatism implies looking inside the modeller's toolbox for the most suitable way to solve specific issues. Modellers could actually switch from one framework to another, introducing surrogate models for computational ease. To take an illustration, one might think it would be convenient to simulate rates through some AR(1) mean reverting Gaussian process up to the typical one year horizon and then compute swaptions or caps intended to hedge convexity risks through a BGM or Black type approach relying on log-normal forward Libor or swap rates. Thus, in many instances, models rely upon different premises. When it comes to probability models, quite often a non-nested, non-meaningful encompassing model is provided which obviously hinders comparisons.

Keeping a minimal degree of overall consistency involves mapping procedures and frequent recalibration of models (one on another, onto market prices). It is more an art than a science. Strong and reliable modelling skills are highly recommended to deal with a large number of interconnected parameters and to keep the business on track. Moreover mapping procedures may not be well suited when it comes to prospective assessment of risk and dynamical approaches of balance sheet items.

Consequently, there is a strong propensity to rely on market practices and well-recognized professional standards within a given business activity, the academic doxa being only in the background²⁰. Within insurance companies, key functions in the system of governance (second pillar of Solvency II) will be involved in setting up standards regarding the management of models. In Europe, EIOPA together with national supervisors is also to be strongly involved. This will parallel the expanding encompassing benchmarking exercises set up in the banking sector, as was the case with the AQR²¹ for the Eurozone banking sector: The AQR was a unique opportunity for a comprehensive review of all pricing models. Major Eurozone banks were required to document accurately their modelling assumptions in a standard format. Nowadays, any supervised entity would need to justify departures from commonly accepted modelling choices. To enforce financial stability, EBA has called for prudent valuation adjustments of fair valued positions. These reserves are deducted from the numerator of capital ratios. Among these adjustments, one is entitled "model risk AVA" (Additional Valuation Adjustment)²². It is noteworthy that as long as there is no divergence among market participants regarding valuation models, there is no need to make an additional valuation adjustment for model risk. This is a strong incentive for converging modelling approaches and the actuarial community is to play a key role for that purpose.

ability does not necessarily justify its costs. Complexity tends to require more data and results in higher costs to develop and maintain the resulting system, for both the insurers and the regulators".

²⁰ In the US, a number of cases regarding the inconsistency of active management practices, thus non-conforming to the efficient market hypothesis went to court. The evidence is mixed. Depending on the context, US courts can presume market efficiency and reject a presumption of prudence. To prevail against the efficient-market defence, participants would have to show active management can triumph over market averages. See <http://www.pionline.com/article/20140707/PRINT/307079997/court-backs-efficient-market>.

It is unclear whether we could face similar issues regarding the computation of reserves in a European context.

²¹ <https://www.eba.europa.eu/regulation-and-policy/market-risk/draft-regulatory-technical-standards-on-prudent-valuation>

<https://www.ecb.europa.eu/pub/pdf/other/assetqualityreviewphase2manual201403en.pdf?e8cc41ce0e4ee40222cbe148574e4af7>

²² <https://www.eba.europa.eu/documents/10180/642449/EBA-RTS-2014-06+RTS+on+Prudent+Valuation.pdf>

4) The recalibration puzzle

In many cases, constraint-driven modelling approaches will need to be closely monitored for changing market environments, models might need to be recalibrated one to another, to remain consistent at every point in time.

To stay in line with the above illustrative example, this would mean calibrating swaptions or cap volatilities depending on the simulated level of rates at a one-year horizon. Such mapping procedure of log-normal volatilities onto, say, the level and slope of interest rates can be achieved by different means, thus we introduce some hidden modelling complexity, with a number of more or less arbitrary and uncontrolled assumptions, while the starting point was ease of computation. It is also worth noting that the impact on interest rate sensitivities and thus ALM policies are likely to be one order of magnitude beyond the best estimate prices.

One of the main issues in the corporate CDO business has been the surge of bespoke CDOs where the underlying credit portfolio was not a standard CDS index, such as iTraxx Europe or CDX NA IG, for which one could easily access liquid market quotes and then rather easily infer a structure of implied base correlations. As for illiquid bespoke CDOs, derivation of the required bespoke correlations has been made thanks to a number of mapping procedures from the above implied correlations. Thus the marking of bespoke CDO tranches involved a blend of market data and modelling choices, difficult to control, due to the illiquidity of the bespoke tranche. As for most insurance liabilities, even though a number of inputs are calibrated to reliable market quotes, a high degree of model uncertainty remains (see chapter on model risk).

On the other hand, model governance process will make it quite difficult to update models when required. Let us go back to our favourite story of normal / log-normal rates. In early 2015 the negative rates region expanded. Not only overnight but also one-month interbank rates became negative, not to speak about core sovereign bond yields. Of course, if one would input a negative-forward Libor rate in a Black formula, this would result in a disruption of the computation... This might be sorted out by, say, using a shifted log-normal model at the price of a recalibration of volatilities. Changes need to be documented, reported and approved. When it comes to supervisory reporting and approval, this might lead to some kind of suspicion regarding modelling choices in the first place. Overall, this leads to what could be called sticky modelling choices.

Regarding risk-models under standard formulas, we already advocated that they might need to be recalibrated under changing market environments²³. The updating process is at regulatory discretion. Clear operational constraints imply that this will remain in the dark, even though this could undermine the outcome of an ORSA exercise.

Eventually, determining the extent to which the use of different models in different places does not break coherence of the global pricing or risk management framework is a matter of human expertise

²³ The low yield exercise conducted within 2014 EIOPA stress tests provides some interesting insights with respect to the recalibration of key parameters under extreme but plausible scenarios: In page 11 of <https://eiopa.europa.eu/Publications/Surveys/LIR%20Stock%20taking%20exercise%202014.pdf> it is stated that “several NSAs reported changes in the valuation approach for technical provisions in the last 2-3 years, of which two introduced the ultimate forward rate (UFR) method while another introduced an optional and temporary floor for the discount rate to calculate the technical provisions. These measures are primarily introduced to give some relief to insurance undertakings in a low yield environment”.

and thus liable to operational risks. One question to be asked is whether every person involved is fully aware and able to assess the consequences of making different models live together, recalibration issues, inconsistent range of possible values. There is legitimate concern that the experts involved in different departments, actuaries, ALM, financial engineers do communicate appropriately and report subsequently to key functions, including AMSB. Everyone should keep in mind the Columbia Shuttle disaster and miscommunication of key points from the bottom up to decision level²⁴. Conversely, top management financial expertise needs to be set at the right level to prevent any malfunctioning and so that black boxes do not run the business (see chapter on the role of models in management decision making by Renaud Dumora and Bernard Bolle-Reddat within this book).

5) Fitting models to models

At this stage, having discussed a number of approaches and issues regarding pricing and risk models, it is worth distinguishing between two approaches to models in insurance or banking businesses:

- Models could be closely related to probability and statistical concepts.
- Models could rather be seen as a processing engine fed by some inputs and intended to produce meaningful outputs. This second view takes its roots from computer science, engineering or systems theory.

When it comes to approximation and the construction of simplified models, a statistician will think in terms of distance between probability cdf under a suitable metric. The approximate model nested within the original or could be related to another probability model under which computations are made easier²⁵, for instance leading to closed-form expressions of insurance liabilities. Also, it is likely that a well-educated modeller will think of prices in terms of expectations and of VaR as a quantile of a loss distribution to be determined. In the same vein, approximation of cash flows will involve, say deviation for the mean and the variance of the error (difference between true and approximate cash-flows).

Under the second approach, the approximate model does not need to be related to what sounds like a properly specified standard academic pricing or risk model. In the theoretical mathematical finance framework, discount bond prices of different maturities are computed as the risk-neutral expectation of a stochastic discount factor. Many approaches developed for yield curve interpolation, such as splines, were not constructed with a dynamic arbitrage-free interest rate model in mind:

²⁴“As information gets passed up an organization hierarchy, from people who do analysis to mid-level managers to high-level leadership, key explanations and supporting information are filtered out. In this context, it is easy to understand how a senior manager might read this PowerPoint slide and not realize that it addresses a life-threatening situation”. Report of Columbia Accident Investigation Board, Volume 1, page 191, http://www.nasa.gov/columbia/home/CAIB_Vol1.html

²⁵ A stochastic volatility model, thus implying several risk factors, possibly with path dependence, can be approximated by a local volatility model, thus with a only one risk factor (underlying asset) and Markovian dynamics. This is related to the so-called Markovian projection technique (see Piterbarg (2006)). Under the approximate model, all call and put options are priced accordingly to the original model. The two models will only differ when it comes to pricing path-dependent options or when considering risk-management issues. This is a typical example where the two models being considered are built under the standard mathematical finance framework. In the interest rate risk context, we refer to Andersen and Piterbarg (2010) for a review of cross-calibration of options prices, with the purpose of dealing with log-normal rates and Black-Scholes type formulas.

- As for the Nelson-Siegel framework, a static parametric interpolation scheme might eventually be made consistent with the above dynamic setting; but this was not the primer modelling purpose.
- A Vasicek formula could be used for yield curve reconstruction. It involves volatility and mean-reversion parameters. When fitting the observed rates, say, thanks to a least square best fit, we may end up with values grossly inconsistent with swaptions prices. Here, we are faced with a model with different parameter inputs depending on the use, computation of bond prices or computation of swaptions. The Vasicek model is then viewed as a proxy of a better but difficult to handle more general interest rate model. Actually, under the objective based inference approach (see Gouriéroux and Laurent (1996)), different parameters may be required when using a misspecified model for different purposes.

In banking regulation, there is now a long story of fitting simple and convenient formulas to more complex models. One can think of the celebrated “maturity adjustment” in the computation of Basel II default risk weights. This is to account that the maturity of loans is greater than the one year prescribed horizon. Thus, non-defaulted loans could suffer from downgrades or credit-spread risk, which is not explicitly taken into account in the banking book²⁶. Different regulatory treatments regarding default and credit spread risk provides huge incentives regarding location of risk and is therefore a clear concern to life insurers.

Another important related issue is the computation of capital charges for default risk of securitization tranches²⁷. The so-called SSFA (Simplified Supervisory Formula Approach) is at the top of the hierarchy, i.e. the one to be used by major financial intermediaries. Given that securitized tranches were deeply involved in the 2008 subprime crisis, such highly technical issues have huge implications regarding the ability to maintain on-balance sheet positions within banks or whether they should be held by end-investors including life insurance investors. Not entering into undue technical details, the capital charge depends on the seniority of the tranche and involves an exponential decay with a prescribed regulatory parameter driving the decay. This simple parametric model is fitted first to another more sophisticated model such as Gordy and Jones (2003) and then final calibration involved QIS to assess the amount of extra capital required²⁸. Here, we are typically in a framework where a simpler model is calibrated to a more complex one. It is rather the ease of use (implementation, monitoring of key parameters by supervisors) that drives the specification of

²⁶ See pages 10, 11 of “An Explanatory Note on the Basel II IRB Risk Weight Functions” (2005), <http://www.bis.org/bcbs/irbriskweight.pdf>: “Maturity adjustments are the ratios of each of these VaR figures to the VaR of a “standard” maturity, which was set at 2.5 years, for each maturity and each rating grade. (...). In order to derive the Basel maturity adjustment function, the grid of relative VaR figures (in relation to 2.5 years maturity) was smoothed by a statistical regression model. (...) The regression formula for the maturity adjustments in the Third Consultative Paper is different from the one in the Revised Framework of June 2004.” Thus, that adjustment is related to the ratio of one year VaR for a one-year maturity loan to the one-year VaR of a 2.5 years maturity loan. This ratio is computed under a structural credit risk model and then regressed onto the logarithms of the default probabilities. One interesting issue regarding this important regulatory feature is the (in)ability to monitor the required recalibrations over a long period of time: It may be that the experts involved first are no longer in place years after implementation of the proxy model, and that details regarding the purpose of the approach and implementation practicalities are lost.

²⁷ <http://www.bis.org/bcbs/publ/d303.pdf>

²⁸ See http://www.bis.org/publ/bcbs_wp22.pdf for details regarding the calibration.

the simpler model, rather than an economic assessment of risks²⁹. Beyond providing comparisons between insurance and banking standard approaches to default risks, it illustrates a regulatory trend.

Whether what we currently see in the banking industry and the building of regulatory risk models is applicable to the life insurance industry is an interesting but difficult-to-answer question: A number of Solvency II issues are still on the agenda, not to speak about the conduct of stress-test, ORSA exercises or systemic risk assessments. EIOPA stress tests, though based on a few meaningful unpleasant but plausible scenarios could provide some alternative or complement ORSA with stochastic scenario generation. Insurance stress tests are thus quite useful to benchmark more sophisticated ORSA models: Easy-to-grasp simplified models can be viewed as sanity checks.

6) Model parameters set at managerial discretion

As discussed previously, multiple models are being used, sometimes with the intent of speeding up computations, sometimes models may have different purposes, say valuation or risk management. Besides consistency issues, it is clear that modelling a number of important quantities such rate of appreciation or the surplus participation rate on outstanding contracts is daring. Thus, it seems wise to directly manage such quantities. Instead of being a model output, it becomes an input, set at managerial discretion. At least, it makes it unequivocal that we will not rely on a black box and we can assert that the managed parameter has a clear and understandable meaning. Moreover, it could lead in further simplifications in the assessment of life insurance liabilities. If, say surplus participation rate would depend on previous rate of returns on assets, we might be faced with a complex path-dependent payoff. Introducing a constant parameter could dramatically ease computations. Let us briefly investigate the potential drawbacks of such a tempting shortcut:

- While the above approach would be certainly meaningful, when the number of such key parameters is small, we could be faced with managing a large number of such parameters. Then, to stay in line with the above example, we would need to introduce some connections among parameters. Say participation rate would be higher in countries where competition among insurers is tougher. Then complexity is back, we now need a model to relate the parameters.
- Regarding compliance and supervisory approval, it puts more pressure on the key functions. Are the chosen parameters set appropriately? Can we rely on expert opinion? What is the decision process regarding updating?

7) Approximation issues for pricing models in the finance and insurance contexts

In this section, we will first recall well-known concepts and approaches of approximation in a general financial context and we will then deal with insurance specific topics. Even though it is dedicated to life insurance modelling issues and we will try to keep technicalities to a minimum, it is a bit more

²⁹ The standard approach to counterparty risk of derivative exposures (SA-CCR, <http://www.bis.org/publ/bcbs279.pdf>) provides some further examples of calibration of models onto models. This is documented in a companion Basel Committee paper: Foundations of the standardized approach for measuring counterparty credit risk exposures http://www.bis.org/publ/bcbs_wp26.pdf. Besides being the standardized capital charge for counterparty credit risk, the model is likely to be used in the now-celebrated leverage ratio. It will thus drive the ability of investment banks to hedge financial risks within life insurance companies via off balance sheet instruments.

academic by nature. It will also provide a flavour of the prevailing dualism regarding modelling approaches: in short, KISS as illustrated in the previous section versus rocket science.

Fortunately enough, most life insurance liabilities with embedded optionality do not involve complex payoffs, such as large number of risk factors and a blend of long/short risks that exacerbate correlation modelling issues and for which approximation methods usually collapse due to the curse of dimensionality. In most cases, insurance liability payoffs involve a smooth degree of path dependence and risks can be adequately captured by the level of current rates³⁰. As a consequence, it may well be that the simplified version of the model grasps the essential features of the original.

There are a number of techniques developed by finance quants and subsequently adapted and expanded to cope with life insurance specificities, among which approximation of cash-flows, the idea behind the replicating portfolios, or approximation of pricing formulas, such as LSMC (Least Square Monte Carlo). We also refer to Planchet and Robert in this book (Chapter from internal to ORSA models) regarding the use of closed-form formulas computed under simplified model assumptions.

The replicating portfolio technique consists in approximating a given payoff by projecting it (i.e. minimizing some suitable distance) on a set of base portfolios/payoffs/risk profiles for which prices are either directly inferred from market observables or easily derived from the pricing model. A linear combination of the base payoffs provides the approximation of the more complex payoff. Then, the approximated price is simply the price of the approximating portfolio. Let us recall that best estimate computation of reserves is associated with a linear pricing rule. Thus, the approximated price is a linear combination of the prices of the base payoffs with the same coefficients as those involved in the approximation of the payoff to be priced.

When the portfolio to be priced can be perfectly replicated, the price of the replicating portfolio is the true price, i.e. there is no pricing error, i.e. no discrepancy between surrogate and true pricing. This idea dates back to the pioneering work of Breeden and Liztenberger (1978). They computed the pricing density for a fixed-time horizon from call option prices with the same maturity. This was further extended to a multi-period framework by Dupire (1994). In the same vein as Breeden and Liztenberger (1978) or Derman et al. (1995), Carr and Madan (2001) provided a strikingly simple result directly relating the payoff $f(S_T)$ of a complex payoff/risk profile to standard call option payoffs $(S_T - K)^+$, where S_T stands for the price of the underlying at option maturity and K the call option strike:

$$f(S_T) = f(0) + f'(0) \times S_T + \int_0^{\infty} f''(K)(S_T - K)^+ dK.$$

Consequently, the price of $f(S_T)$ at time zero (we assume for simplicity that the underlying asset does not pay any dividend) is given by:

³⁰ Day-one shocks on interest rates as considered in a number of stress-tests clearly do not involve complex dynamics. For a number of life insurance companies, a long-lasting period of extremely low rates (“Japanese style”), followed by a taper tantrum, as experienced in the 2013 US market (delayed “inverse scenario” following EIOPA terminology) would be damaging, as it would be likely to be associated with a sharp increase in bond spreads in the periphery (both on sovereigns and corporates). This shows that path-dependency cannot be formally ruled out. We refer to EIOPA 2014 stress tests for further discussion:

<https://eiopa.europa.eu/Publications/Surveys/Stress%20Test%20Report%202014.pdf>.

$$\text{Price}(f) = f(0)e^{-rT} + f'(0) \times S_0 + \int_0^{\infty} f''(K)C(K,T)dK,$$

where r stands for the default-free short term rate and $C(K,T)$ for the price of a call option with strike K and maturity T . This idea of using the prices of simple options to compute trickier ones, as those involved in insurance liabilities (capital guarantees, triggered returns and redeemable features), dates back to Ross (1978). Pelsser (2003) provides an application in an insurance context.

An appealing feature of this approach is the direct connection between prices of complex liabilities and market prices of more standard traded options. The prices of the latter would be obtained directly for the market, leading to a model-free valuation. This bypasses using the notion of rather abstract pricing densities and risk-neutral probability, as mentioned earlier for digital caps.

However, as can be seen from the previous formula, the approach requires knowing call option prices for all strikes. The set of observable liquid option prices is quite small and some form of interpolation and, more importantly, arbitrary extrapolation technique of option prices is required to get a complete set of options prices. It is well known that rather standard interest rate derivatives such as CMS (constant maturity swaps) are quite sensitive to the chosen extrapolation scheme³¹. CMS are involved when coupon rates are indexed on long-term rates and allow to dealing with exposures to the change of the slope of the yield curve.

Also, while the Carr and Madan (2001) decomposition formula is straightforward when dealing with a single risk factor, it becomes trickier in greater dimensions. Bakshi & Madan (2000), Nachman (1988), Ross (1976), Zhang (1998) argue that correlation derivatives are required to decompose hybrid payoffs. However, since such complex options are not routinely traded, we are faced with a severe limitation of the approach. Going back to simplicity needs to project insurance liability payoffs onto say the current (at the same time the payoff is being paid) level of the short-term interest rates.

This is more or less the idea behind the replicating portfolio approach: It involves an approximation of the payoff through a linear combination of base payoffs that could be easily priced or calibrated to market quotes. As mentioned earlier, the practical scope seems rather limited but well-suited to most life insurance liabilities and embedded interest rate optionality. Among actuarial studies dedicated to this approach, we can mention Boekel et al (2009), Botvinnik et al (2014), Natolski and Werner (2014), Oechslein et al (2007) and Schrager (2008).

Among the issues at hand, when dealing with replicating portfolios, we could consider polynomials such as of Hermite family instead of call option payoffs. This can make sense if the payoff to be approximated is a smooth function. Whenever this payoff depends upon a Gaussian variable, such as the short rate in the Vasicek model, Hermite polynomials appear as a reasonable choice: It is known that these polynomials are orthogonal under the Gaussian measure and form a base of the space of all potential payoffs; among which of course, the insurance liabilities to be evaluated should rely. The theory that underpins the approach is not part of the core syllabus of actuarial studies but, nevertheless, linear operators are quite standard and well known to mathematicians. Payoffs can be computed through series expansion that is truncated to provide an easy-to-price approximation.

³¹ We refer to Andersen and Piterbarg (2010) for the static replication of CMS rates thanks to swaptions. The approach dates back to Amblard and Lebuchoux (2000).

While everything is neat from a mathematical perspective, practical implementation needs to address the following items:

- What is the most suitable choice of polynomials? There are a number of competitors to the Hermite family mentioned above.
- What is the correct level of truncation, i.e. the finite number of terms in the expansion?
- Can we control the approximation error, without of course having to compute the true price? In other words, could we provide ex-ante bounds to the difference between the approximation and the true prices.

As mentioned earlier, in today's applied finance and insurance modelling, pragmatism heads theoretical concerns. Models are implemented first and it is only in a second stage that their empirical performance is assessed. We refer to Li (2014), Beutner et al. (2015) for an investigation of above issues.

A dual approach, initiated in the finance world by Longstaff and Schwartz (2001), known as LSMC or Least Squares Monte Carlo, involves an approximation of the prices rather than of the payoffs³². We recall that computation of capital requirements, based on internal models, involves evaluating liabilities for every simulation node, which is computationally intensive. Thus, some form of interpolation from a number of well-sampled forward values of liabilities is required. This can be achieved through functional regression and kernel based estimation. While the approach performs quite well under one-factor interest rate models and when payoffs are not path-dependent, computational performance collapses in higher dimensions: Thus, the need of projecting a one-factor interest rate model³³ and the payoff onto the factor, whenever the payoff exhibits some form of path-dependence³⁴. We refer to Bauer et al. (2010) for applications to the life insurance context.

A general approach to the approximation issue relies on the analysis of the pricing operator³⁵. In that framework, Darolles and Laurent (2000) look for optimal approximations of payoffs and pricing formulas and show the duality in the two approaches. In the case of interest rate models, they deal with the case of Vasicek type models (mean-reversion of the short rate), where the Hermite polynomials are actually optimal and the case of a Brownian motion with reflecting barriers (corridor type dynamics). While this kind of stationary dynamics is suitable to approximation, it is also shown that standard Black-Scholes dynamics does not comply with the technical requirements for the

³² See also Stentoft (2004), Glasserman and Yu (2004).

³³ Markovian projection technique is theoretically appealing but leads to non-linear interest rate dynamics. On the other hand, the use of easy to deal with one-factor interest rate models (say of Vasicek type) implies dealing with recalibration at node points.

³⁴ In a banking context, Adam et al. (2009), consider the pricing and hedging of non-maturing deposits. Depending on the return on money market funds, deposits can be redeemed. Dynamic hedging of interest rate risks leads to some path-dependence in the optimal hedging portfolio. Approximation of the corresponding payoff by means of computing conditional expectation of the payoff on current Libor rates is investigated. This leads to a replicating portfolio based on caps and floors and eliminates the path-dependency issue at the cost of a loss of accuracy.

³⁵ See Ait-Sahalia et al. (2008) and the references therein for a review of methods and issues when using linear pricing operators in a Markovian setting. Hansen & Jagannathan (1997) also provide some background regarding approximation methods and pricing kernels.

approximation methodologies to behave properly. We also refer to Pelsser and Schweizer (2015) who compare LSMC and replicating portfolio techniques in an insurance context.

Rather than trying to approximate payoffs, as in the replicating portfolio approach, or pricing formulas as in LSMC, a simpler model could be considered from scratch. This is standard in the financial world. We already mentioned the log-normal approximations of forward swap rates in a BGM setting (see Andersen and Piterbarg (2010)). The computation of CVA (Credit Valuation Adjustments) for portfolios of complex interest rate derivatives within trading books of investment banks follows the same lines. CVAs are required to account for counterparty credit risk, especially in the case of uncollateralized trades. Under the standard set-up, the valuation adjustment involves the expectation of the positive exposure (EPE). Positive exposure is the maximum of zero and the present value of the considered book. Stated slightly differently, we need to compute a call option price (with zero strike) on a book of interest rate derivatives, which can include tricky options. This computation is usually done under a much simpler model, say, a Vasicek-type, than the official internal model used to price exotic interest rate derivatives. Bonnin et al (2014) have been quite successfully using this approach: they could dramatically speed up ORSA computations thanks to the use of closed formulas for best estimate of savings contracts under a proxy model.

Up-to-now, we have more or less discussed numerical issues. Still focusing on embedded optionality in life insurance contracts, we need to assess the relevance of the ESG and whether management actions or customer behavior is adequately described. The ability to efficiently monitor participation benefits' features and surrender options are of first importance to mitigate the short-term negative effects of rate increases on fixed income investments. These are partly model-based and involve a mix of customer behavior and of management actions. Commitment from management to follow the actions as set up in the model is not formal. Depending on difficult to quantify parameters, such as the regulatory environment at the time decisions need to be made, the intensity of competition among insurers, it may be optimal to deviate from the originally scheduled actions. Eventually, such models need to be articulated with the dynamics of default-free rates to provide trustworthy best estimates of liabilities and associated duration measures. Clearly, there is much more model risk as we move away from the field of actively traded fixed income products³⁶.

We have a poor view of the magnitude of the errors introduced at the various stages of the computation. This section illustrates the technical complexities involved and the bottom-up communication issues mentioned previously about the Columbia Shuttle disaster. Regarding business processes and operational risks, we need to rely on experts to assess the reliability of the approaches and the robustness of the outputs. This is likely to displease ASMB or regulators, but this the way things are.

8) Conclusion

Regarding the development and the use of models within life insurance business, the learning curve involves a trial and error process and, almost as usual in finance and insurance modelling, empiricism

³⁶ Would it mean that we might drop our guard with respect to, say default-free rates? Unfortunately, understating volatility of long-maturity rates would just add to the modelling maze: Errors add-up.

heads theory. Model multiplicity involves more or less ad-hoc recalibration procedures. It is quite difficult to make ex-ante assessment of potential inconsistencies. From a management perspective, a strong reliance on the skills of actuaries and quantitative modellers is required. Operational risks could arise and are difficult to monitor through standard auditing procedures (see chapter on the threat of model risk by Christian Robert for insurance companies). Special emphasis should be put on validation standards, sanity checks and bottom-up information so that ASMB is aware of the key modelling and business challenges (see chapter by Bolle-Reddat and Dumora within this book).

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