Managing counterparty risk in an extended Basel II approach

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Addressed points

- Looking for a convenient framework for modelling default correlation
- Aggregating credit portfolios
  - *Multiple factors and diversification effects*
- Choosing an appropriate risk measure
  - *VaR versus Expected Shortfall: a quantitative assessment*
- Assessing the Gaussian copula approach
- Dealing with correlation between recovery rates and default events

Overall purpose

- Default probabilities are given
- Copula:
  - *dependence structure between default events or default dates*
- Aim is to study how credit risk depends upon correlation
- Provide a framework to study diversification effects
  - *On loss distributions*
  - *Risk measures*
  - *CDO tranche premiums*

Overlook

- Gaussian copulas
- One factor Gaussian copulas
  - *Correlation sensitivities*
- More general correlation structures
  - *Intra and inter sector correlations*
  - *VaR and Expected Shortfall*
  - *CDO tranches*
- Beyond Gaussian copulas
  - *Clayton, Student t and multivariate exponential models*
- Correlation between recovery rates and default events
  - *Two factor model*
  - *Credit portfolios and CDO tranches*

Default dates: Gaussian copula

- CreditMetrics [1997], *Li* [2000]
- $i = 1, \ldots, n$: names
- $\tau_1, \ldots \tau_n$: default dates
- $N_1(t) = 1_{\{\tau_1 \le t\}}, \ldots, N_n(t) = 1_{\{\tau_n \le t\}}$: default indicators
- $F_1(t) = Q(\tau_1 \le t), \ldots, F_n(t) = Q(\tau_n \le t)$: default probabilities
- $V_1, \ldots, V_n$: Gaussian vector with covariance matrix Σ
- $\tau_i = F_i^{-1}\left(\Phi\left(V_i\right)\right)$, where Φ Gaussian cdf
- Full specification of joint dependence of default dates

Default dates: one factor Gaussian copula

- Basel II, *Vasicek* (1997)
- $V_i = \rho_i V + \sqrt{1-\rho_i^2}\,\bar{V}_i,$
- where $V, \bar{V}_1, \ldots, \bar{V}_n$ are independent Gaussian variables,
- V : common factor, $\bar{V}_1, \ldots, \bar{V}_n$: idiosyncratic risk
- $\rho_1, \ldots \rho_n$ correlation parameters
- $\tau_i = F_i^{-1}\left(\Phi\left(V_i\right)\right)$, where Φ Gaussian cdf
- Independence between default dates given factor V

Pros

- Parsimonious (*n* parameters)
- Explicit losses for large portfolios
- Benchmark Basel II
- Analytical computations for VaR and Expected Shortfall
- Analytical computations of CDO tranches

Cons

- Constrained correlation matrix
- Gaussian copula?
- Computation of correlation sensitivities

Correlation parameters

- Regulatory correlations:
  - $\rho = 0.12 \times \frac{1-e^{-50\times PD}}{1-e^{-50}} + 0.24 \times \left(1 - \frac{1-e^{-50\times PD}}{1-e^{-50}}\right)$ for corporate exposures
  - Varies between 24% for $PD = 0\%$ to to 12% for $PD = 100\%$
  - $\rho = 0.03 \times \frac{1-e^{-35\times PD}}{1-e^{-35}} + 0.16 \times \left(1 - \frac{1-e^{-35\times PD}}{1-e^{-35}}\right)$ for retail exposures
- Use of implied correlations from CDOs
  - Friend & Rogge [2004] report implied correlation between 5% and 19% on Euro Triboxx tranches on Nov. 13, 2003

- Use of historical data from default events (Schmit [2004]), or credit spreads (KMV) or asset returns (Pitts [2004])

Correlation sensitivities

- Prices of CDO tranches, one factor Gaussian copula, as a function of correlation

ρ	equity	mezzanine	senior
0 %	5341	560	0.03
10 %	3779	632	4.6
30 %	2298	612	20
50 %	1491	539	36
70 %	937	443	52
100%	167	167	91

CDO margins (bp pa) Gaussian copula

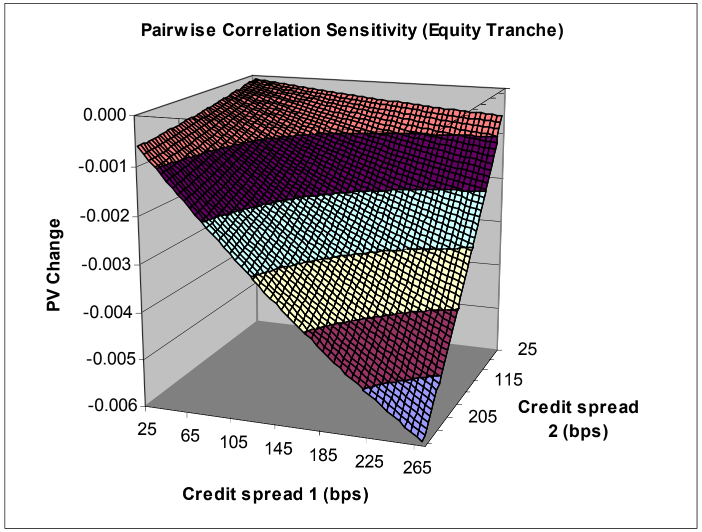
- 5 years, 100 names, credit spreads = 100bp,
- $\delta = 40\%$, attachment points: 4%, 10%
- Increase in correlation leads to fatter tails (see senior tranche)
- Intermediate losses (mezzanine tranche) not very sensitive to ρ

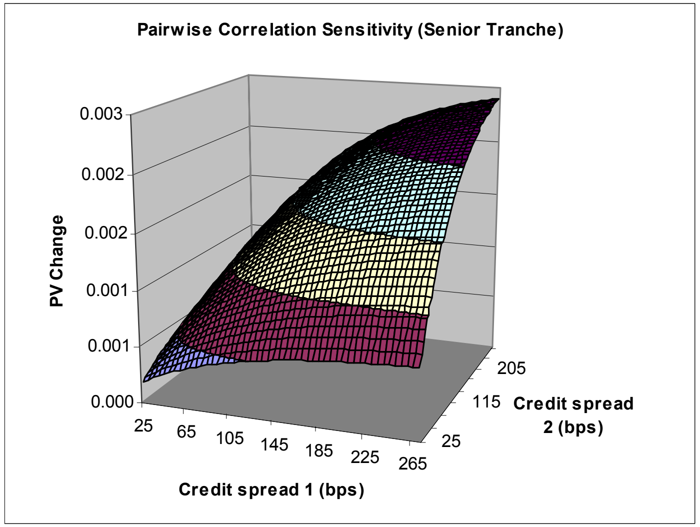
Pairwise correlation sensitivities

- 5 year CDO tranches: senior, mezzanine, equity
- Attachment points: 4%, 15%
- 50 names, credit spreads = 25, 30, 35,... up to 270 basis points.
- Recovery rates = 40%
- Constant correlation = 25%
- Pairwise correlation bumped from 25% to 35%
- Changes in the PV of the tranches (*buyer of credit protection*):

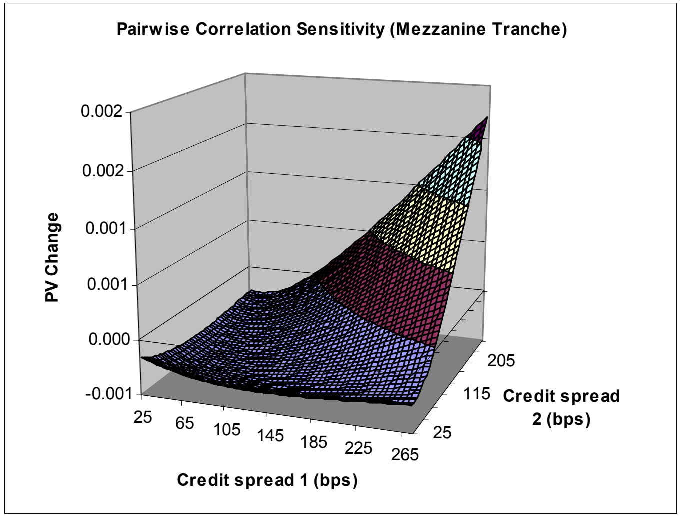
Correlation sensitivities

- Senior tranche has a positive correlation sensitivity
  - *higher correlation means poorer diversification*
  - *higher volatility on aggregated losses*
  - *senior tranche has positive vega (long call)*
- More pronounced effects for higher spread names
- Equity tranche has negative sensitivity





Pairwise correlation sensitivities



- Mezzanine tranche has smaller sensitivity with respect to correlation parameters
  - However positive correlation sensitivities for high credit spreads
- Analytical computations
  - Gregory & Laurent [2004], "in the core of correlation", www.defaultrisk.com

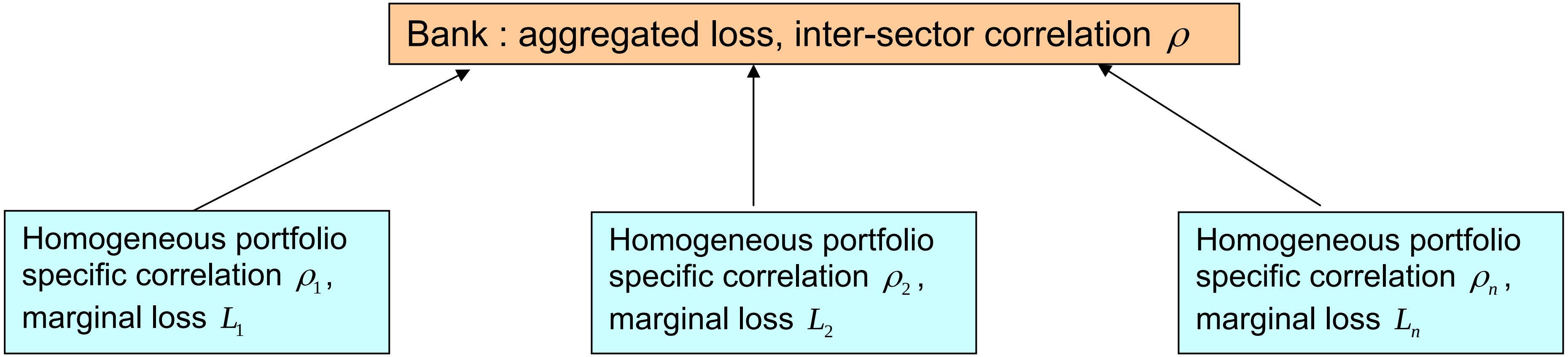
Correlation matrix with inter and intra-sector correlation

- i, name, $k(i)$ sector
- $V_i = \rho_{k(i)} W_{k(i)} + \sqrt{1-\rho_{k(i)}^2}\,\overline{V}_i$, $W_{k(i)}$ sector factor, $\overline{V}_i$ specific risk,
- $W_{k(i)} = \rho W + \sqrt{1-\rho^2}\,\overline{W}_{k(i)}$, W global factor.
- ρ, systemic or inter-sector correlation
- Number of factors = number of sectors + 1

- Correlation matrix $$\begin{pmatrix} 1 & \beta_1 & \beta_1 & & & & & & & \\ \beta_1 & 1 & \beta_1 & & & & & & \gamma & \\ \beta_1 & \beta_1 & 1 & & & & & & & \\ & & & 1 & & & & & & \\ & & & & . & & & & & \\ & & & & & . & & & & \\ & & & & & & 1 & & & \\ & & & & & & & 1 & \beta_m & \beta_m \\ & \gamma & & & & & & \beta_m & 1 & \beta_m \\ & & & & & & & \beta_m & \beta_m & 1 \end{pmatrix}$$

Correlation matrix with inter and intra-sector correlation

- One factor models within a sector + inter-sector correlation ρ
  - 100% sector correlation leads to Basel II



- Credit retail type portfolio:

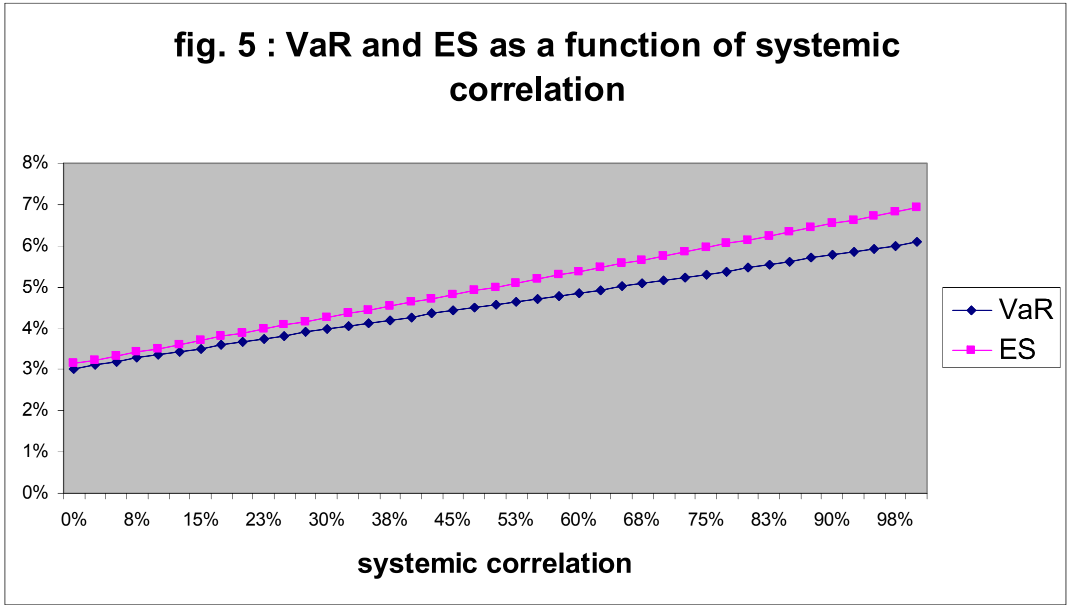
Line	1	2	3	4	5	6	7	8	9	10	11	12	13	14
EAD_J	14%	20%	7%	10%	10%	7%	8%	2%	6%	1%	1%	5%	7%	3%
PD_J	0.06%	0.18%	0.24%	0.42%	0.60%	0.84%	1.44%	3.18%	3.24%	4.56%	7.20%	7.33%	16%	55%
ρ_J	16.7%	16.1%	15.8%	14.9%	14.2%	13.2%	11.1%	6.9%	6.8%	5.0%	3.2%	3.2%	2.1%	2.0%

Correlation matrix with inter and intra-sector correlation

- VaR based risk measure
  - $\xi_\alpha(L) = \inf \left(x, P\left[L - E(L) \leq x\right] \geq \alpha\right),$
  - quantile based on unexpected losses
  - $\alpha = 99.9\%$
- Expected shortfall: $ES_\alpha(L) = E^P\left[L - E(L) \middle| L > VaR_\alpha(L)\right]$
  - average magnitude of unexpected losses given losses are greater than VaR

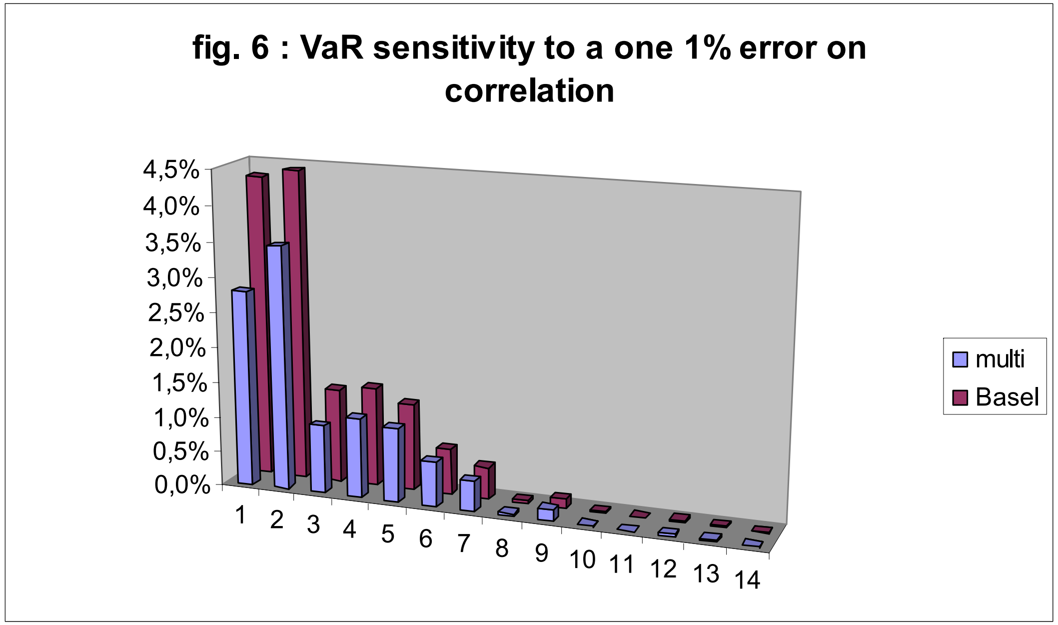
	ζ (VaR)	κ (Expected Shortfall)
$\rho = 100\%$ (Basel II)	6,1%	6,9%
$\rho = 50\%$ (multifactor model)	4,6%	5,0%
Relative variation	-25%	-27%

VaR, Expected Shortfall and systemic correlation ρ



- Risk measures change almost linearly wrt to systemic correlation
- Basel II: $\rho = 100\%$ no sector diversification
- Sector diversification lessens capital requirements
    - See "Aggregation and credit risk measurement in retail banking", Chabaane et al [2003]

Dependence of VaR upon intra-sector correlation



- Elasticity of VaR wrt intra-sector correlation parameters:
  - $\frac{\rho_J}{\zeta} \times \frac{\partial \zeta}{\partial \rho_J}$
- Lines 1 and 2 correspond to subportfolios with highest credit quality

CDO tranches as a function of intra-sector correlation

- TRAC-X Europe index, 5 sectors, Inter-sector correlation = 20%

	0-3%	3-6%	6-9%	9-12%	12-22%
20%	1274	287	93	33	6
30%	1227	294	103	40	7
40%	1169	303	114	47	10
50%	1100	314	128	56	13
60%	1020	326	144	67	17
70%	929	337	167	81	22
80%	822	349	188	99	27

Bp pa

- Increase in intra-sector correlation means less diversification:
  - Thus higher volatility of credit losses
  - Senior tranche (buy protection): call on credit losses
  - Positive vega
  - Increase in senior tranche premiums

Gaussian copula for default times?

- Other standard dependence models:
  - Clayton, Student *t*, Multivariate exponential copulas
- Set a comparison approach:
  - Parameters of other models are calibrated to the Gaussian copula equity tranche
  - 5 years, 100 names, credit spreads = 100bp,
  - $\delta = 40\%$, attachment points: 4%, 10%
- Then reprice mezzanine (intermediate losses) and senior tranches (large losses)

Gaussian copula for default times?

ρ	0%	10%	30%	50%	70%	100%
Gaussian	560	633	612	539	443	167
Clayton	560	637	628	560	464	167
Student (6)	676	676	637	550	447	167
Student (12)	647	647	621	543	445	167
MO	560	284	144	125	134	167

mezzanine tranche (bp pa)

ρ	0%	10%	30%	50%	70%	100%
Gaussian	0.03	4.6	20	36	52	167
Clayton	0.03	4.0	18	33	50	167
Student (6)	7.7	7.7	17	34	51	167
Student (12)	2.9	2.9	19	35	52	167
MO	0.03	25	49	62	73	167

senior tranche (bp pa)

Correlation between default dates and recovery rates

- Gaussian variables with one factor structure for default events:

$$\Psi_i = \sqrt{\rho}\Psi + \sqrt{1-\rho}\bar{\Psi}_i \ ,$$

- Default event if $\Psi_i < \Phi^{-1}\left(PD_i\right)$,

- Where PD_i= default probability, Φ, Gaussian cdf

- Losses Given Default (LGD) also have a one factor structure:

$$\xi_i = \sqrt{\beta}\xi + \sqrt{1-\beta}\bar{\xi}_i,$$

- ξ_i Gaussian latent variable driving LGD, ξ factor for LGD
    - o Chabaane, Laurent & Salomon, “Double Impact”, www.defaultrisk.com

Correlation between default dates and recovery rates

- Merton type LGD: $\max\left(0,1-e^{\mu+\sigma\xi_i}\right)$, μ,σ asset value parameters

- A two factor model with factors Ψ,ξ

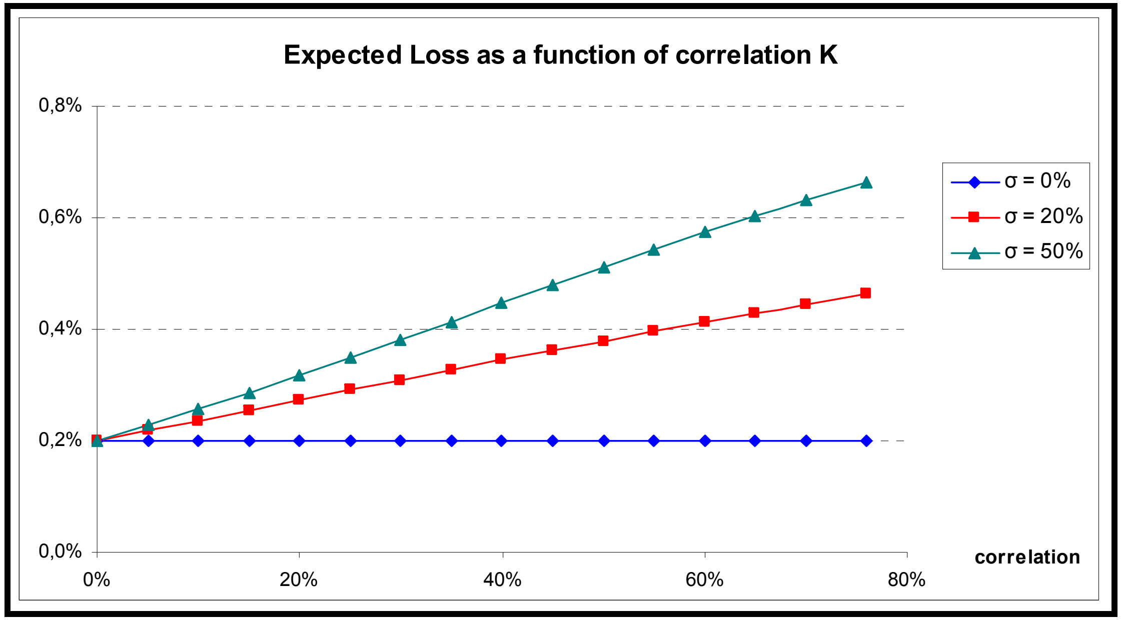
- Correlation structure between latent variables

	Ψ	ξ	$\bar{\Psi}_i$	$\bar{\Psi}_j$	$\bar{\xi}_i$	$\bar{\xi}_j$
Ψ	1	η	0	0	0	0
ξ	η	1	0	0	0	0
$\bar{\Psi}_i$	0	0	1	0	γ	0
$\bar{\Psi}_j$	0	0	0	1	0	γ
$\bar{\xi}_i$	0	0	γ	0	1	0
$\bar{\xi}_j$	0	0	0	γ	0	1

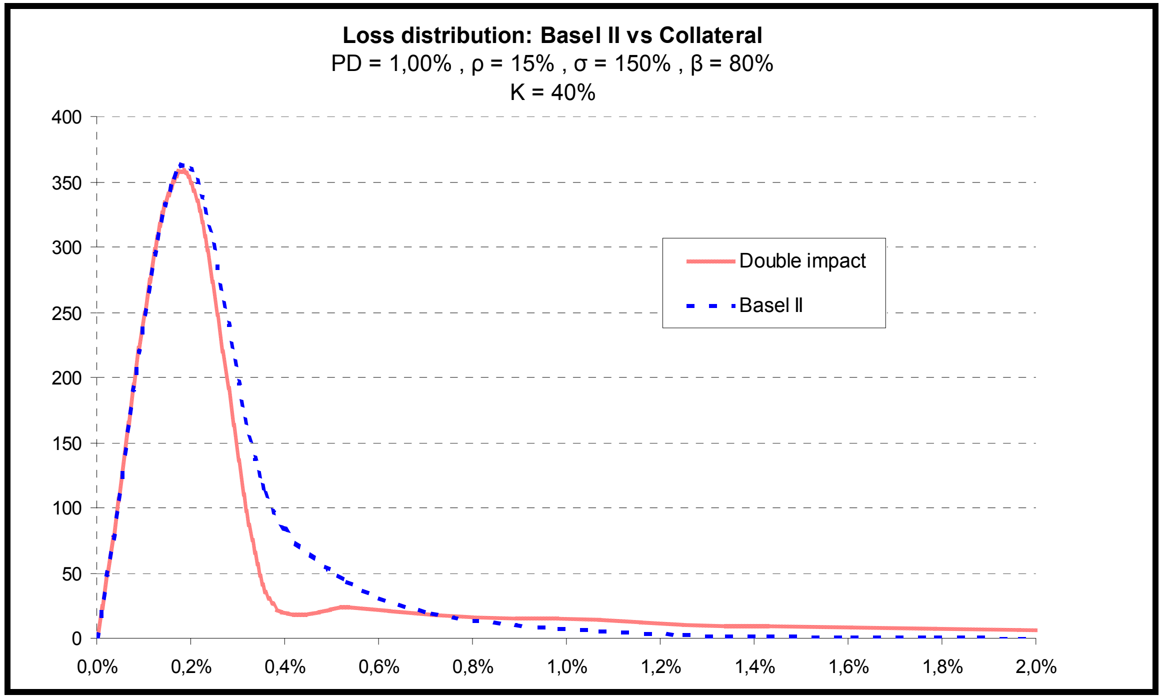
- Correlation between defaults and recoveries and amongst recoveries

Risk measures

- Expected loss as a function of correlation between default events and recovery rates
- Default probability = 1%, expected loss in Basel 2 = 0.2%



Risk measures



Loss Distribution: Comparison between Basel II and the extended approach

- fatter tails, less intermediate losses

Risk measures

η \ β	0%	20%	40%	60%	80%	100%
0%	158,9% *154,8%*	161,0% *160,2%*	164,2% *165,4%*	162,5% *164,7%*	159,3% *162,4%*	145,9% *152,1%*
20%	157,5% *153,9%*	175,4% *175,6%*	182,6% *183,7%*	186,8% *188,6%*	186,0% *192,5%*	172,8% *179,8%*
40%	160,2% *156,0%*	194,1% *196,6%*	207,9% *211,6%*	211,8% *218,7%*	212,6% *219,5%*	205,7% *217,2%*
60%	158,2% *155,2%*	207,4% *210,3%*	227,0% *231,1%*	238,9% *243,0%*	240,8% *249,2%*	234,1% *243,4%*
80%	159,6% *156,0%*	223,1% *229,4%*	244,1% *249,4%*	257,4% *265,1%*	264,5% *271,2%*	260,5% *273,4%*
100%	158,1% *153,9%*	238,9% *246,4%*	262,7% *268,0%*	276,5% *287,3%*	283,3% *296,3%*	286,8% *296,6%*

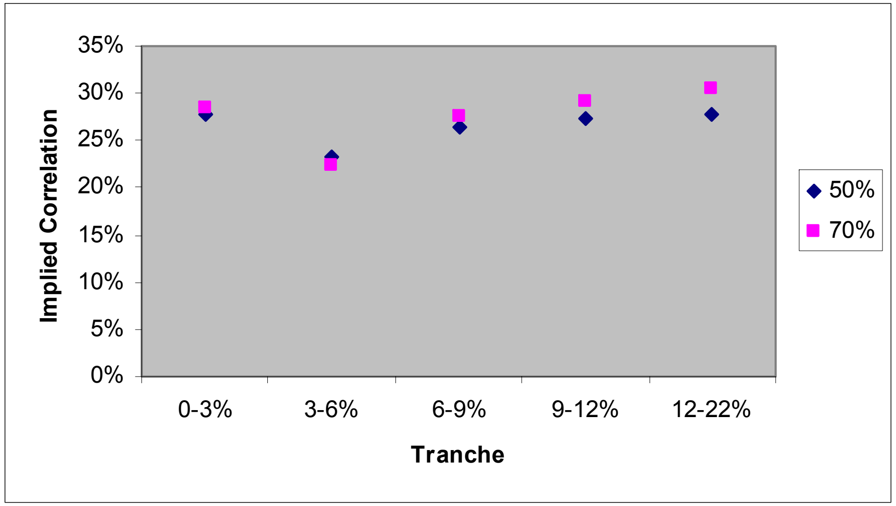
$\overline{\mathrm{VaR}}$ **and** $\overline{\mathrm{ES}}$ **(in italic) (γ = 50%) as a function of correlation parameters**

- Taking into account correlation between default events and LGD leads to a substantial increase in VaR and Expected Shortfall

CDO tranches

- Modelling of default dates τ_i and losses given default M_i

- $$\begin{cases} \tau_i = F_i^{-1}\left(\Phi(V_i)\right) \\ M_i = N_i \times \sum_{k=0}^{K}\left(1-\delta_k\right) 1_{b_{i,k} \le \xi_i < b_{i,k+1}} \end{cases}$$ Still a two factor model



Correlation smile implied from the correlated recovery rates

- Higher prices of senior tranches means fatter tails for credit loss distributions

Conclusion

- One factor Gaussian copula too simple
- Aggregating different sub-portfolios without 100% correlation
- Modelling with intra and inter-sector correlation accounts better for diversification effects
- Correlation between recovery rates and default events is an important feature
- Leads to higher credit risk
- Model risk: apart for country or systemic risk, Gaussian copula is a reasonable assumption