Contagion effects and the risk management of CDOs

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Presentation related to the papers

Hedging default risks of CDOs in Markovian contagion models **(2008)**

Available on www.defaultrisk.com

with Areski Cousin (Univ. Lyon) and Jean-David Fermanian (BNP Paribas)

And

Hedging issues for CDOs **(with Areski Cousin)**

Overview

- CDO Business and modeling context
  - Risks at hand in synthetic CDOs
  - Decline of the one factor Gaussian copula model for risk management purposes?
  - Recent correlation crisis
  - Unsatisfactory credit deltas for CDO tranches?
  - Relating credit deltas to structural models: “break-even correlation”
- “Tree approach” to hedging defaults
  - From theoretical ideas
  - To practical implementation of hedging strategies
- Empirical work
  - Robustness of the approach?
  - Contagion models, reduced-form models
- CDO of subprimes and SIVs

CDO business and modeling context

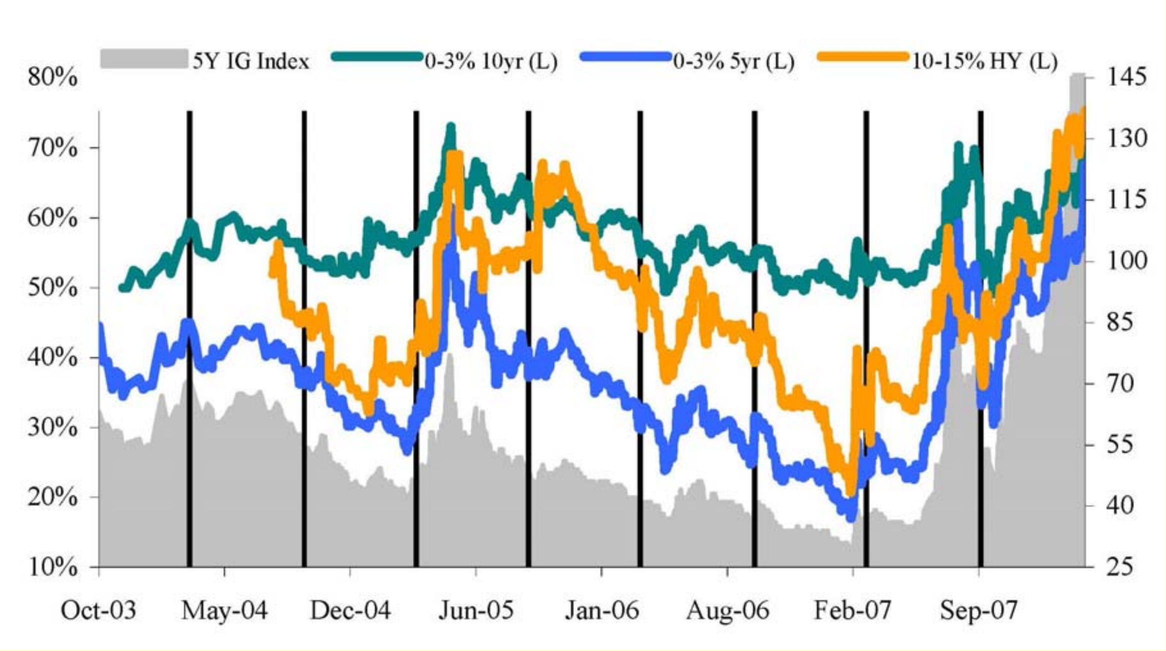
- Default risk
  - Default bond price jumps to recovery value at default time.
  - Drives the CDO cash-flows
- Credit spread risk
  - Changes in defaultable bond prices prior to default
    - Due to shifts in credit quality or in risk premiums
  - Changes in the marked to market of tranches
- Interactions between credit spread and default risks
  - Increase of credit spreads increases the probability of future defaults
  - Arrival of defaults may lead to jump in credit spreads
    - Contagion effects: Jarrow & Yu (2001)
    - Not consistent with the reduced-form approach

CDO business and modeling context

- Contagion effects and historical data
  - Das, Duffie, Kapadia and Saita : "*Common failings: how corporate defaults are correlated*" (2007)
    - Tends to show that there are contagion (or "frailty") effects on top of macroeconomic factors to explain the clustering of defaults
    - Case studies: Enron, Parmalat show mixed evidence
  - Jarrow, Guo and Lin: "*Distressed debt prices and recovery rate estimation*" (2008)
    - Question the notion of "economic date" which is usually before the legal default date (or "default event")
    - Jumps in spreads related to default and contagion effects should be considered at the "economic default date"
    - This may change the picture about the significance of contagion

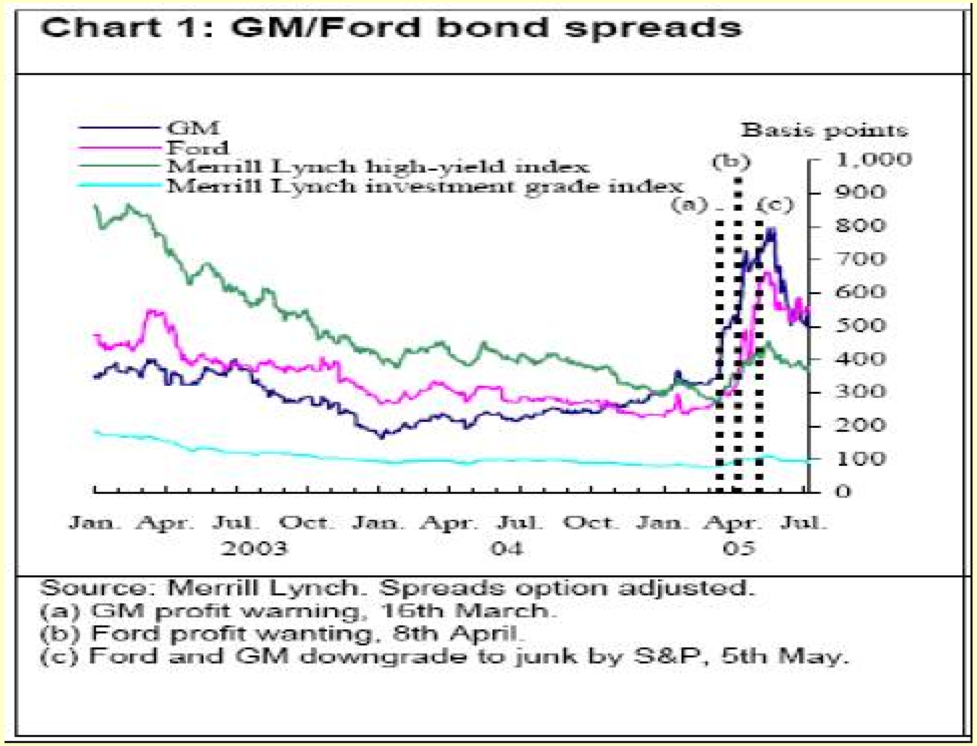
CDO business and modeling context

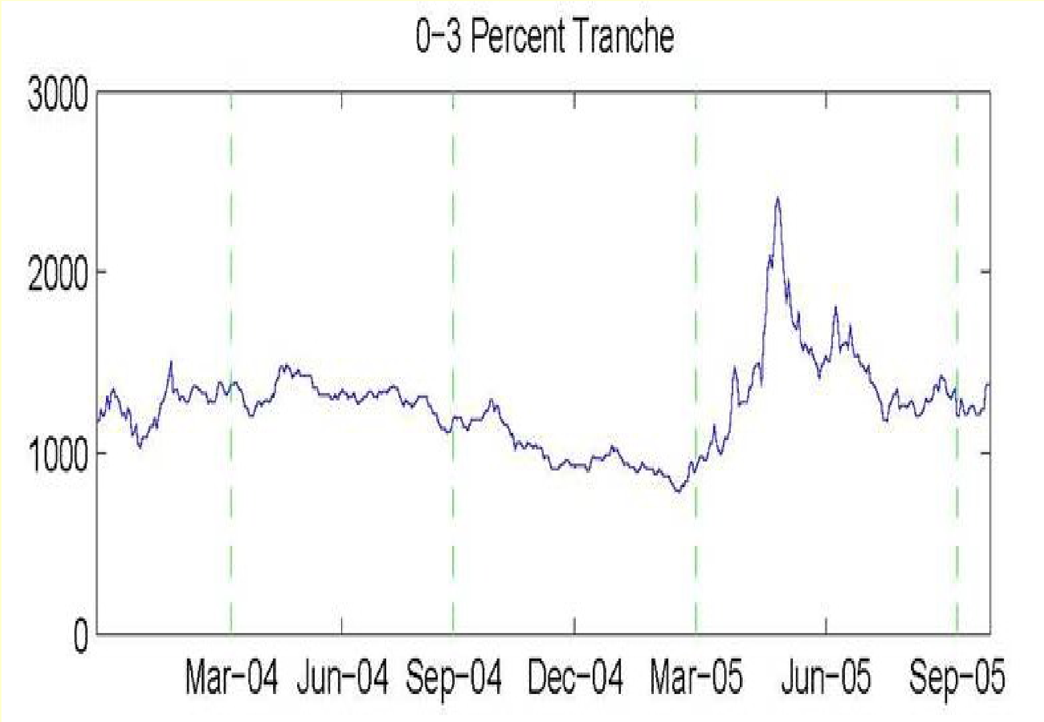
- Parallel shifts in credit spreads
  - As can be seen from the current crisis
  - On March 10, 2008, the 5Y CDX IG index spread quoted at 194 bp pa
  - starting from 30 bp pa on February 2007
    - ➢See grey figure
  - this is also associated with a surge in equity tranche premiums



CDO business and modeling context

- Idiosyncratic shift of a credit spread of a given name
    - Correlation crisis in May 2005 due to Ford and GM downgrades
    - Increase in the heterogeneity of the reference credit portfolio
    - Increase in equity tranche premiums





CDO business and modeling context

- Changes in the dependence structure between default times
  - In the Gaussian copula world, change in the correlation parameters in the copula
  - The present value of the default leg of an equity tranche decreases when correlation increases
- Dependence parameters and credit spreads may be highly correlated

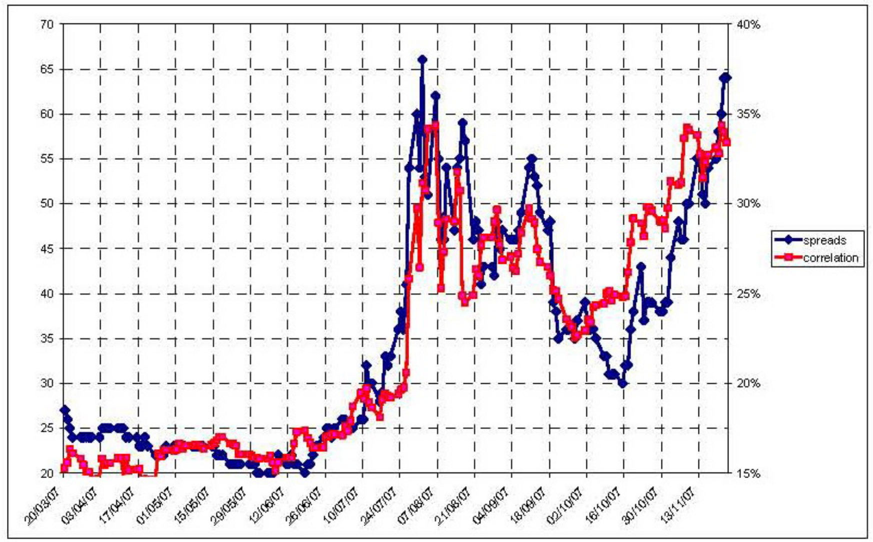
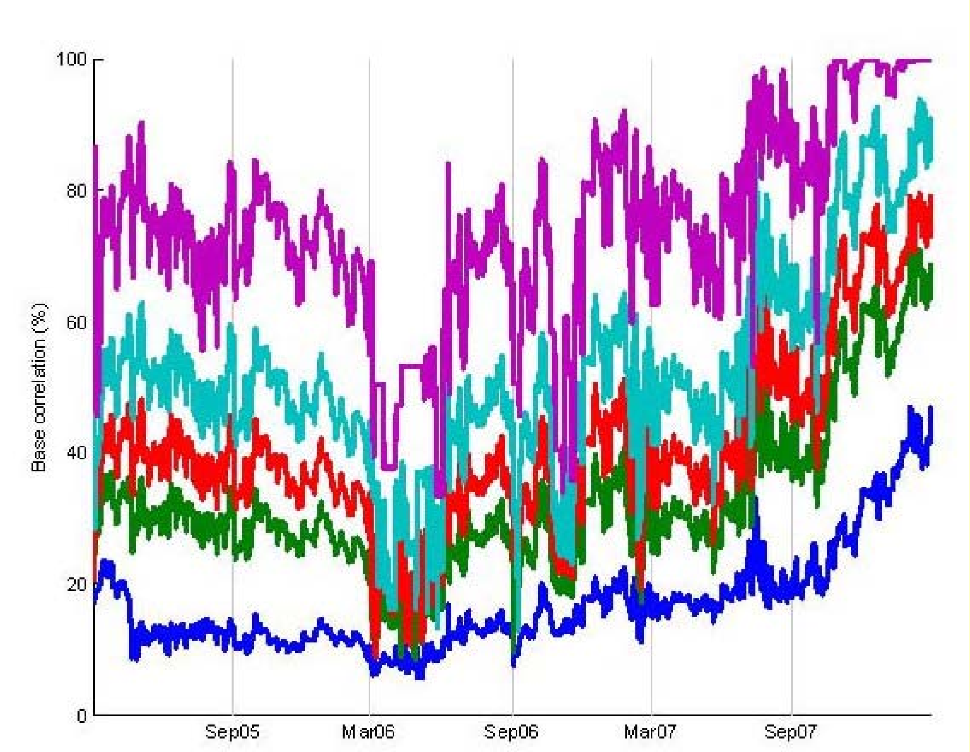


Figure 9. Credit spreads on the five years iTraxx index (Series 7) in bps on the left axis. Implied correlation on the equity tranche on the right axis

CDO business and modeling context

- Implied base correlation fluctuates through time
- Correlation skew:
  - implied correlation usually increases with detachment point
  - Reflecting fat tails in loss distributions
  - Cross-sectional effects



CDX base correlations
From C. Finger (2008)
RiskMetrics Group

CDO business and modeling context

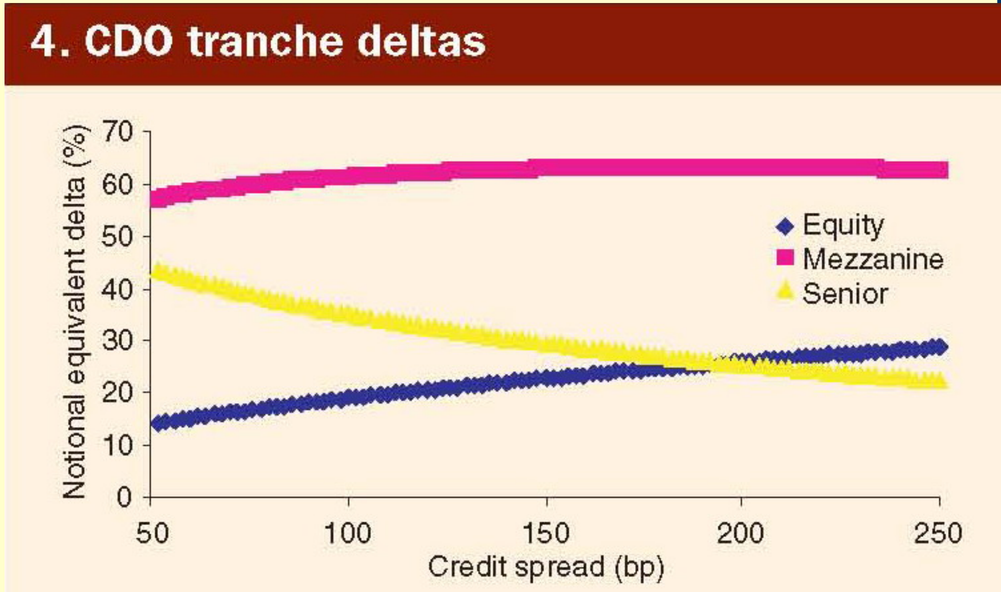
- One factor Gaussian copula remains the benchmark for pricing and risk managing synthetic CDOs
  - A very short reminder
  - $V, \bar{V}_i,\ i = 1, \ldots, n$ independent standard Gaussian variables

$$V_i = \rho V + \sqrt{1-\rho^2}\,\bar{V}_i$$

  - Default times $\tau_i = F_i^{-1}(\Phi(V_i))$
  - F_i risk-neutral marginal distribution function of default time *i*
  - Provided by calibration onto credit default swap (CDS) quotes
    - ➢ Given some recovery rate assumption
  - Analytical techniques for pricing tranches, large pool approximations, uniqueness of base correlations…

CDO business and modeling context

- CDS hedge ratios are computed by bumping the marginal credit curves
  - In 1F Gaussian copula framework
  - Focus on credit spread risk
  - individual name effects
  - Bottom-up approach
  - Smooth effects
  - Pre-crisis…



From “I will survive” (2003), RISK

- Poor theoretical properties
  - Does not lead to a replication of CDO tranche payoffs
  - Not a hedge against defaults…
  - Unclear issues with respect to the management of correlation risks

CDO business and modeling context

- We are still within a financial turmoil
  - Lots of restructuring and risk management of trading books
  - Collapse of highly leveraged products (CPDO)
  - February and March 2008 crisis on iTraxx and CDX markets
    - Surge in credit spreads
    - Extremely high correlations
    - Trading of [60-100%] tranches
    - Emergence of recovery rate risk
  - Questions about the pricing of bespoke tranches
  - Use of quantitative models?
  - The decline of the one factor Gaussian copula model

CDO business and modeling context

Morgan Stanley

MORGAN STANLEY RESEARCH

March 10, 2008
Structured Credit Analytics

CDX and iTraxx – Correlation Analysis and Delta Neutral Return

CDX Series 9

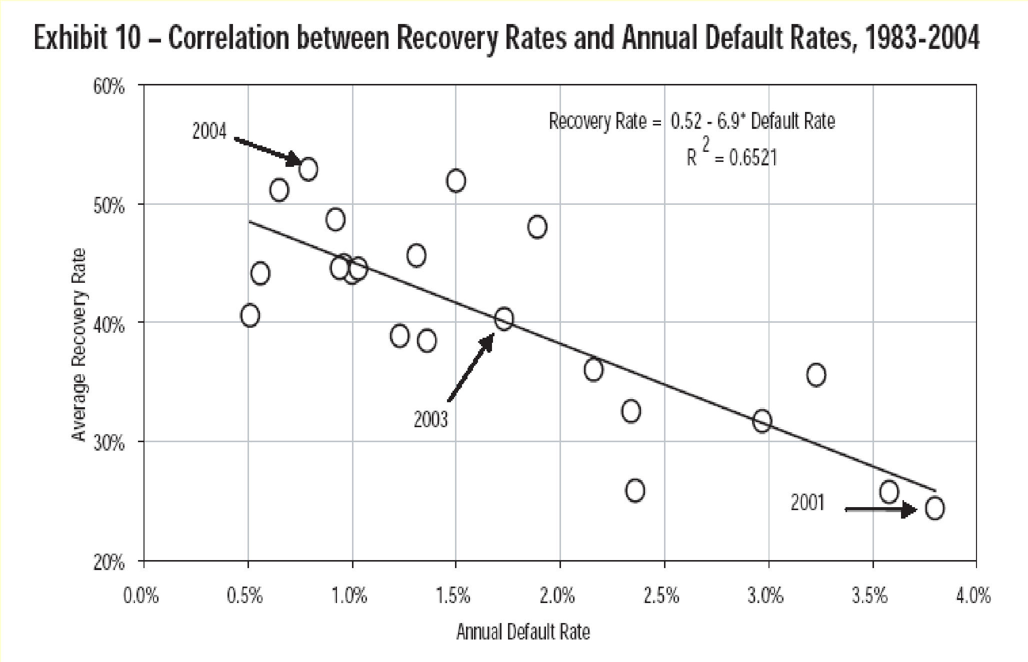
Index	Tranche	Bid	Ask	Mid 1 Week Change	Mid 1 Month Change	Index Level	Delta	Detach Corr.[1]	Corr. Skew	1 Week Change	Corr. Skew Z Score	Corr. Skew Rel. Value	CDX vs. iTRAXX Corr Skew	Value vs. iTraxx	Delta Neutral Returns 1 Week	1 Month	1 Year
5yr	Index	185.0	185.0	25.0	69.0												
5yr[2]	0-3%	67.5%	68.5%	10.4%	13.0%	185 bp	2.7	38%		-7.4%	1.19	RICH	(1.10)	CHEAP	-6.6%	0.6%	50.0%
5yr	3-7%	731	738	138	219		3.0	64%	25%	2.7%	1.07	RICH	0.97	FAIR	-2.1%	3.2%	9.0%
5yr	7-10%	405	415	75	141		1.8	73%	9%	0.5%	(0.31)	FAIR	0.21	FAIR	-1.2%	1.4%	-4.7%
5yr	10-15%	207	214	12	73		1.3	87%	14%	2.1%	1.15	RICH	0.96	FAIR	0.6%	1.2%	-1.8%
5yr	15-30%	126	130	13	62		1.0	108%	21%	1.2%	(0.01)	FAIR	0.18	FAIR	0.4%	-0.3%	-1.3%
5yr	30-100%	78	80	10	45		0.7								0.3%	-0.6%	-1.6%
7yr	Index	178.0	178.0	20.0	62.0												
7yr[2]	0-3%	71.0%	72.0%	8.0%	11.3%	178 bp	2.0	39%		-4.2%	1.38	RICH			-5.0%	1.8%	42.9%
7yr	3-7%	785	795	130	193		3.2	62%	23%	1.6%	1.00	FAIR			-2.6%	5.7%	18.2%
7yr	7-10%	452	460	79	106		2.5	71%	8%	0.1%	(0.66)	FAIR			-1.8%	5.4%	-2.8%
7yr	10-15%	256	265	21	73		1.6	83%	12%	1.6%	(0.09)	FAIR			0.5%	3.0%	-2.0%
7yr	15-30%	139	144	14	65		1.1	104%	21%	0.3%	(0.73)	FAIR			0.3%	-0.1%	-1.7%
7yr	30-100%	81	83	12	48		0.7								0.1%	-0.9%	-2.1%
10yr	Index	174.0	174.0	18.0	57.0												
10yr[2]	0-3%	74.0%	74.8%	7.9%	9.9%	174 bp	1.5	39%		-5.5%	1.33	RICH	1.31	RICH	-5.2%	1.5%	31.6%
10yr	3-7%	910	920	110	204		3.4	57%	18%	3.3%	3.13	RICH	3.16	RICH	-1.6%	7.5%	37.4%
10yr	7-10%	523	533	67	98		2.5	64%	7%	0.4%	(0.64)	FAIR	(0.64)	FAIR	-0.9%	8.5%	1.8%
10yr	10-15%	300	308	22	95		1.8	76%	12%	1.1%	(0.60)	FAIR	(0.60)	FAIR	1.0%	2.6%	3.0%
10yr	15-30%	150	155	14	63		1.2	99%	23%	0.3%	(1.99)	CHEAP	(2.01)	CHEAP	0.5%	0.0%	-1.6%
10yr	30-100%	79	82	10	45		0.7								0.2%	-1.2%	-2.9%
HY	Index	718.8	718.8	23.6	83.1												
HY[3]	0-10%	92.1%	92.6%	1.6%	2.5%	719 bp	0.7	44%		-2.4%	1.05	RICH			-1.1%	-0.4%	10.2%
HY	10-15%	75.3%	75.8%	2.5%	6.1%		1.8	49%	5%	1.2%	(0.12)	FAIR			-1.4%	-1.4%	8.0%
HY	15-25%	1,290	1,305	81	222		2.3	72%	23%	1.4%	2.73	RICH			-1.0%	-1.5%	1.6%
HY	25-35%	680	695	48	145		1.5	92%	19%	0.2%	(0.21)	FAIR			-0.6%	-1.6%	-6.3%
HY	35-100%	202	208	(2)	32		0.6								0.5%	0.0%	-2.1%
LCDX	Index	440.0	440.0	7.8	29.6												
LCDX[3]	0-5%	86.6%	87.5%	3.9%	-0.2%	440 bp	1.6	78%		-6.2%					-3.5%	3.0%	
LCDX[3]	5-8%	66.9%	67.6%	4.3%	1.0%		2.7								-3.6%	3.6%	
LCDX	8-12%	1140.0	1152.0	30.0	35.0		3.5								0.0%	3.0%	
LCDX	12-15%	740.0	748.0	-7.0	107.0		2.4								0.9%	-1.4%	
LCDX	15-100%	206.0	210.0	-13.5	20.0		0.6								0.7%	-0.8%	

[1]Correlation of tranche with 0% attachment and the same detachment point as the benchmark tranche, implied from market prices of benchmark tranches
[2]Points upfront plus 500 bp running
[3]Points upfront plus 0 bp running
Source: Morgan Stanley

MS provided implied correlations for senior tranches above 100%

CDO business and modeling context

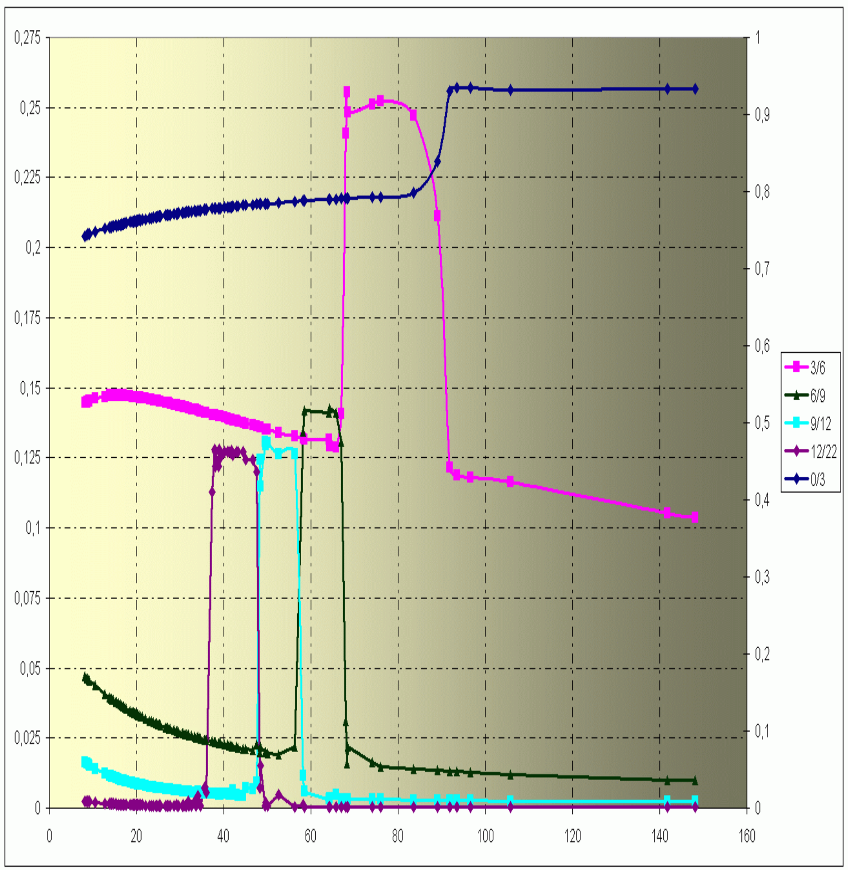
- **Recovery rates**
  - **Market agreement of a fixed recovery rate of 40% is inadequate**



  - **Currently a major issue in the CDO market**
  - **Use of state dependent stochastic recovery rates will dramatically change the credit deltas**

CDO business and modeling context

- Decline of the one factor Gaussian copula model
- Credit deltas in “high correlation states”
  - Morgan & Mortensen: “*CDO Hedging Anomalies in the Base Correlation Approach*”, Lehman Brothers (2007)
  - Close to comonotonic default dates (current market situation)
  - Deltas are equal to zero or one depending on the level of spreads
    - ➢ Individual effects are too pronounced
    - ➢ Unrealistic gammas



From Burtschell, Gregory & Laurent
Journal of Credit Risk (2007)

CDO business and modeling context

- The decline of the one factor Gaussian copula model + base correlation
  - This is rather a practical than a theoretical issue
- Negative tranche deltas frequently occur
  - Which is rather unlikely for out of the money call spreads
    - Though this could actually arise in an arbitrage-free model
    - Schloegl, Mortensen & Morgan, Lehman Brothers WP (2008)
  - Especially with steep base correlations curves
    - In the base correlation approach, the deltas of base tranches are computed under different correlations
  - And with thin tranchelets
    - Often due to “numerical” and interpolation issues

CDO business and modeling context

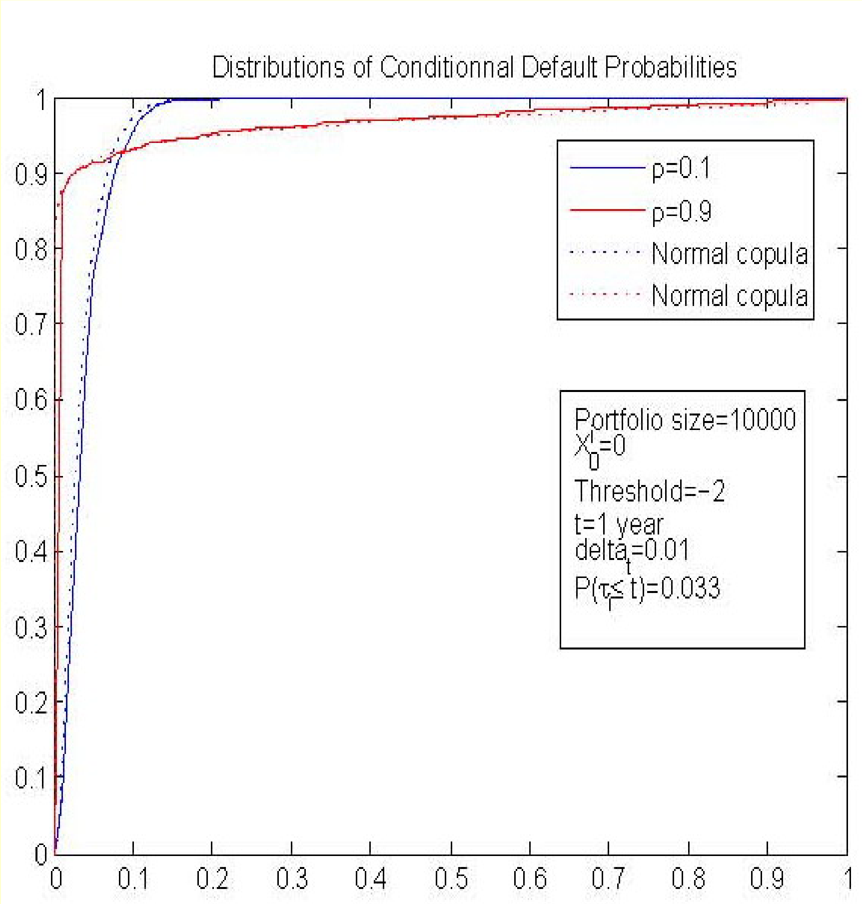
- No clear agreement about the computation of credit deltas in the 1F Gaussian copula model
  - Sticky correlation, sticky delta?
  - Computation with respect to credit default swap index, individual CDS?
- Weird effects when pricing and risk managing bespoke tranches
  - Price dispersion due to “projection” techniques
  - Negative deltas effects magnified
  - Sensitivity to names out of the considered basket

CDO business and modeling context

- Amongst all these issues, some good news might eventually occur for the one factor Gaussian copula
  - “break-even” correlation: Fermanian and Vigneron (2008)
  - Prior to default, perfect replication of a CDO tranche when using Gaussian copula deltas,
  - Provided that the Gaussian copula correlation is equal to the spread correlation
- How can we explain this?
  - Hull, Predescu and White: “*The Valuation of Correlation-Dependent Credit Derivatives Using a Structural Model”* (2005)
  - Cousin and Laurent: “*Comparison results for homogeneous credit portfolios*” (2008)
  - Houdain and Guegan: “*hedging tranche index products: illustration of the model dependency*” (2006)

CDO business and modeling context

- Hull *et al.* (2005) show that multivariate structural models provide almost the same CDO tranche quotes as the 1F Gaussian copula
  - First hitting times of some barriers by correlated Brownian motions
- Cousin and Laurent (2008) explain this by the nearness of conditional default probabilities which determine CDO tranche quotes
- This should extend to credit deltas
- The above multivariate structural model is associated with replicating deltas
- But lack of tail dependence between assets:
  - use of multivariate NIG processes
  - Houdain and Guegan (2006) actually use NIG type copulas



Cousin & Laurent (2008)

Tree approach to hedging defaults

- The “ultimate step” : complete markets
  - As many risks as hedging instruments
  - News products are only designed to save transactions costs and are used for risk management purposes
  - Assumes a high liquidity of the market
- Perfect replication of payoffs by dynamically trading a small number of « underlying assets »
  - Black-Scholes type framework
  - Possibly some model risk
- This is further investigated in the presentation
  - Dynamic trading of CDS to replicate CDO tranche payoff

Tree approach to hedging defaults

- What are we trying to achieve?
- Show that under some (stringent) assumptions the market for CDO tranches is complete
  - CDO tranches can be perfectly replicated by dynamically trading CDS
  - Exhibit the building of the unique risk-neutral measure
- Display the analogue of the local volatility model of Dupire (1994) or Derman & Kani (1994) for credit portfolio derivatives
  - One to one correspondence between CDO tranche quotes and model dynamics (continuous time Markov chain for losses)
- Show the practical implementation of the model with market data
  - Deltas correspond to “sticky implied tree”

Tree approach to hedging defaults

- Main theoretical features of the complete market model
  - No simultaneous defaults
    - **Unlike multivariate Poisson models**
  - Credit spreads are driven by defaults
    - ➢ Contagion model
      - **Jumps in credit spreads at default times**
    - ➢ Credit spreads are deterministic between two defaults
  - Bottom-up approach
    - ➢ Aggregate loss intensity is derived from individual loss intensities
  - Correlation dynamics is also driven by defaults
    - ➢ Defaults lead to an increase in dependence

Tree approach to hedging defaults

- Without additional assumptions the model is intractable
  - Homogeneous portfolio
    - Only need of the CDS index
    - No individual name effect
    - Top-down approach
      - **Only need of the aggregate loss dynamics**
  - Markovian dynamics
    - Pricing and hedging CDO tranches within a binomial tree
    - Easy computation of dynamic hedging strategies
  - Perfect calibration the loss dynamics from CDO tranche quotes
    - Thanks to forward Kolmogorov equations
  - Practical building of dynamic credit deltas
  - Meaningful comparisons with practitioner’s approaches

Tree approach to hedging defaults

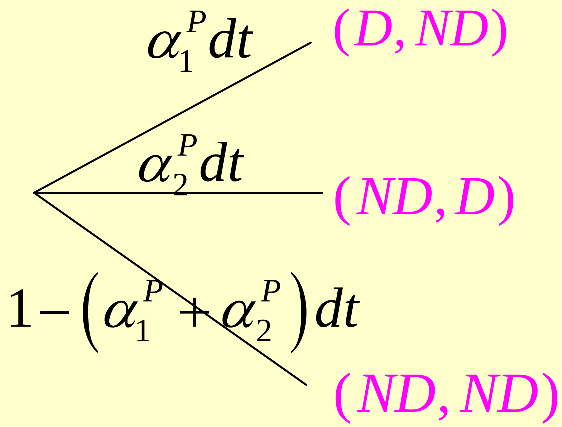
- We will start with two names only
- Firstly in a static framework
  - Look for a First to Default Swap
  - Discuss historical and risk-neutral probabilities
- Further extending the model to a dynamic framework
  - Computation of prices and hedging strategies along the tree
  - Pricing and hedging of tranchelets
- Multiname case: homogeneous Markovian model
  - Computation of risk-neutral tree for the loss
  - Computation of dynamic deltas
- Technical details can be found in the paper:
  - "hedging default risks of CDOs in Markovian contagion models"

Tree approach to hedging defaults

- Some notations :
  - τ_1, τ_2 default times of counterparties 1 and 2,
  - $\mathcal{H}_t$ available information at time *t*,
  - P historical probability,
  - α_1^P, α_2^P : (historical) default intensities:
    - $P\left[\tau_i \in [t, t+dt[\,\middle|\, H_t\right] = \alpha_i^P dt,\ i = 1, 2$
- Assumption of « local » independence between default events
  - Probability of 1 and 2 defaulting altogether:
    - $P\left[\tau_1 \in [t, t+dt[, \tau_2 \in [t, t+dt[\,\middle|\, H_t\right] = \alpha_1^P dt \times \alpha_2^P dt \text{ in } (dt)^2$
  - Local independence: simultaneous joint defaults can be neglected

Tree approach to hedging defaults

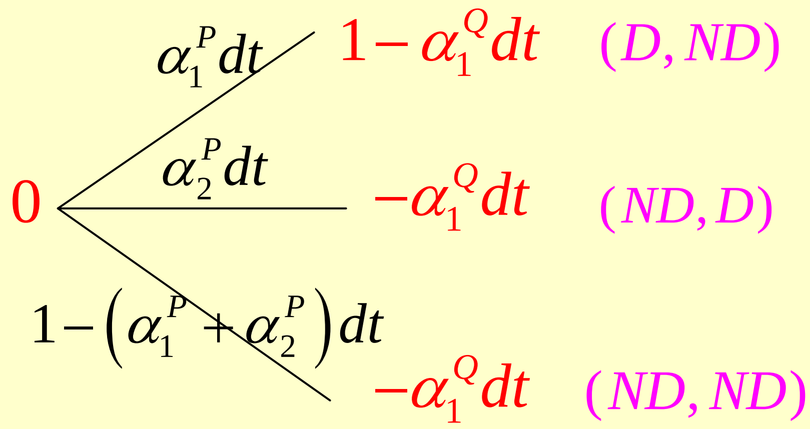
- Building up a tree:
  - Four possible states: (D,D), (D,ND), (ND,D), (ND,ND)
  - Under no simultaneous defaults assumption $p_{(D,D)}=0$
  - Only three possible states: (D,ND), (ND,D), (ND,ND)
  - Identifying (historical) tree probabilities:



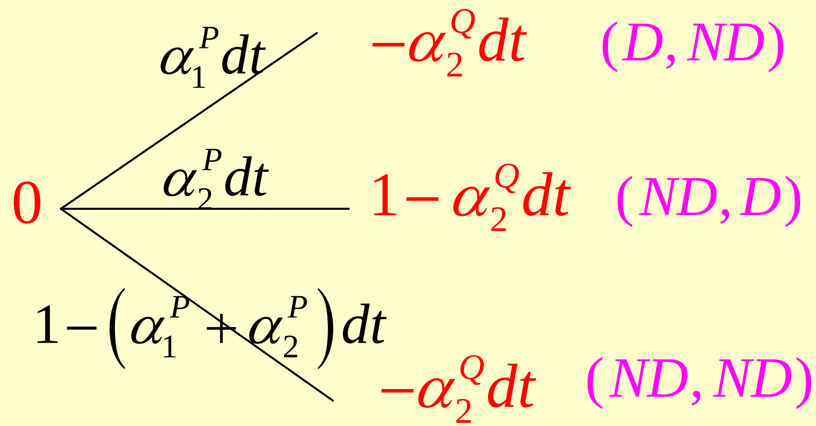
$$\begin{cases} p_{(D,D)} = 0 \Rightarrow p_{(D,ND)} = p_{(D,D)} + p_{(D,ND)} = p_{(D,.)} = \alpha_1^P dt \\ p_{(D,D)} = 0 \Rightarrow p_{(ND,D)} = p_{(D,D)} + p_{(ND,D)} = p_{(.,D)} = \alpha_2^P dt \\ p_{(ND,ND)} = 1 - p_{(D,.)} - p_{(.,D)} \end{cases}$$

Tree approach to hedging defaults

- Stylized cash flows of short term digital CDS on counterparty 1:
  - $\alpha_1^Q dt$ CDS 1 premium



- Stylized cash flows of short term digital CDS on counterparty 2:



Tree approach to hedging defaults

- Cash flows of short term digital first to default swap with premium $\alpha_F^Q dt$:

$$0 \longrightarrow \begin{cases} \alpha_1^P dt: & 1-\alpha_F^Q dt \quad (D, ND) \\ \alpha_2^P dt: & 1-\alpha_F^Q dt \quad (ND, D) \\ 1-\left(\alpha_1^P+\alpha_2^P\right)dt: & -\alpha_F^Q dt \quad (ND, ND) \end{cases}$$

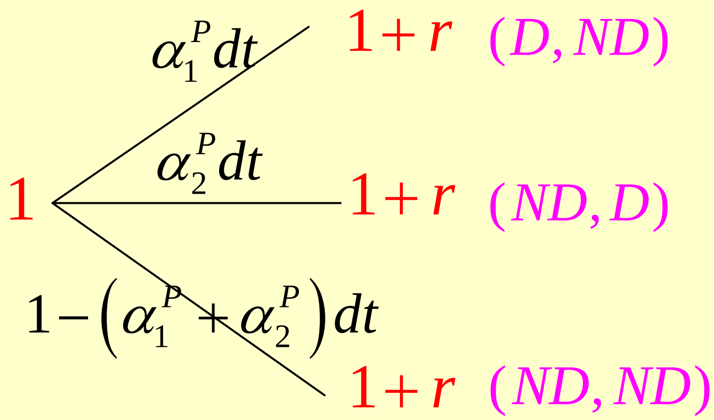
- Cash flows of holding CDS 1 + CDS 2:

$$0 \longrightarrow \begin{cases} \alpha_1^P dt: & 1-\left(\alpha_1^Q+\alpha_2^Q\right)dt \quad (D, ND) \\ \alpha_2^P dt: & 1-\left(\alpha_1^Q+\alpha_2^Q\right)dt \quad (ND, D) \\ 1-\left(\alpha_1^P+\alpha_2^P\right)dt: & -\left(\alpha_1^Q+\alpha_2^Q\right)dt \quad (ND, ND) \end{cases}$$

- Perfect hedge of first to default swap by holding 1 CDS 1 + 1 CDS 2
  - Delta with respect to CDS 1 = 1, delta with respect to CDS 2 = 1

Tree approach to hedging defaults

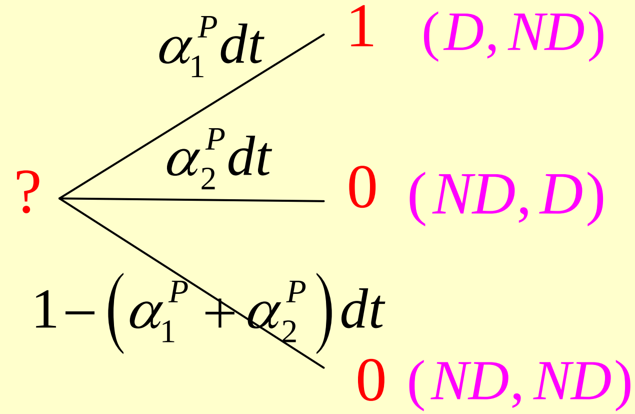
- Absence of arbitrage opportunities imply:
  - $\alpha_F^Q = \alpha_1^Q + \alpha_2^Q$
- Arbitrage free first to default swap premium
  - Does not depend on historical probabilities α_1^P, α_2^P
- Three possible states: (D,ND), (ND,D), (ND,ND)
- Three tradable assets: CDS1, CDS2, risk-free asset

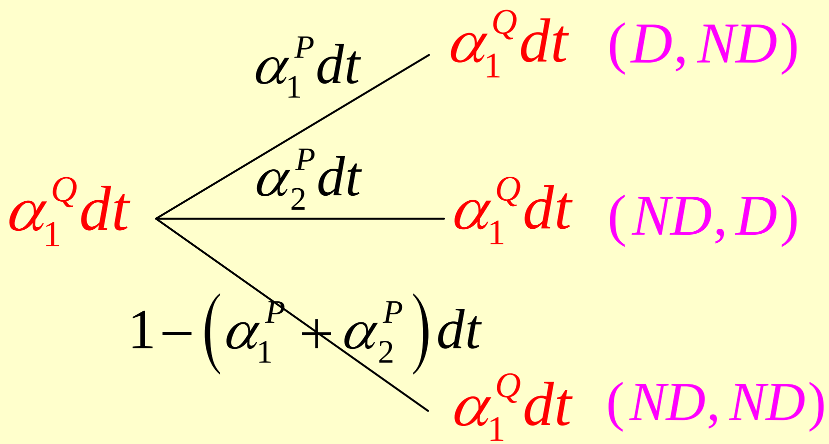


- For simplicity, let us assume $r = 0$

Tree approach to hedging defaults

- Three state contingent claims
  - Example: claim contingent on state (D, ND)
  - Can be replicated by holding
  - 1 CDS 1 + $\alpha_1^Q dt$ risk-free asset





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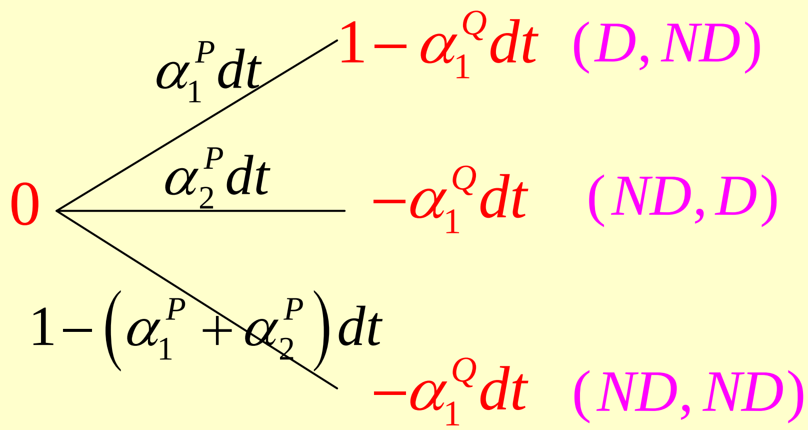
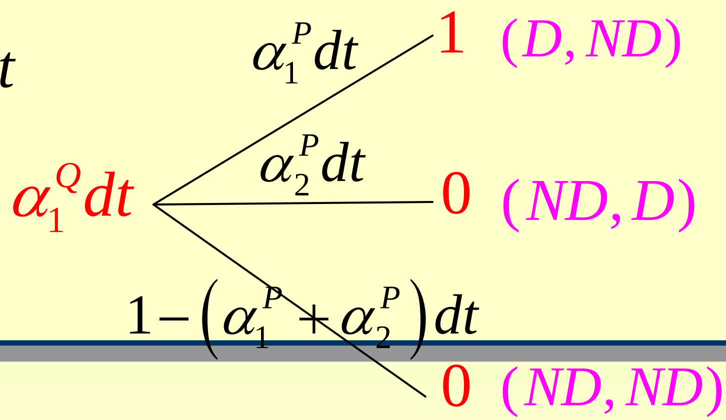
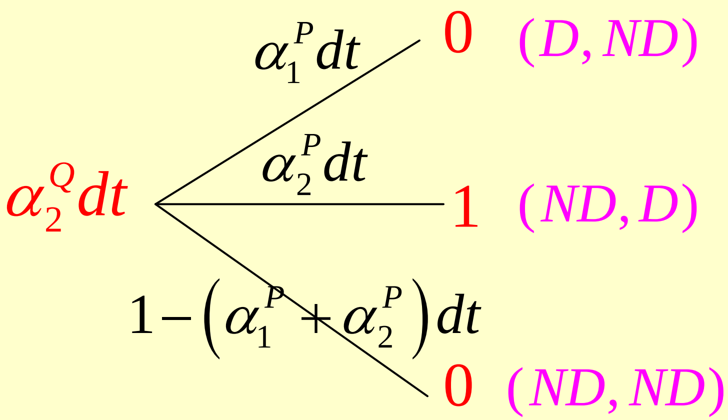


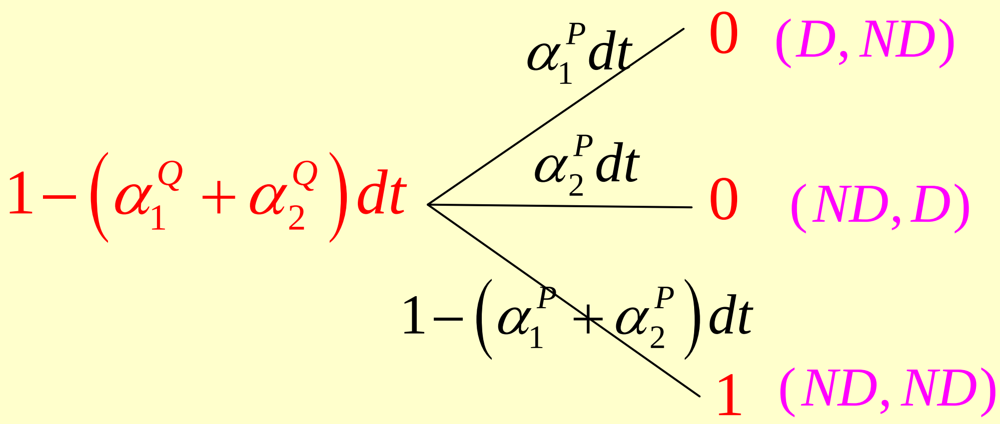
  - Replication price = $\alpha_1^Q dt$



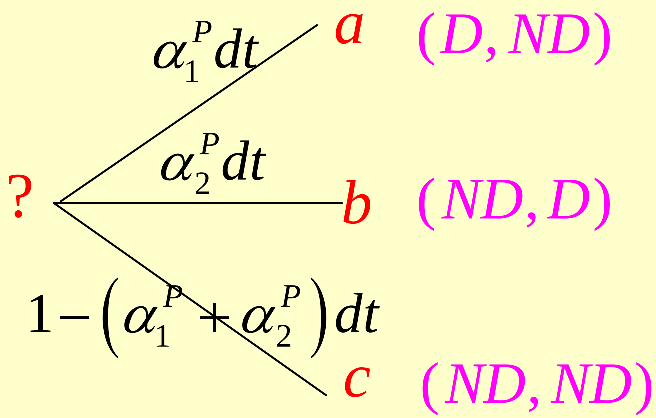
Tree approach to hedging defaults

- Similarly, the replication prices of the (ND, D) and (ND, ND) claims





- Replication price of:

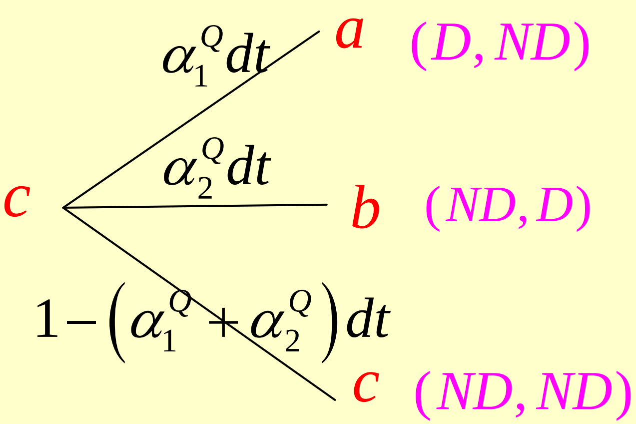


- Replication price = $\alpha_1^Q dt \times a + \alpha_2^Q dt \times b + \left(1-(\alpha_1^Q+\alpha_2^Q)dt\right)c$

Tree approach to hedging defaults

- Replication price obtained by computing the expected payoff
  - Along a risk-neutral tree

$$\alpha_1^Q dt \times a + \alpha_2^Q dt \times b + \left(1 - (\alpha_1^Q + \alpha_2^Q)dt\right)c$$



- Risk-neutral probabilities
  - Used for computing replication prices
  - Uniquely determined from short term CDS premiums
  - No need of historical default probabilities

Tree approach to hedging defaults

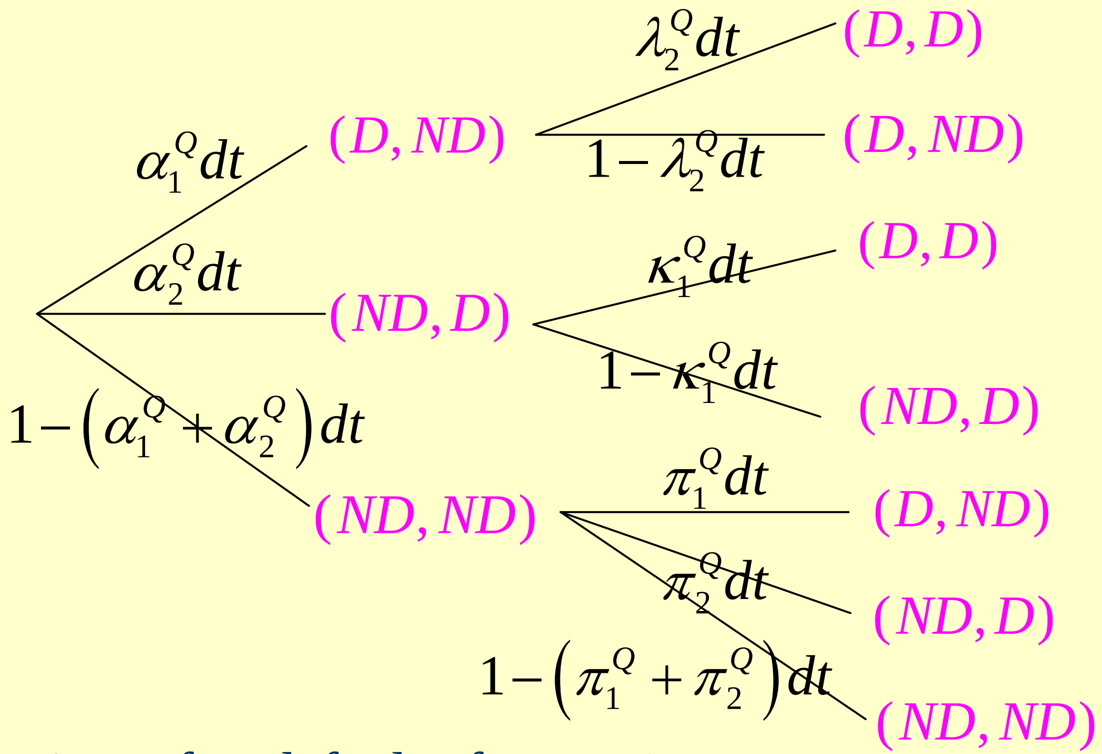
- Computation of deltas
  - Delta with respect to CDS 1: δ_1
  - Delta with respect to CDS 2: δ_2
  - Delta with respect to risk-free asset: p
    - p also equal to up-front premium

$$\begin{cases} a = p + \delta_1 \times \overbrace{\left(1 - \alpha_1^Q dt\right)}^{\text{payoff CDS 1}} + \delta_2 \times \overbrace{\left(-\alpha_2^Q dt\right)}^{\text{payoff CDS 2}} \\ b = p + \delta_1 \times \left(-\alpha_1^Q dt\right) + \delta_2 \times \left(1 - \alpha_2^Q dt\right) \\ c = p + \delta_1 \times \underbrace{\left(-\alpha_1^Q dt\right)}_{\text{payoff CDS 1}} + \delta_2 \times \underbrace{\left(-\alpha_2^Q dt\right)}_{\text{payoff CDS 2}} \end{cases}$$

  - As for the replication price, deltas only depend upon CDS premiums

Tree approach to hedging defaults

- Dynamic case:



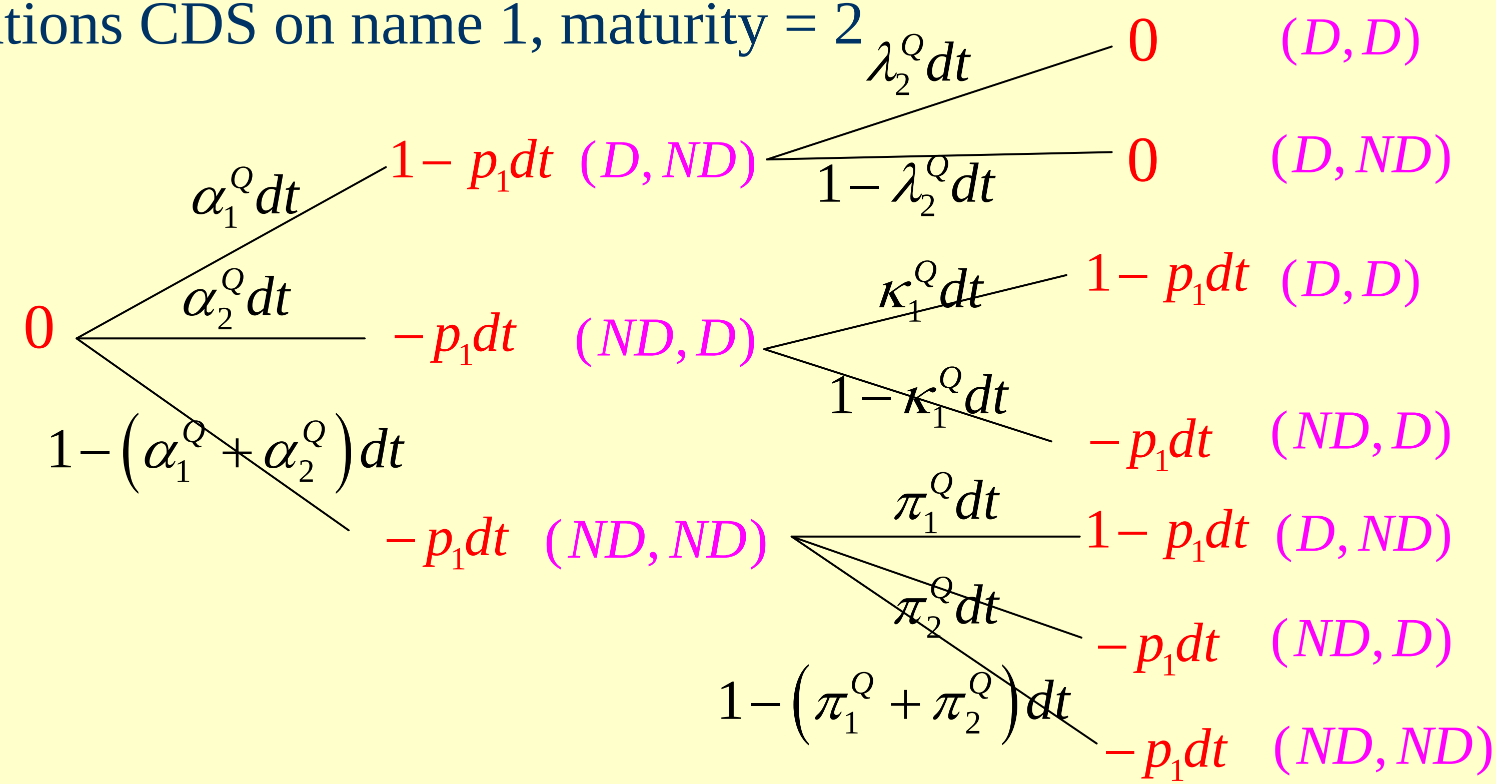
  - $\lambda_2^Q dt$ CDS 2 premium after default of name 1
  - $\kappa_1^Q dt$ CDS 1 premium after default of name 2
  - $\pi_1^Q dt$ CDS 1 premium if no name defaults at period 1
  - $\pi_2^Q dt$ CDS 2 premium if no name defaults at period 1
- Change in CDS premiums due to contagion effects
  - Usually, $\pi_1^Q < \alpha_1^Q < \kappa_1^Q$ and $\pi_2^Q < \alpha_2^Q < \lambda_2^Q$

Tree approach to hedging defaults

- Computation of prices and hedging strategies by backward induction
  - use of the dynamic risk-neutral tree
  - Start from period 2, compute price at period 1 for the three possible nodes
  - + hedge ratios in short term CDS 1,2 at period 1
  - Compute price and hedge ratio in short term CDS 1,2 at time 0
- Example: term structure of credit spreads
  - computation of CDS 1 premium, maturity = 2
  - $p_1 dt$ will denote the periodic premium
  - Cash-flow along the nodes of the tree

Tree approach to hedging defaults

- Computations CDS on name 1, maturity = 2

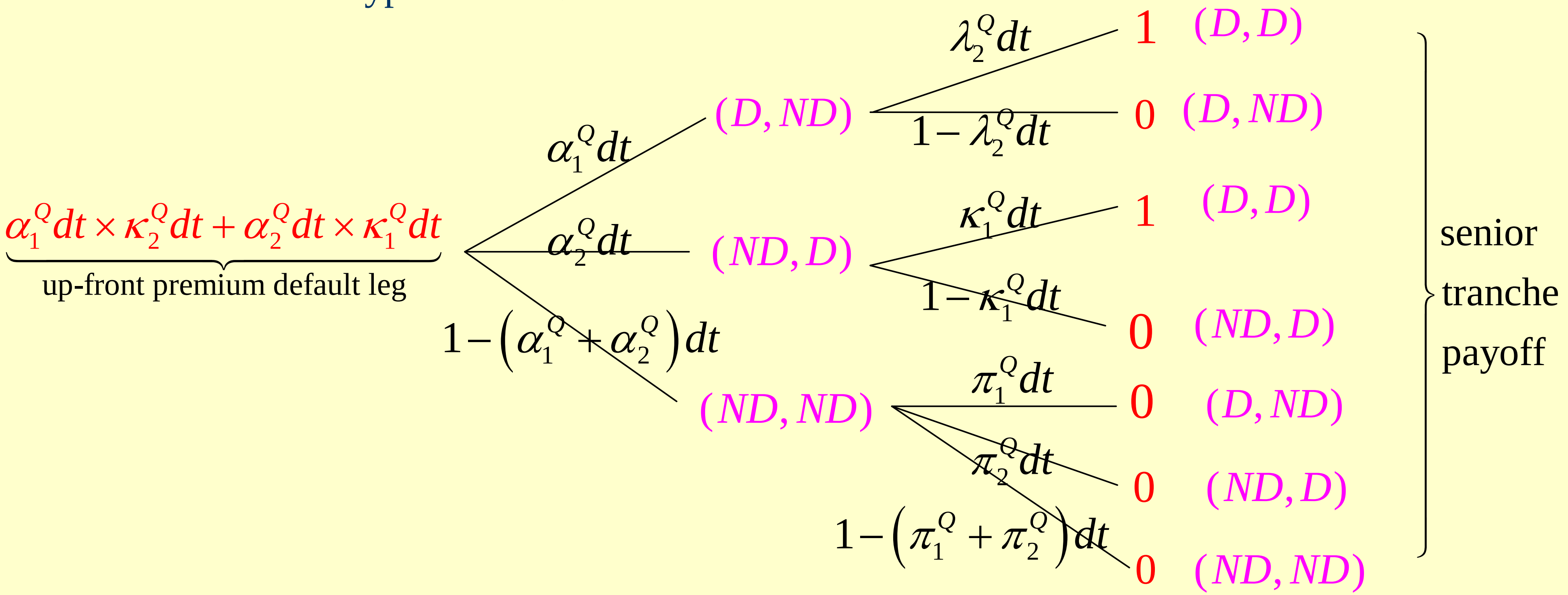


- Premium of CDS on name 1, maturity = 2, time = 0, p_1dt solves for:

$$0 = (1-p_1)\alpha_1^Q + \left(-p_1 + (1-p_1)\kappa_1^Q - p_1\left(1-\kappa_1^Q\right)\right)\alpha_2^Q$$
$$+ \left(-p_1 + (1-p_1)\pi_1^Q - p_1\pi_2^Q - p_1\left(1-\pi_1^Q-\pi_2^Q\right)\right)\left(1-\alpha_1^Q-\alpha_2^Q\right)$$

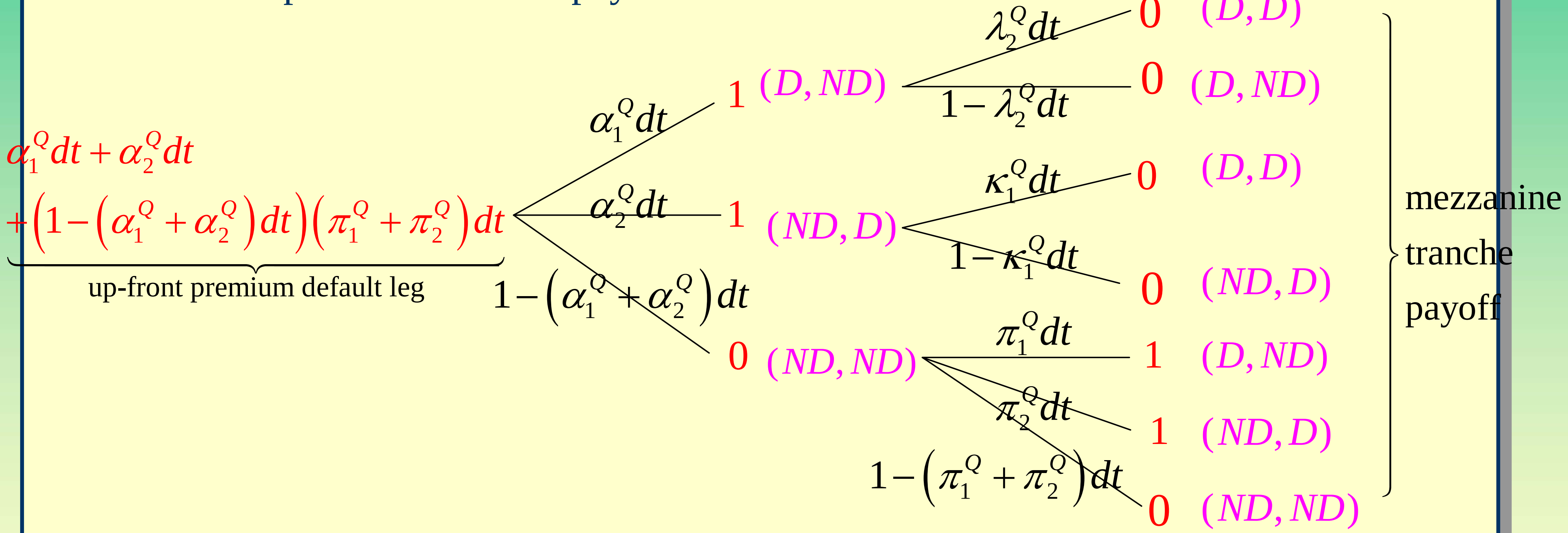
Tree approach to hedging defaults

- **Stylized example: default leg of a senior tranche**
  - Zero-recovery, maturity 2
  - Aggregate loss at time 2 can be equal to 0,1,2
    - Equity type tranche contingent on no defaults
    - Mezzanine type tranche : one default
    - Senior type tranche : two defaults



Tree approach to hedging defaults

- **Stylized example: default leg of a mezzanine tranche**
  - Time pattern of default payments



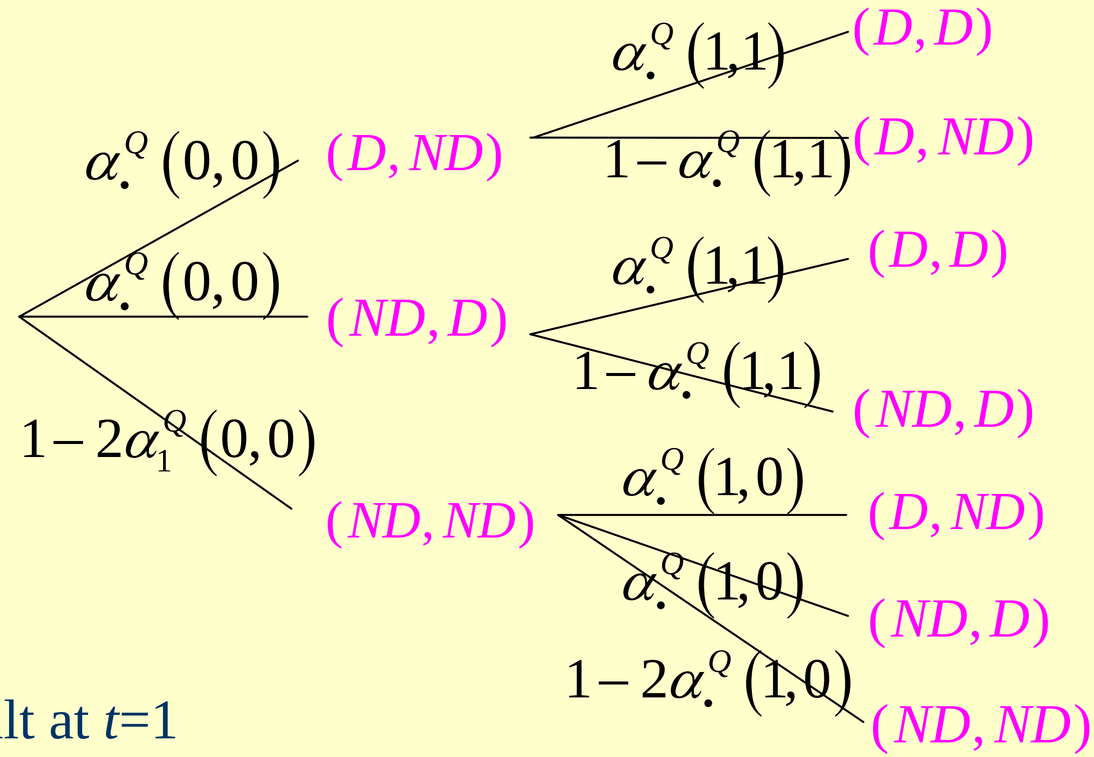
  - Possibility of taking into account discounting effects
  - The timing of premium payments
  - Computation of dynamic deltas with respect to short or actual CDS on names 1,2

Tree approach to hedging defaults

- In theory, one could also derive dynamic hedging strategies for standardized CDO tranches
  - Numerical issues: large dimensional, non recombining trees
  - Homogeneous Markovian assumption is very convenient
    - CDS premiums at a given time *t* only depend upon the current number of defaults $N(t)$
  - CDS premium at time 0 (no defaults) $\alpha_1^Q dt = \alpha_2^Q dt = \alpha_\bullet^Q \left(t = 0, N(0) = 0\right)$
  - CDS premium at time 1 (one default) $\lambda_2^Q dt = \kappa_1^Q dt = \alpha_\bullet^Q \left(t = 1, N(t) = 1\right)$
  - CDS premium at time 1 (no defaults) $\pi_1^Q dt = \pi_2^Q dt = \alpha_\bullet^Q \left(t = 1, N(t) = 0\right)$

Tree approach to hedging defaults

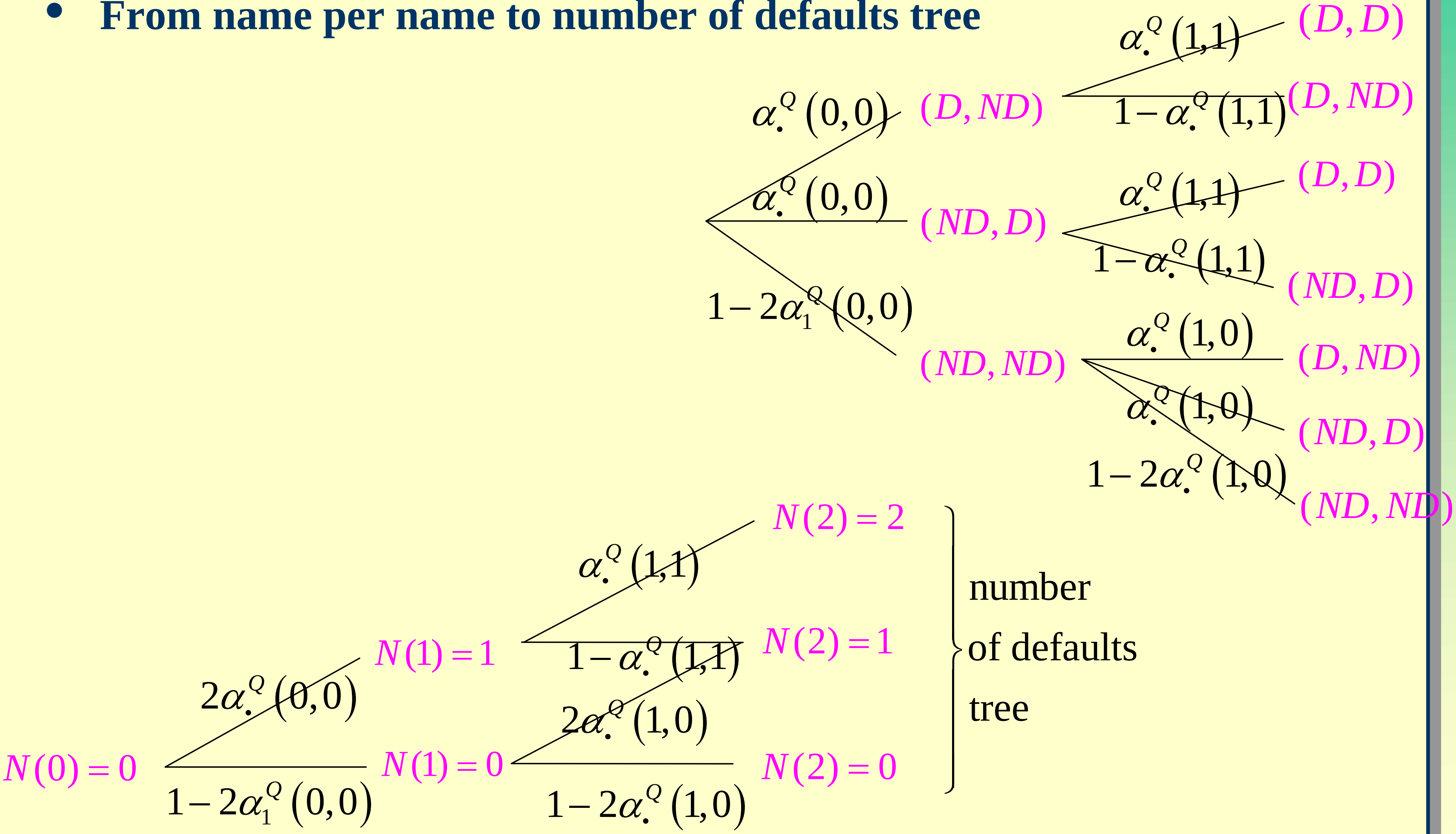
- Tree in the homogeneous case



  - If we have $N(1)=1$, one default at t=1
  - The probability to have $N(2)=1$, one default at t=2…
  - Is $1-\alpha_{\bullet}^{Q}(1,1)$ and does not depend on the defaulted name at t=1
  - $N(t)$ is a **Markov process**
  - **Dynamics of the number of defaults can be expressed through a binomial tree**

Tree approach to hedging defaults

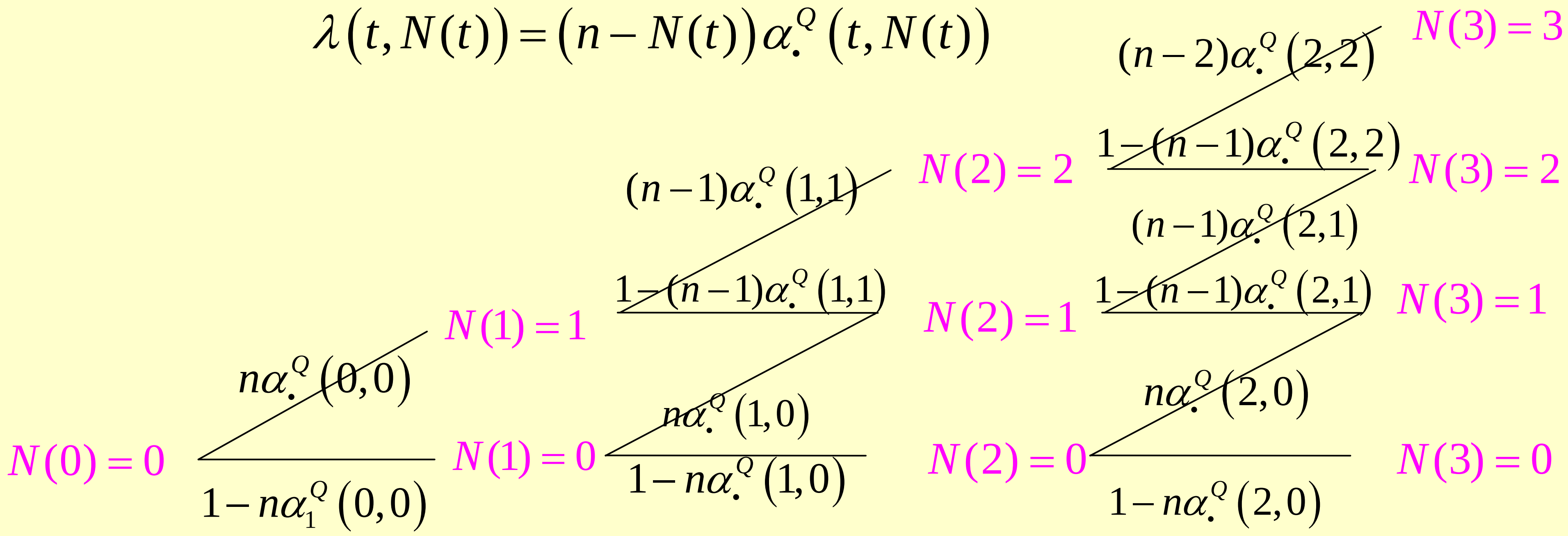
- **From name per name to number of defaults tree**



Tree approach to hedging defaults

- Easy extension to *n* names
  - Predefault name intensity at time *t* for $N(t)$ defaults: $\alpha_{\cdot}^{Q}\left(t,N(t)\right)$
  - Number of defaults intensity : sum of surviving name intensities:

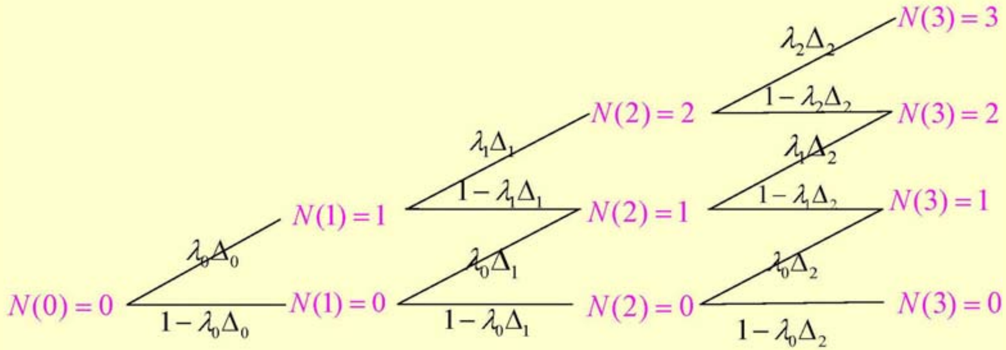
$$\lambda\left(t,N(t)\right)=\left(n-N(t)\right)\alpha_{\cdot}^{Q}\left(t,N(t)\right)$$



  - $\alpha_{\cdot}^{Q}(0,0),\alpha_{\cdot}^{Q}(1,0),\alpha_{\cdot}^{Q}(1,1),\alpha_{\cdot}^{Q}(2,0),\alpha_{\cdot}^{Q}(2,1),\ldots$ can be easily calibrated
  - on marginal distributions of $N(t)$by forward induction.

Empirical results

- Calibration of the tree example
  - Number of names: 125
  - Default-free rate: 4%
  - 5Y credit spreads: 20 bps
  - Recovery rate: 40%



3%	6%	9%	12%	22%
18%	28%	36%	42%	58%

Table 8. Base correlations with respect to attachment points.

- Loss intensities with respect to the number of defaults
  - For simplicity, assumption of time homogeneous intensities
  - Increase in intensities: contagion effects
  - Compare flat and steep base correlation structures

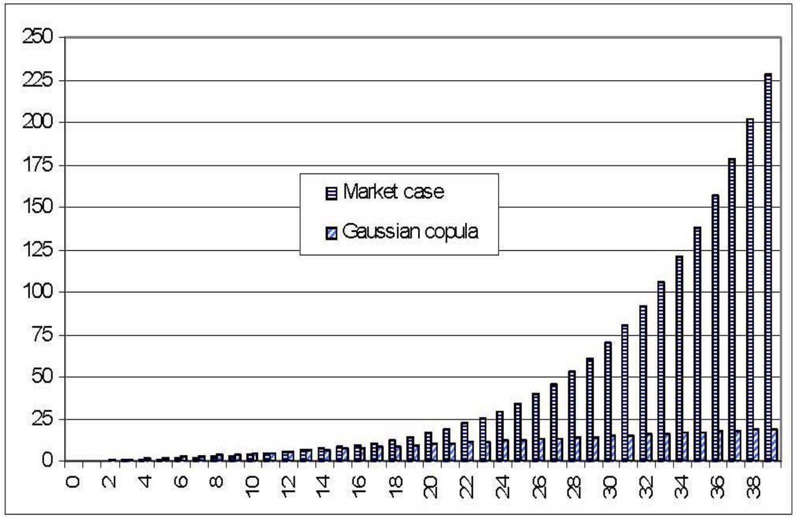


Figure 6. Loss intensities for the Gaussian copula and market case examples. Number of defaults on the $x-$ axis.

Empirical results

- **Dynamics of the credit default swap index in the tree**

Nb Defaults	Weeks			
	0	**14**	**56**	**84**
0	20	19	17	16
1	0	31	23	20
2	0	95	57	43
3	0	269	150	98
4	0	592	361	228
5	0	1022	723	490
6	0	1466	1193	905
7	0	1870	1680	1420
8	0	2243	2126	1945
9	0	2623	2534	2423
10	0	3035	2939	2859

Table 9. Dynamics of credit default swap index spread $s_{IS}(i,k)$ in basis points per annum.

  – The first default leads to a jump from 19 bps to 31 bps
  – The second default is associated with a jump from 31 bps to 95 bps
  – Explosive behavior associated with upward base correlation curve

Empirical results

- What about the credit deltas?
  - In a homogeneous framework, deltas with respect to CDS are all the same
  - Perfect dynamic replication of a CDO tranche with a credit default swap index and the default-free asset
  - Credit delta with respect to the credit default swap index
  - = change in PV of the tranche / change in PV of the CDS index

Nb Defaults	OutStanding Nominal	Weeks			
		0	14	56	84
0	**3.00%**	0.541	0.617	0.823	0.910
1	**2.52%**	0	0.279	0.510	0.690
2	**2.04%**	0	0.072	0.166	0.304
3	**1.56%**	0	0.016	0.034	0.072
4	**1.08%**	0	0.004	0.006	0.012
5	**0.60%**	0	0.002	0.002	0.002
6	**0.12%**	0	0.001	0.000	0.000
7	**0.00%**	0	0	0	0

Table 11. Delta of the default leg of the $[0,3\%]$ equity tranche with respect to the credit default swap index ($\delta_d(i,k)$).

Empirical results

- Dynamics of credit deltas:

Nb Defaults	OutStanding Nominal	Weeks			
		0	14	56	84
0	**3.00%**	0.541	0.617	0.823	0.910
1	**2.52%**	0	0.279	0.510	0.690
2	**2.04%**	0	0.072	0.166	0.304
3	**1.56%**	0	0.016	0.034	0.072
4	**1.08%**	0	0.004	0.006	0.012
5	**0.60%**	0	0.002	0.002	0.002
6	**0.12%**	0	0.001	0.000	0.000
7	**0.00%**	0	0	0	0

Table 11. Delta of the default leg of the $[0,3\%]$ equity tranche with respect to the credit default swap index ($\delta_d(i,k)$).

  - Deltas are between 0 and 1
  - Gradually decrease with the number of defaults
    - ➢Concave payoff, negative gammas
  - When the number of defaults is > 6, the tranche is exhausted
  - Credit deltas increase with time
    - ➢Consistent with a decrease in time value

Empirical results

- Market and tree deltas at inception
- Market deltas computed under the Gaussian copula model
  - Base correlation is unchanged when shifting spreads
  - "Sticky strike" rule
  - Standard way of computing CDS index hedges in trading desks

	[0-3%]	[3-6%]	[6-9%]	[9-12%]	[12-22%]
market deltas	27	4.5	1.25	0.6	0.25
model deltas	21.5	4.63	1.63	0.9	NA

- Smaller equity tranche deltas for in the tree model
  - How can we explain this?

Empirical results

- **Smaller equity tranche deltas in the tree model (cont.)**
  - **Default is associated with an increase in dependence**
    - ➢Contagion effects

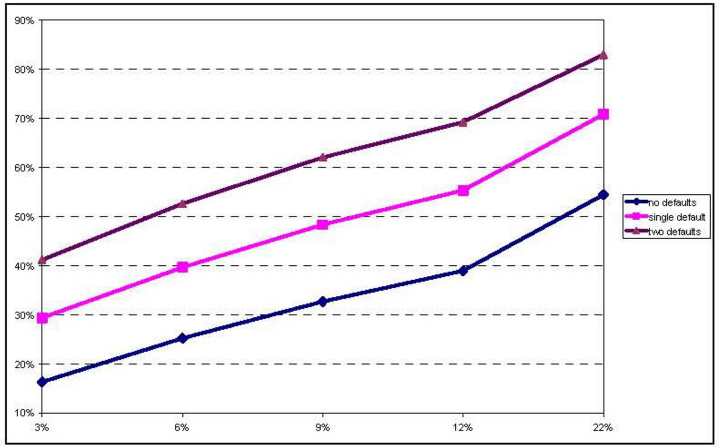


Figure 8. Dynamics of the base correlation curve with respect to the number of defaults. Detachment points on the x–axis. Base correlations on the y–axis.

  - **Increasing correlation leads to a decrease in the PV of the equity tranche**
    - ➢Sticky implied tree deltas
  - **Recent market shifts go in favour of the contagion model**

Empirical results

- The current crisis is associated with joint upward shifts in credit spreads
  - Systemic risk
- And an increase in base correlations

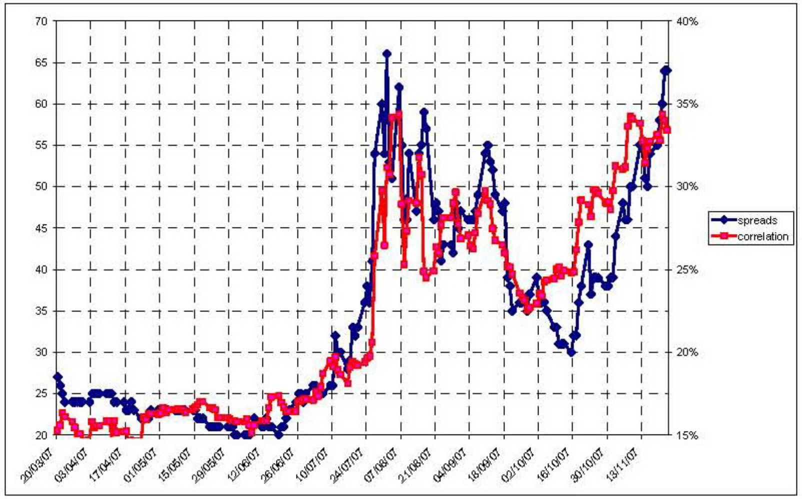


Figure 9. Credit spreads on the five years iTraxx index (Series 7) in bps on the left axis. Implied correlation on the equity tranche on the right axis

- Sticky implied tree deltas are well suited in regimes of fear
  - Derman: “regimes of volatility” (1999)

Empirical results

- Comparing with results provided by:
  - Arnsdorf and Halperin "*BSLP: Markovian Bivariate Spread-Loss Model for Portfolio Credit Derivatives*" Working Paper, JP Morgan (2007), Figure 7

	[0-3%]	[3-6%]	[6-9%]	[9-12%]	[12-22%]
market deltas	26.5	4.5	1.25	0.65	0.25
model deltas	21.9	4.81	1.64	0.79	0.38

  - Computed in March 2007 on the iTraxx tranches
  - Two dimensional Markov chain, shift in credit spreads

	[0-3%]	[3-6%]	[6-9%]	[9-12%]	[12-22%]
market deltas	27	4.5	1.25	0.6	0.25
model deltas	21.5	4.63	1.63	0.9	0.6

  - Note that our results, related to default deltas, are quite similar
    - ➢ Equity tranche deltas are smaller in contagion models than Gaussian copula credit deltas

Empirical results

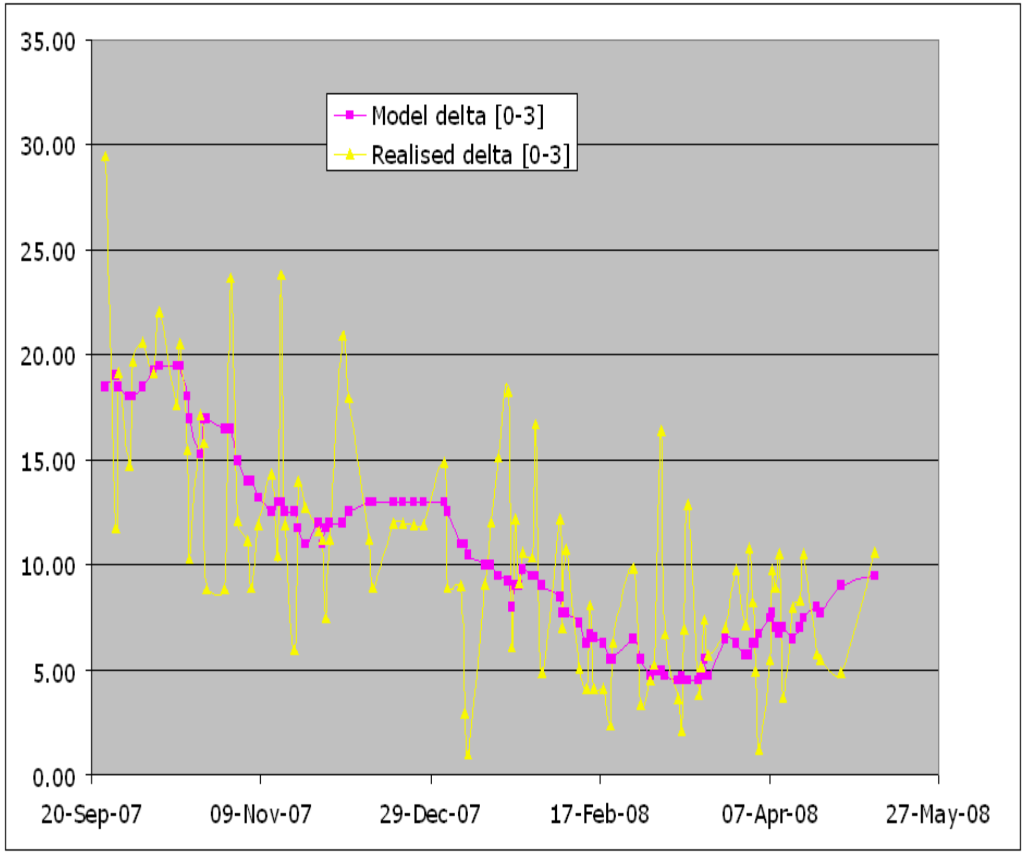
- Cont and Kan: "*Dynamic hedging of portfolio credit derivatives*" (2008)
- **Spread deltas**
  - Gaussian copula model
  - Local intensity corresponds to our contagion model
  - BSLP corresponds to Arnsdorf and Halperin (2007)
  - GPL: generalized Poisson loss model of Brigo *et al*. (2006)
- This shows some kind of robustness
- Picture becomes more complicated when considering other hedging criteria…

Tranche	Gauss	Local	BSLP	GPL
0 - 3	24.48	24.52	24.79	24.48
3 - 6	5.54	5.45	5.30	5.54
6 - 9	1.79	1.80	1.80	1.79
9 - 12	0.87	0.85	0.88	0.87
12 - 22	0.35	0.35	0.32	0.35
22 - 100	0.08	0.08	0.09	0.08

<u>Spread deltas</u> computed for 5Y Europe iTraxx on 20 September 2006

Empirical results

- Back-test study on iTraxx Series 8 equity tranche
- Comparison of realized spread deltas on the equity tranche and model (implied tree) deltas
- Good hedging performance compared with the Gaussian copula model
  - **During the credit crisis**
  - **Discrepancy with results of Cont and Kan (2008)?**



Source: S. Amraoui BNP Paribas

Empirical results

- Cont and Kan (2008) show rather poor performance of "jump to default" deltas
  - Even the recent crisis period
- However, unsurprisingly, the credit deltas ("jump to default") seem to be rather sensitive to the calibration of contagion parameters on quoted CDO tranches
- Right pictures represent aggregate loss intensities
  - Huge contagion effects for the first six defaults in Cont *et al*. (2008)
  - Much smaller contagion effects for the first defaults in Laurent *et al*. (2007)

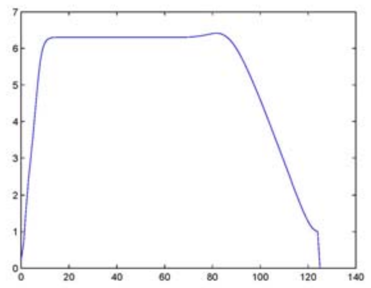


Figure 2: Dependence of default intensity on number of defaults for $t = 1$ year: ITRAXX Europe Series 6, March 15 2007..

Cont, Minca and Savescu (2008)

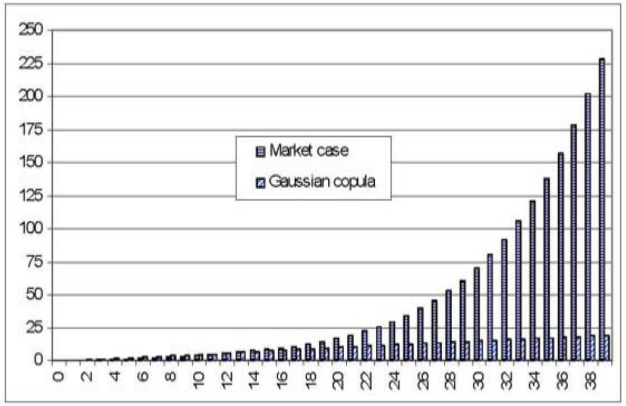


Figure 6. Loss intensities for the Gaussian copula and market case examples. Number of defaults on the x – axis.

Laurent, Cousin and Fermanian (2007)

Empirical results

- Frey and Backhaus: "*Dynamic hedging of synthetic CDO tranches with spread risk and default contagion*" (2007)

Tranche	[0,3]	[3,6]	[6,9]	[9,12]	[12,22]
Spread	26 %	84 bp	24 bp	14 bp	11 bp
Tranche Correlation	17.30 %	3.22 %	9.93 %	15.81 %	27.46 %
Gauss Cop. Δ	$\boxed{0.61}$	0.23	0.06	0.03	0.07

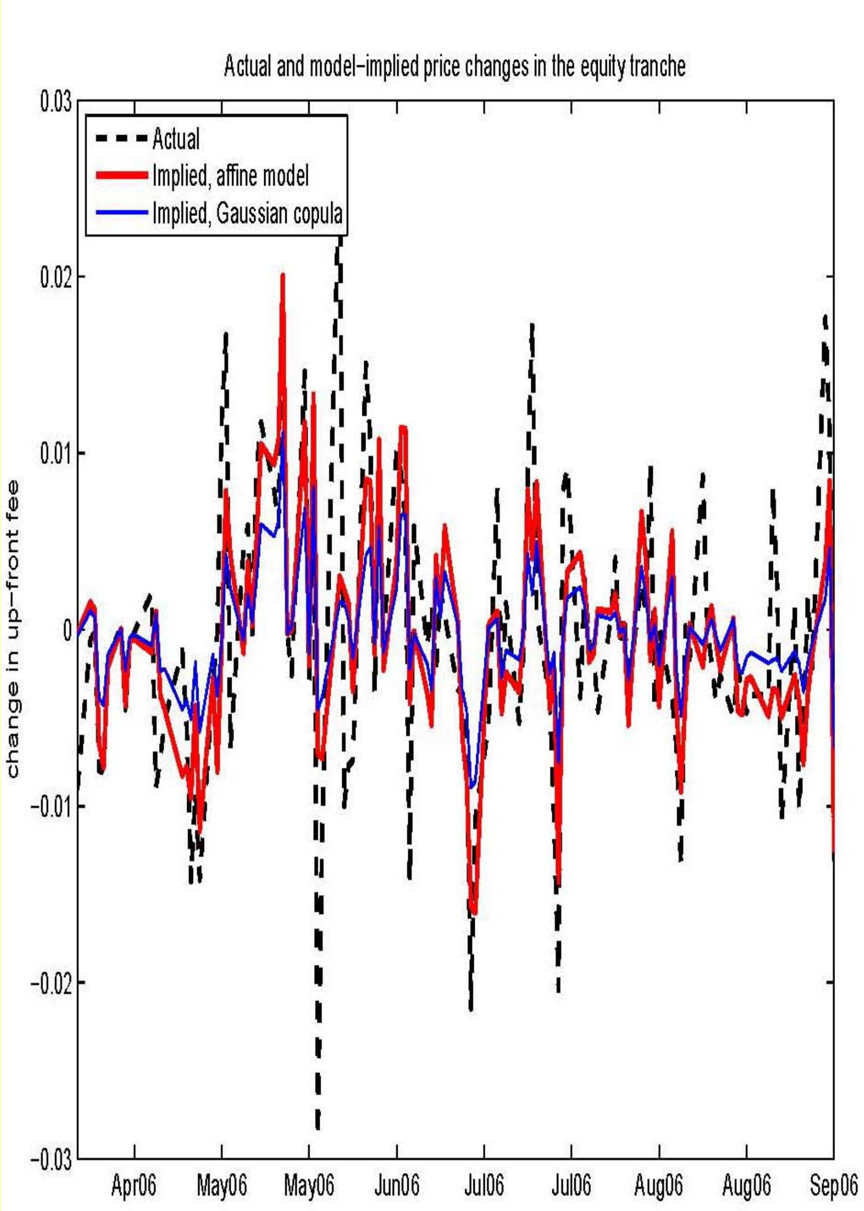
VOD: Value on default

	VOD in the Markov model	VOD in the Copula model
$[0, 3]$	$\boxed{0.344}$	1.002
$[3, 6]$	0.138	0.171
$[6, 9]$	0.058	0.023
$[9, 12]$	0.039	0.008
$[12, 22]$	0.107	0.010

Much smaller deltas in the contagion model than in Gaussian copula model

Empirical results

- Laurent: “A note on the risk management of CDO” (2007)
  – provides a theoretical framework for hedging credit spread risk only while default risk is diversified at the portfolio level
  – no default contagion, correlation between defaults are related to “correlation” between credit spreads
- Feldhütter: “An empirical investigation of an intensity-based model for pricing CDO tranches” (2008)
  – comparison of hedging performance of a Duffie and Garleanu (2001) reduced-form model and one factor Gaussian copula
  – Use of information at time *t*+1 to compute hedge ratios at time *t*
  – Higher deltas for the equity tranche in the affine model compared with the 1F Gaussian copula (market deltas)



Empirical results

- Consistent results with the affine model of Eckner (2007) based on December 2005 CDX data

Tranches	[0-3%]	[3-7%]	[7-10%]	[10-15%]	[15-30%]
market deltas	18.5	5.5	1.5	0.8	0.4
AJD deltas	21.7	6.0	1.1	0.4	0.1
contagion model deltas	17.9	6.3	2.5	1.3	0.8

  – Market deltas, “intensity” model credit deltas in Eckner (2007) and contagion model deltas
  – Goes into the opposite direction when comparing with the contagion model
- Note that Feldhütter (2008) and Eckner (2007) are pre-crisis
- And are according to a “sticky delta rule” (Derman) which is reflects irrational exuberance or greed
  – And might be appropriate for the pre-crisis period

Empirical results: other work in progress

- Individual credit deltas in the above Markov chain (or tree) models
    - ➢ Giesecke, Halperin: forthcoming
  - – Use of "random thinning" to compute individual name deltas
- Discrimination of credit deltas might improve hedging efficiency as compared with hedging with the credit default swap index only
  - – Credit deltas of names with high spreads are likely to be higher when considering an equity tranche
    - ➢ Improvement of hedging efficiency should be related to the dispersion between spreads of names in the underlying portfolio
    - ➢ Empirical studies remain to be conducted…

Empirical results

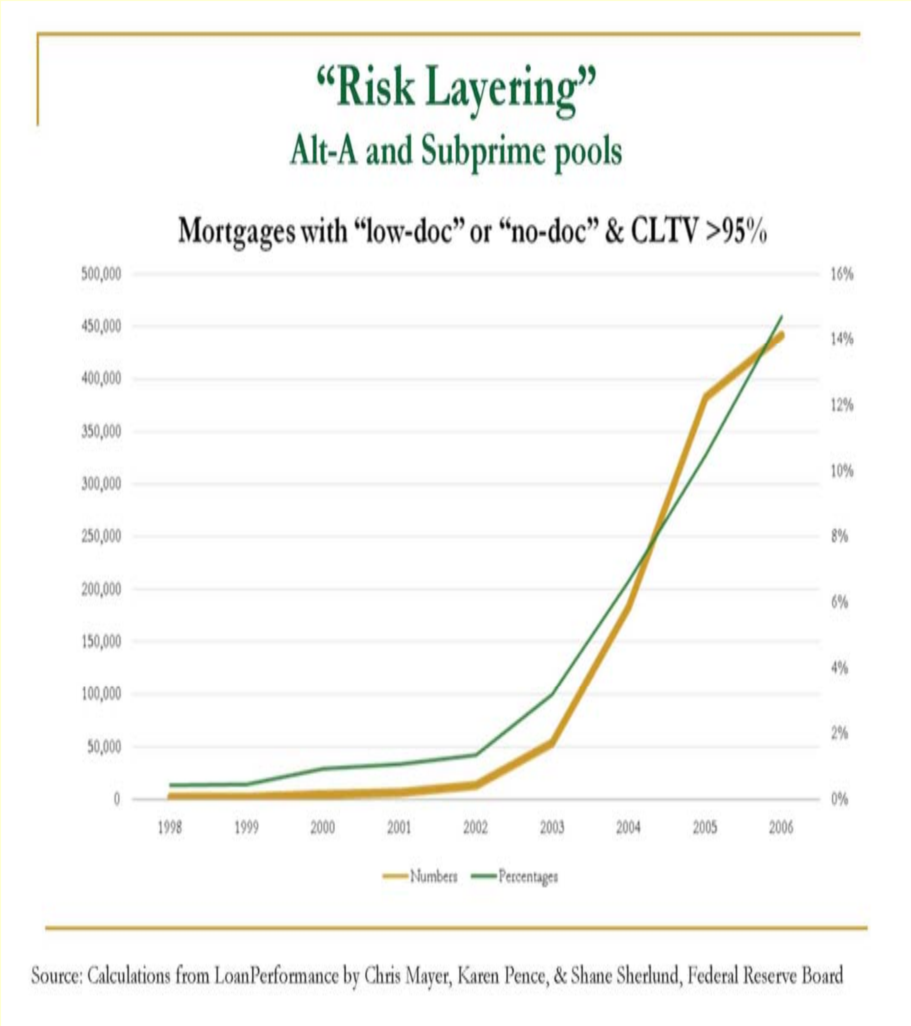
- What do we learn from the previous approaches?
  - Thanks to stringent assumptions:
    - credit spreads driven by defaults
    - homogeneity
    - Markov property
  - It is possible to compute a dynamic hedging strategy
    - Based on the CDS index
  - That fully replicates the CDO tranche payoffs
    - Model matches market quotes of liquid tranches
    - Very simple implementation
    - Credit deltas are easy to understand
  - Improve the computation of default hedges
    - Since it takes into account credit contagion
    - Provide some meaningful results in the current credit crisis

Empirical results

- What we still need to learn (selected items)?
  - Contagion models seem to show lack of robustness
    - Calibration of contagion parameters?
  - Do not properly deal with heterogeneity
    - ➢ See May 2005 idiosyncratic crisis due to the downgrading of GMAC
    - ➢ "idiosyncratic Gamma" is not properly dealt with
  - May not suitable in all market conditions
    - see previous results on reduced-form models
  - Reduced form models may still be of interest: Feldhütter (2008)
    - What is the correct regime?
  - Firm value models and therefore copula models may still be of interest
    - ➢ Could provide a relevant "complete markets" framework
    - ➢ Further need of empirical research in that direction
    - ➢ Take into account tail dependence for asset returns

CDO of Subprimes and SIVs

- CDO of subprimes, RMBS (residential mortgage backed securities)
  - Obvious issues related to fraud and due diligence on mortgages
  - Legal issues in the US with respect to lender's protection
    - ➢ At some point in time, the lender can only claim for the underlying house and not for the borrower's income
- As compared with synthetic STCDOs on corporate issuers, there are usually extra-protection
  - Overcollateralization
  - Non pass-through structure: part of the interest income is retained in the SPV
  - Which is fair enough, but…

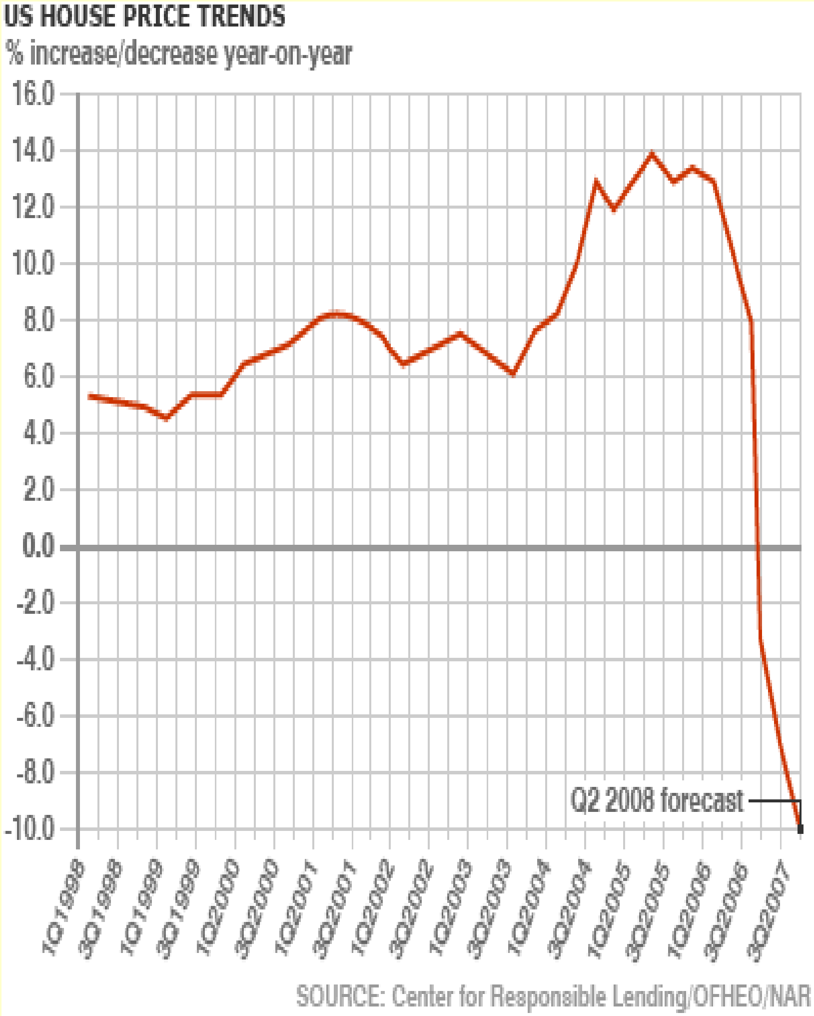


CLTV: combined loan to value

CDO of Subprimes and SIVs

- CDO of subprimes are actually CDO squared:
  - Crouhy and Turnbull: *"The Subprime Credit Crisis of 07"* (2008)
  - Ashcraft and Schuermann: *"Understanding the Securitization of Subprime Mortgage Credit"* (2008)
    - ➢ The mini-tranches, usually rated BBB or A have already well-diversified idiosyncratic risk
    - ➢ The housing market in the US is the common factor
- Since the attachment points of the mini-tranches were rather similar and related to the same underlying risk
- Defaults of the mini-tranches became almost simultaneous
  - Simultaneous defaults rather than contagion effects
  - Comonotonicity: as in Basel II, measures of risk are <u>**additive**</u>
  - The rating of the most senior tranches had to be the same as the ratings of the constituents (say A or BBB) instead of AAA

CDO of Subprimes and SIVs



One common factor: housing market

Collapse of CDOs of subprimes and failure of rating agencies

CDO of Subprimes and SIVs

- A SIV is actually a **synthetic bank**
  - Long-term illiquid and difficult to value assets such as RMBS
  - Short-term funding by issuing commercial paper, usually with the best rating…
  - Huge and obvious liquidity issues
- But SIVs were not submitted to bank regulation
- Issues, especially with SIVs sponsored by banks
  - Off-balance sheet agreements to guarantee SIVs liquidity
  - Explicit or implicit is still unclear
    - **"Partnerships" in case of Enron?**
  - Off-balance sheet commitments should be guaranteed with the capital of the sponsor
  - Late application of Basel II in the US
    - **Controversial issue**
  - To what extend, Fed and department of Treasury were involved?

CDO of Subprimes and SIVs

- Eventually, the collapse of SIVs plus “reintermediation” within the balance sheet of the sponsors led to a fear of systemic risk
- Usual mechanisms in bank crises
  - Increase of short-term spreads
  - Credit crunch in the longer part of the interbank lending market
  - Increased by the opacity of the assets and the dissemination of risks throughout the world (dynamic money funds)
  - Collapse of some financial intermediaries
  - Central banks as lenders of last resort: providing liquidity guaranteed by illiquid securitized assets
- Eventually, contagion effects similar to those discussed above in synthetic CDOs