Hedging credit spread & default risk in CDO tranches

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Hedging credit spread & default risk in CDO tranches

• Context
  – Almost all relevant literature is related to the pricing of CDO tranches
    ➢ Almost no theoretical or practical investigation of hedging issues
    ➢ In interest rate or equity markets, the pricing is related to the cost of the hedge
      – Complete markets
    ➢ Not a similar approach in credit markets
  – Need to relate pricing & hedging
    ➢ Business model for CDOs
Hedging credit spread & default risk in CDO tranches

- **Purpose**
  - Overview of issues in the risk management of CDO tranches
  - Many pricing models
    - Emphasis is put on some specific risks
  - Provide a framework for the risk management of CDO tranches on large indices
    - Thought provocative result
    - Concentrate on the dynamic hedging of credit spread risk
      - Idiosyncratic and parallel credit spreads movements
    - Default risk is statically hedged by diversification
      - Since default events are conditionally independent upon credit spreads
      - Insurance idea
Hedging credit spread & default risk in CDO tranches

- Risks within CDO tranches
- A first approach to default risk hedging
  - Long maturity CDS vs roll-over of short maturity CDS
  - FTD example
- Risks as seen from different models
  - Name per name or individual models
  - Aggregate loss or collective models
- Hedging in different models
  - Structural models
  - Copula and contagion models
  - Aggregate loss models
  - Multiple defaults
  - Hedging in intensity models
Risk within CDO tranches

- Default risk
  - Default bond price jumps to recovery value at default time.
  - Drives the CDO cash-flows
  - Possibility of multiple defaults

- Credit spread risk
  - Changes in defaultable bond prices prior to default, due to shifts in credit quality or in risk premiums.
  - Changes in the marked to market of tranches
  - Increase or decrease the probability of future defaults
  - Changes in the level, the dispersion of credit spreads, the correlation between credit spreads

- Recovery risk
  - Magnitude of aggregate loss jumps is random
Model free approach to default risk hedging

- **Purpose:**
  - Introduction to dynamic trading of default swaps
  - Illustrates how default and credit spread risk arise

- **Arbitrage between long and short term default swaps**
  - Sell one long-term default swap
  - Buy a series of short-term default swaps

- **Example:**
  - Default swaps on a FRN issued by BBB counterparty
  - 5 years default swap premium: 50bp, recovery rate = 60%

Credit derivatives dealer

If default, 60%

Until default, 50 bp

Client
Model free approach to default risk hedging

• Rolling over short-term default swap
  – at inception, one year default swap premium: 33bp
  – cash-flows after one year:

Credit derivatives dealer \[\rightarrow\] 33 bp \[\leftarrow\] Market

\[\leftarrow\] 60% if default

• Buy a one year default swap at the end of every yearly period, if no default:
  – Dynamic strategy,
  – future premiums depend on future credit quality
  – future premiums are unknown

Credit derivatives dealer \[\rightarrow\] ?? bp \[\leftarrow\] Market

\[\leftarrow\] 60% if default
Risk analysis of rolling over short term against long term default swaps

Exchanged cash-flows:
- Dealer receives 5 years (fixed) credit spread,
- Dealer pays 1 year (variable) credit spread.

Full one to one protection at default time
- the previous strategy has eliminated one source of risk, that is default risk
- Recovery risk has been eliminated too.
Model free approach to default risk hedging

- Negative exposure to an increase in short-term default swap premiums
  - if short-term premiums increase from 33bp to 70bp
  - reflecting a lower (short-term) credit quality
  - and no default occurs before the fifth year

- Loss due to negative carry
  - long position in long term credit spreads
  - short position in short term credit spreads
Consider a basket of $M$ defaultable bonds
  - multiple counterparties

First to default swaps
  - protection against the first default

Hedging and valuation of basket default swaps
  - involves the joint (multivariate) modeling of default arrivals of issuers in the basket of bonds.
  - Modeling accurately the dependence between default times is a critical issue.
Hedging First to default swaps: introduction

- Hedging Default Risk in Basket Default Swaps
- Example: first to default swap from a basket of two risky bonds.
  - If the first default time occurs before maturity,
  - The seller of the first to default swap pays the non recovered fraction of the defaulted bond.
- Assume that the two bonds cannot default simultaneously
  - We moreover assume that default on one bond has no effect on the credit spread of the remaining bond.
- How can the seller be protected at default time?
  - The only way to be protected at default time is to hold two default swaps with the same nominal than the nominal of the bonds.
  - The maturity of underlying default swaps does not matter.
Some notations:
- $\tau_1$, $\tau_2$ default times of counterparties 1 and 2,
- $\mathcal{H}_t$ available information at time $t$,
- $P$ historical probability,
- $\lambda_1$, $\lambda_2$: (historical) risk neutral intensities:
  \[ P\left[ \tau_i \in [t, t + dt] \mid \mathcal{H}_t \right] = \lambda_i dt, \ i = 1, 2 \]

Assumption: « Local » independence between default events
- Probability of 1 and 2 defaulting altogether:
  \[ P\left[ \tau_1 \in [t, t + dt], \tau_2 \in [t, t + dt] \mid \mathcal{H}_t \right] = \lambda_1 dt \times \lambda_2 dt \text{ in } (dt)^2 \]
- Local independence: simultaneous joint defaults can be neglected
Building up a tree:

- Four possible states: $(D,D)$, $(D,ND)$, $(ND,D)$, $(ND,ND)$
- Under no simultaneous defaults assumption $p_{(D,D)}=0$
- Only three possible states: $(D,ND)$, $(ND,D)$, $(ND,ND)$
- Identifying (historical) tree probabilities:

$$
\begin{align*}
&D, ND \\
\lambda_1 dt &\quad (D, ND) \\
\lambda_2 dt &\quad (ND, D) \\
1-(\lambda_1 + \lambda_2) dt &\quad (ND, ND)
\end{align*}
$$

\[
\begin{cases}
 p_{(D,D)} = 0 \Rightarrow p_{(D,ND)} = p_{(D,D)} + p_{(D,ND)} = p_{(D,.)} = \lambda_1 dt \\
 p_{(D,D)} = 0 \Rightarrow p_{(ND,D)} = p_{(D,D)} + p_{(ND,D)} = p_{(.D)} = \lambda_2 dt \\
 p_{(ND,ND)} = 1 - p_{(D,.)} - p_{(.D)}
\end{cases}
\]
Hedging First to default swaps: introduction

- Cash flows of (digital) CDS on counterparty 1:
  - $\lambda_1 \phi_1 dt$ CDS premium, $\phi_1$ default risk premium

\[
\begin{align*}
\lambda_1 dt & \quad 1 - \lambda_1 \phi_1 dt & \quad (D, ND) \\
\lambda_2 dt & \quad -\lambda_1 \phi_1 dt & \quad (ND, D) \\
1 - (\lambda_1 + \lambda_2) dt & \quad -\lambda_1 \phi_1 dt & \quad (ND, ND)
\end{align*}
\]

- Cash flows of (digital) CDS on counterparty 1:

\[
\begin{align*}
\lambda_1 dt & \quad -\lambda_2 \phi_2 dt & \quad (D, ND) \\
\lambda_2 dt & \quad 1 - \lambda_2 \phi_2 dt & \quad (ND, D) \\
1 - (\lambda_1 + \lambda_2) dt & \quad -\lambda_2 \phi_2 dt & \quad (ND, ND)
\end{align*}
\]
Hedging First to default swaps: introduction

- Cash flows of (digital) first to default swap (with premium $p_F$):
  \[
  \lambda_1 dt \quad 1 - p_F dt \quad (D, ND)
  \]
  \[
  \lambda_2 dt \quad 1 - p_F dt \quad (ND, D)
  \]
  \[
  1 - (\lambda_1 + \lambda_2) dt \quad -(p_F) dt \quad (ND, ND)
  \]

- Cash flows of holding CDS 1 + CDS 2:
  \[
  \lambda_1 dt \quad 1 - (\lambda_1 \phi_1 + \lambda_2 \phi_2) dt \quad (D, ND)
  \]
  \[
  \lambda_2 dt \quad 1 - (\lambda_1 \phi_1 + \lambda_2 \phi_2) dt \quad (ND, D)
  \]
  \[
  1 - (\lambda_1 + \lambda_2) dt \quad -(\lambda_1 \phi_1 + \lambda_2 \phi_2) dt \quad (ND, ND)
  \]

- Absence of arbitrage opportunities imply:
  - $p_F = \lambda_1 \phi_1 + \lambda_2 \phi_2$
  - Perfect hedge of first to default swap by holding 1 CDS 1 + 1 CDS 2
Three possible states: $(D, ND)$, $(ND, D)$, $(ND, ND)$

Three tradable assets: CDS1, CDS2, risk-free asset

The market is still « complete »

Risk-neutral probabilities
- Used for computing prices
- Consistent pricing of traded instruments
- Uniquely determined from CDS premiums

\[ p_{(D, D)} = 0, \quad p_{(D, ND)} = \lambda_1 \phi_1 dt, \quad p_{(ND, D)} = \lambda_2 \phi_2 dt, \quad p_{(ND, ND)} = 1 - (\lambda_1 \phi_1 + \lambda_2 \phi_2) dt \]
Hedging First to default swaps: introduction

- **hedge ratios** for first to default swaps
- Consider a first to default swap associated with a basket of two defaultable loans.
  - Hedging portfolios based on standard underlying default swaps
  - Hedge ratios if:
    - simultaneous default events
    - Jumps of credit spreads at default times
- Simultaneous default events:
  - If counterparties default *altogether*, holding the complete set of default swaps is a conservative (and thus expensive) hedge.
  - In the extreme case where default *always* occur altogether, we only need a single default swap on the loan with largest nominal.
  - In other cases, holding a *fraction* of underlying default swaps does not hedge default risk (if only one counterparty defaults).
**Hedging First to default swaps: introduction**

- Default hedge ratios for first to default swaps and contagion
- What occurs if there is a *jump in the credit spread* of the second counterparty after *default* of the first?
  - default of first counterparty means *bad news* for the second.
  - Contagion effects
- If hedging with short-term default swaps, *no capital gain* at default.
  - Since PV of short-term default swaps is not *sensitive* to credit spreads.
- This is not the case if hedging with long term default swaps.
  - If credit spreads *jump*, PV of long-term default swaps *jumps*.
- Then, the amount of hedging default swaps can be *reduced*.
  - This reduction is *model-dependent*.
Hedging First to default swaps: introduction

- Default hedge ratios for first to default swaps and stochastic credit spreads
- If one uses short maturity CDS to hedge the FTD?
  - Sell protection on FTD
  - Buy protection on underlying CDS
  - Short maturity CDS: no contagion
  - But, roll-over the hedge until first to default time
  - Negative exposure to an increase in CDS spreads
- If one uses long maturity CDS to hedge the FTD
  - unknown cost of unwinding the remain CDS
  - Credit spreads might have risen or decreased
• Pricing at the cost of the hedge:
  – If some risk can be hedged, its price should be the cost of the hedge.
  – Think of a plain vanilla stock index call. Its replication price is 10% (say).
  – One given investor is ready to pay for 11% (He feels better of with such an option, then doing nothing). Should he really give this 1% to the market?

• The feasibility of hedging
  – «completeness» of credit markets?
  – incomplete markets, multiplicity of risk-neutral measures.
Models for multivariate credit risk analysis

• Structural models
• Intensity models
  – Cox or doubly stochastic Poisson processes, conditionally independent defaults
• Contagion models
• Copula models
• Multivariate Poisson models
• Aggregate loss models
Models for multivariate credit risk analysis

- Structural models
  - modeling of firm’s assets
  - First time passage below a critical threshold
  - Similar to ruin models in insurance theory

- CDS as a barrier option on asset value
  - CDS appear as deep out of the money options

- Log-normal or normal asset dynamics
  - Very similar to Gaussian copula
    ➢Hull, Pedrescu & White

- Model can improved by introducing stochastic volatility, jumps in asset values
  - Numerical issues, Monte Carlo simulation
Multivariate Poisson models

- Shock models
- Default indicators are driven by a multivariate Poisson model
  - Lindskog & McNeil, Elouerkhaoui, Duffie & Singleton
- Common and idiosyncratic shocks
- Common shocks can be fatal or non-fatal
  - A name can survive a non-fatal shock
- There might be multiple defaults
  - This drives the dependence
- High degree of incompleteness
  - Armageddon risk
  - Possibly large values for senior tranches
- Intensities are deterministic between two shocks
Models for multivariate credit risk analysis

• Intensity models
  – Default arrivals are no longer predictable
  – Model conditional local probabilities of default $\lambda(t) \, dt$
  – $\tau$: default date, $\lambda(t)$ risk intensity or hazard rate
    
    $\lambda_i(t) \, dt = P[\tau_i \in [t, t+dt] | \tau_i > t]$
  – Marginal default intensity

• Multivariate case: no simultaneous defaults
  – Model starts from specifying default intensities

• Multivariate Cox processes
  – Credit spreads do not jump at default times
  – Duffie Singleton, Lando, …

• Contagion models (interacting intensities)
  – Jumps of credit spreads of survival names at default times
  – Jarrow & Yu, Yu, Frey & Backhaus
Copula models

- Starting point: copula of default times
- Copula specification states the dependence between default times
- Marginal default time distributions are self-calibrated onto credit spread curves
- Intensities in copula models
- Related to partial derivatives of the copula
- May be difficult to compute
- Default intensities are deterministic between two default times
- Jump at default times
- Contagion effects in copula models
Models for multivariate credit risk analysis

• Structural models, multivariate Cox processes, contagion models (interacting intensities), copula models, multivariate Poisson models,…

• Name per name (individual) models

• The aggregate loss on a portfolio is obtained by summation of individual losses
  – Bottom-up approach

• Aggregate loss models (collective models)
  – Direct specification of loss dynamics
  – CDO tranches only involve European options on aggregate loss
  – Aggregate loss : Marked Point Process
Models for multivariate credit risk analysis

• Aggregate loss models (following)
  – Increasing Market Point Process
  – Aggregate loss intensity = sum of name default intensities
  – Magnitude of jumps = 1 – recovery of defaulted name

• Markovian models
  – SPA, Schonbucher
  – Markov chain (or more general) processes for the aggregate loss

• Non Markovian
  – Giesecke & Goldberg
  – Self-exciting processes, Hawkes, ACD type
  – Loss intensity only depends upon past losses

• Top-down approach?
  – Individual intensities need to be equal for self-consistency
  – Homogeneous models
  – Assessment of credit spread homogeneity wrt to CDO tranche hedging
Econometric approach to credit spread hedging

Hedging liquid tranches with the index
  - iTraxx or CDX
  - Look for historical data on tranche premiums and index credit spread
  - Try to relate through some regression analysis changes in tranche premiums to changes in spreads
  - Check the hedging performance of different models
    - Houdain & Guegan
    - Similar ideas in equity derivatives markets
    - Baskhi, Cao & Chen
Hedging in different modeling framework

- Structural models
- Multiname credit derivatives can be perfectly hedged in a Black-Cox framework
- Defaults are predictable
- Only one kind of risk
  - Credit spread or asset price risk
  - Stock prices and credit spreads are perfectly correlated
  - Complete markets
- Equity derivatives type hedge
Multivariate Poisson models

- Possibility of simultaneous defaults
  - Name 1 and 2 may default altogether
  - Name 1 and 3 may default altogether
  - Name 2 and 3 may default altogether
  - Name 1, 2, and 3 may default altogether
- $2^n$ states of the world
- $n$ hedging instruments (single name CDS)
- High degree of default risk incompleteness
- Intensities are deterministic between two shocks
  - Not really any credit spread risk
Copula and contagion models: theory

- Default intensities are only related to past defaults
- In other words, credit spread risk derives from default risk

Smooth copula precludes simultaneous defaults

- In previous models, perfect hedge of multiname credit derivatives with single name CDS
- Complete markets
- Representation theorems for multivariate point processes
- Only default risk, no “true” credit spread risk
- Work in progress

Bielecki, Jeanblanc & Rutkowski
Copula models: practice very different from theory
Practical implementation of hedging strategies
Focus on credit spread risk only
Price of a CDO tranche depends upon marginal credit curves and the copula
Compute CDS hedge ratio by bumping the marginal credit curves and compute the CDO price increment
Local sensitivity analysis
  - Model dependent
  - No guarantee that local hedging leads to a correct global hedge
  - Does gamma effects offset theta effects?
Copula models: gamma effects

Homogeneous portfolio
  - Gamma matrix of a CDO tranche (wrt credit spreads)

\[
\begin{pmatrix}
I & B & B & B & B \\
B & I & B & B & B \\
B & B & I & B & B \\
B & B & B & I & B \\
B & B & B & B & I \\
\end{pmatrix}
\]

- \((s_1, \ldots, s_n)\) change in credit spreads
  - Assume credit delta hedging with CDS
  - First order change in PV are equal to zero
Hedging in different modeling framework

- Copula models: gamma effects
  - Assume \( s_2 = \cdots = s_n = 0 \)
    - Change in PV \( \frac{I}{2}s_1^2 \) idiosyncratic gamma effect
  - Assume \( s_1 = \cdots = s_n = s \)
    - Change in PV \( \frac{n}{2}(I+(n-1)B)s^2 \) parallel gamma

- Homogeneous portfolio
  - Credit spread covariance matrix
    \[
    \sigma^2 \Delta t \begin{pmatrix}
    1 & \rho & \rho & \rho & \rho \\
    \rho & 1 & \rho & \rho & \rho \\
    \rho & \rho & 1 & \rho & \rho \\
    \rho & \rho & \rho & 1 & \rho \\
    \rho & \rho & \rho & \rho & 1
    \end{pmatrix}
    \]

- Expected gamma P&L
  \[
  n \frac{\sigma^2 \Delta t}{2} \left( (1 - \rho)I + \rho(I + (n - 1)B) \right),
  \]

- \((n-1)B\rho\) high spread correlation sensitivity
Hedging CDO tranches in the base correlation approach
  – Tranchelets on standard indices
  – Bespoke portfolios

Correlation depends upon the expected loss of the tranche

Change in credit spreads changes the marginal credit curves and the implied correlation parameter
  – Sticky deltas

Still main focus upon credit spread hedging
  – Still dispersion risk (idiosyncratic gamma) and parallel spread risk
Hedging in aggregate loss models

- No notion of idiosyncratic gamma
- Individual credit spreads are perfectly correlated
- Jumps in aggregate loss process (default risk)
- Change in loss intensity: parallel Gamma

Hedging on a name per name basis

Or based upon the index: same hedge ratios for all names

- Hedging equity tranche with an aggregate loss model can become problematic
- High sensitivity to heterogeneity between credit spreads
- Hedge ratios for riskier names are likely to be higher
- Does not take into account idiosyncratic gamma
Hedging in different modeling framework

- **Hedging CDO tranches with liquid tranches**
  - Case of tranchelets on iTraxx or CDX
  - Not the same hedging instruments

- **Entropic calibration**
  - Perfect copula type approach
  - Start from some specification of conditional default probabilities
  - $g_0$ a priori density function of conditional default probabilities
  - Look for some a posteriori density function of cdp:

$$\min_g \int g(p) \ln \frac{g(p)}{g_0(p)} dp$$

- consistency constraints with liquid tranches prices

$$\int_0^1 (p - k_i)^+ g(p) dp = \pi_i$$

$$g(p) = g_0(p) \exp \left( \lambda + \sum_{i=0}^I \lambda_i (p - k_i)^+ \right)$$

- Hedge ratios: compute partial derivatives of tranchelets wrt $\pi_i$
Hedging credit spread risk for large portfolios

- When dealing with the risk management of CDOs, traders concentrate upon credit spread and correlation risk.
- What about default risk?
  - For large indices, default of one name has only a small effect on the aggregate loss.
- Model framework
  - Given probability $Q$ such that:
    - Defaultable bond prices are martingales.
    - Default times follow a multivariate Cox process.
    - $Q$ equivalent to historical probability $P$.
    - Bounded risk premiums.
Hayden credit spread risk for large portfolios

- No simultaneous defaults
- No contagion effects
  - Credit spreads drive defaults but defaults do not drive credit spreads
  - For a large portfolio, default risk is perfectly diversified
  - Only remains credit spread risks: parallel & idiosyncratic
- Technical background
  - Projection of default indicators on the information generated by credit spreads
  - Smooth projection of the aggregate loss
  - No default risk in the market with incomplete information
  - Only credit spread risk
Example

- Assume that credit spreads follow a multivariate CIR process
- Framework similar to interest rate models
- Step 1: approximate the CDO tranche payoff
  - Replace actual aggregate loss by its smoothed projection on credit spreads
- Step 2: consider some pseudo CDS
  - Similar to well diversified portfolios
    - Björk & Naslund, de Donno
  - Diversification of default risk of a CDS at the name level
- Smoothed or shadow market only involves credit spread risk
**Hedging credit spread risk for large portfolios**

- Shadow market is complete and Markovian
  - Step 3: compute perfect hedge ratios
    - With respect to pseudo CDS 1, … , n
    - Technicalities are left aside
      - High dimensionality
      - Use of semi-analytical techniques
      - Not detailed in the paper
  - Step 4: apply the hedging strategy to the true CDS

- Main result
  - Bound on the hedging error following the previous hedging strategy
  - When hedging an actual CDO tranche with actual CDS
  - Hedging error decreases with the number of names
  - Default risk diversification
Hedging credit spread risk for large portfolios

• Provides a hedging technique for CDO tranches
  – Known theoretical properties
  – Takes into account idiosyncratic and parallel gamma risks
  – Good theoretical properties rely on no simultaneous defaults, no contagion effects assumptions
  – Empirical work remains to be done
  – Comparison with standard base correlation approaches

• Thought provocative
  – To construct a practical hedging strategy, do not forget default risk
  – Equity tranche [0,3%]
  – iTraxx or CDX first losses cannot be considered as smooth
Hedging credit spread risk for large portfolios

- Linking pricing and hedging?
- The black hole in CDO modeling?
- Standard valuation approach in derivatives markets
  - Complete markets
  - Price = cost of the hedging/replicating portfolio
- Mixing of dynamic hedging strategies
  - for credit spread risk
- And diversification/insurance techniques
  - For default risk