Hedging credit spread & default risk in CDO tranches

IAFE meeting
London 1st November 2006

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Hedging credit spread & default risk in CDO tranches

- **Context**
  - **Almost all relevant literature is related to the pricing of CDO tranches**
    - ➢Almost no theoretical or practical investigation of hedging issues
    - ➢In interest rate or equity markets, the pricing is related to the cost of the hedge
      - **Complete markets**
    - ➢Not a similar approach in credit markets
  - **Need to relate pricing & hedging**
    - ➢Business model for CDOs

Hedging credit spread & default risk in CDO tranches

- **Purpose**
  - **Overview of issues in the risk management of CDO tranches**
  - **Many pricing models**
    - ➢ Emphasis is put on some specific risks
  - **Provide a framework for the risk management of CDO tranches on large indices**
    - ➢ Thought provocative result
    - ➢ Concentrate on the dynamic hedging of credit spread risk
      - **Idiosyncratic and parallel credit spreads movements**
    - ➢ Default risk is statically hedged by diversification
      - **Since default events are conditionally independent upon credit spreads**
      - **Insurance idea**

Hedging credit spread & default risk in CDO tranches

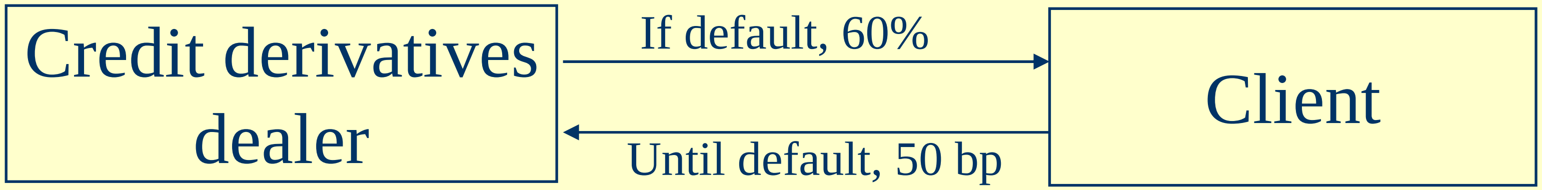
- Risks within CDO tranches
- A first approach to default risk hedging
  - Long maturity CDS vs roll-over of short maturity CDS
  - FTD example
- Risks as seen from different models
  - Name per name or individual models
  - Aggregate loss or collective models
- Hedging in different models
  - structural models
  - Copula and contagion models
  - aggregate loss models
  - Multiple defaults
  - Hedging in intensity models

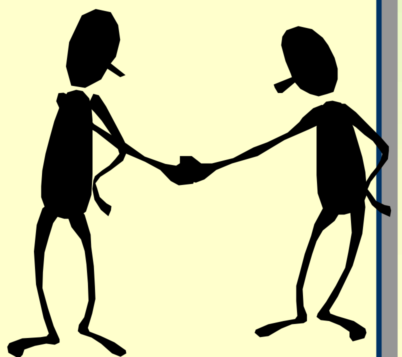
Risk within CDO tranches

- Default risk
  - Default bond price jumps to recovery value at default time.
  - Drives the CDO cash-flows
  - Possibility of multiple defaults
- Credit spread risk
  - Changes in defaultable bond prices prior to default, due to shifts in credit quality or in risk premiums.
  - Changes in the marked to market of tranches
  - Increase or decrease the probability of future defaults
  - Changes in the level, the dispersion of credit spreads, the correlation between credit spreads
- Recovery risk
  - Magnitude of aggregate loss jumps is random

Model free approach to default risk hedging

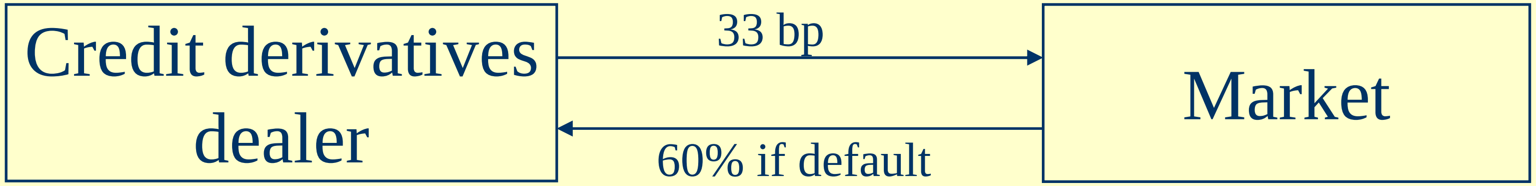
- Purpose:
  - Introduction to dynamic trading of default swaps
  - Illustrates how default and credit spread risk arise
- Arbitrage between long and short term default swaps
  - sell one long-term default swap
  - buy a series of short-term default swaps
- Example:
  - default swaps on a FRN issued by BBB counterparty
  - 5 years default swap premium : 50bp, recovery rate = 60%



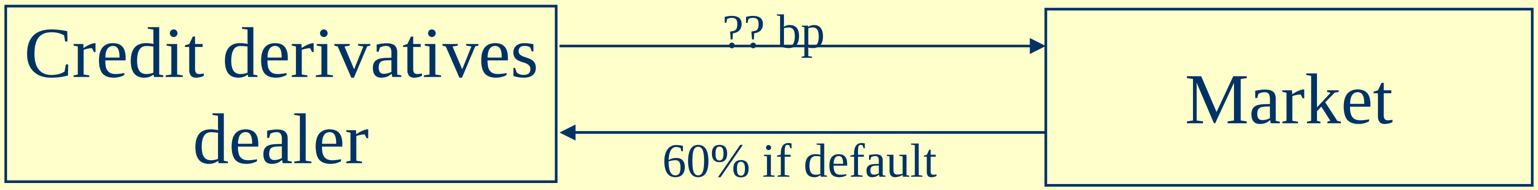


Model free approach to default risk hedging

- Rolling over short-term default swap
  - at inception, one year default swap premium : 33bp
  - cash-flows after one year:

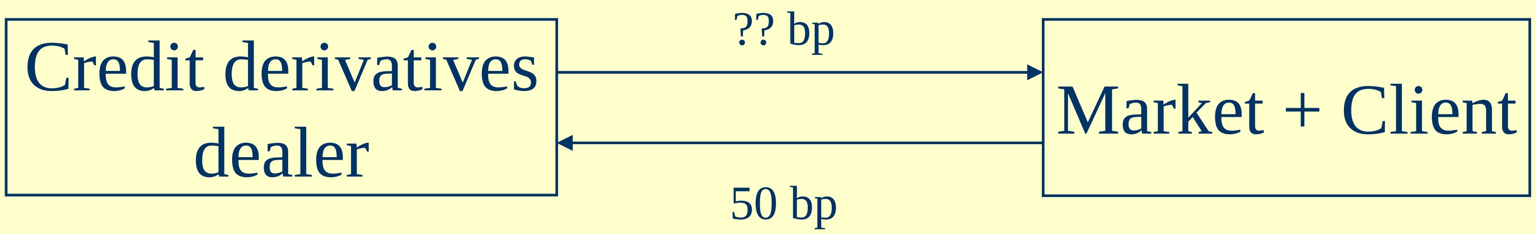


- Buy a one year default swap at the end of every yearly period, if no default:
  - Dynamic strategy,
  - <u>future</u> premiums depend on <u>future</u> credit quality
  - future premiums are <u>unknown</u>



Model free approach to default risk hedging

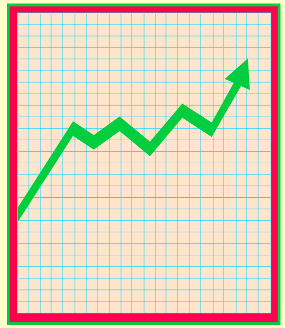
- *Risk analysis* of rolling over short term against long term default swaps

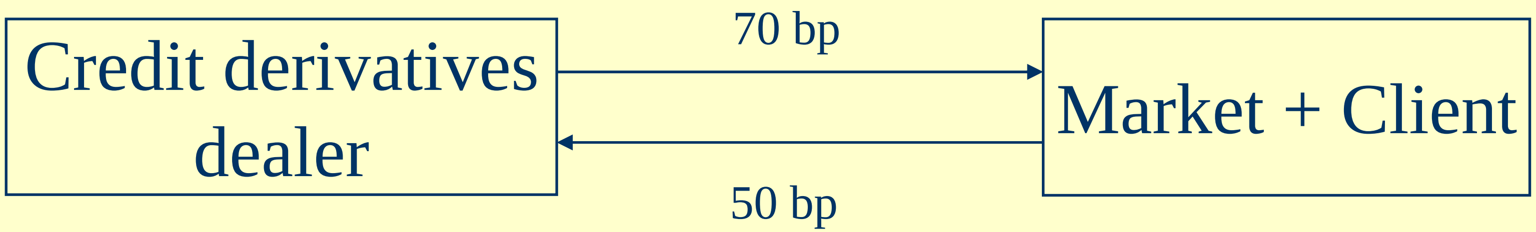


- Exchanged cash-flows :
  - Dealer receives 5 years (fixed) credit spread,
  - Dealer pays 1 year (variable) credit spread.
- <u>Full one to one protection at default time</u>
  - the previous strategy has <u>eliminated</u> one source of risk, that is <u>default risk</u>
  - <u>Recovery risk</u> has been eliminated too.

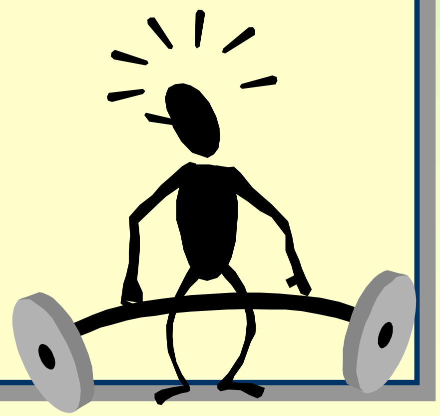
Model free approach to default risk hedging

- Negative exposure to an increase in short-term default swap premiums
  - if short-term premiums increase from 33bp to 70bp
  - reflecting a lower (short-term) credit quality
  - and no default occurs before the fifth year





- Loss due to negative carry
  - long position in long term credit spreads
  - short position in short term credit spreads



Hedging First to default swaps: introduction

- Consider a basket of *M* defaultable bonds
  - multiple counterparties
- First to default swaps
  - protection against the first default
- Hedging and valuation of basket default swaps
  - involves the joint (multivariate) modeling of default arrivals of issuers in the basket of bonds.
  - Modeling accurately the dependence between default times is a critical issue.

Hedging First to default swaps: introduction

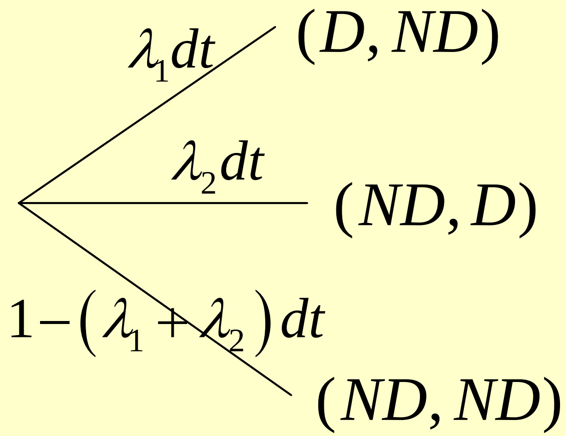
- Hedging Default Risk in Basket Default Swaps
- Example: first to default swap from a basket of two risky bonds.
  - If the first default time occurs before maturity,
  - The seller of the first to default swap pays the non recovered fraction of the defaulted bond.
- Assume that the two bonds cannot default simultaneously
  - We moreover assume that default on one bond has *no effect* on the credit spread of the remaining bond.
- How can the seller be protected *at default time* ?
  - The only way to be protected at default time is to hold two default swaps with the *same nominal* than the *nominal* of the bonds.
  - The *maturity* of underlying default swaps does not matter.

Hedging First to default swaps: introduction

- Some notations :
  - τ_1, τ_2 default times of counterparties 1 and 2,
  - $\mathcal{H}_t$ available information at time t,
  - P historical probability,
  - λ_1 , λ_2 : (historical) risk neutral intensities:
    - $P\left[\tau_i \in [t, t+dt[\,\middle|\, H_t\right] = \lambda_i dt, \; i = 1,2$
- Assumption : « Local » independence between default events
  - Probability of 1 and 2 defaulting altogether:
    - $P\left[\tau_1 \in [t, t+dt[, \tau_2 \in [t, t+dt[\,\middle|\, H_t\right] = \lambda_1 dt \times \lambda_2 dt \text{ in } (dt)^2$
  - Local independence: simultaneous joint defaults can be neglected

Hedging First to default swaps: introduction

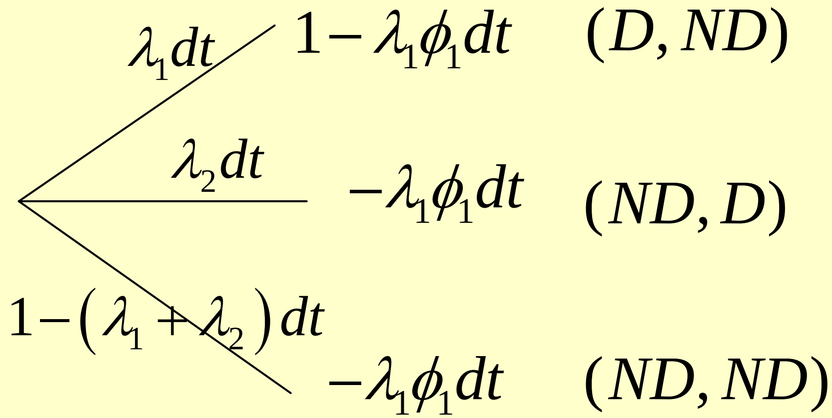
- Building up a tree:
  - Four possible states: (D,D), (D,ND), (ND,D), (ND,ND)
  - Under no simultaneous defaults assumption $p_{(D,D)}=0$
  - Only three possible states: (D,ND), (ND,D), (ND,ND)
  - Identifying (historical) tree probabilities:



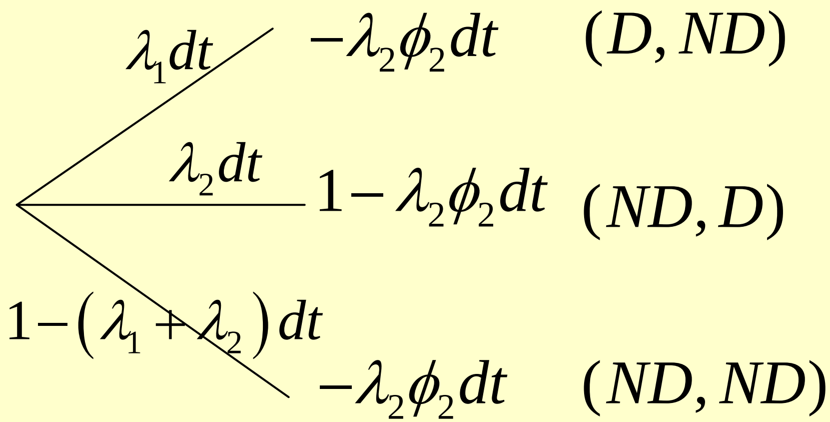
$$\begin{cases} p_{(D,D)} = 0 \Rightarrow p_{(D,ND)} = p_{(D,D)} + p_{(D,ND)} = p_{(D,.)} = \lambda_1 dt \\ p_{(D,D)} = 0 \Rightarrow p_{(ND,D)} = p_{(D,D)} + p_{(ND,D)} = p_{(.,D)} = \lambda_2 dt \\ p_{(ND,ND)} = 1 - p_{(D,.)} - p_{(.,D)} \end{cases}$$

Hedging First to default swaps: introduction

- Cash flows of (digital) CDS on counterparty 1:
  - $\lambda_1 \phi_1 dt$ CDS premium, ϕ_1 default risk premium



- Cash flows of (digital) CDS on counterparty 1:



Hedging First to default swaps: introduction

- Cash flows of (digital) first to default swap (with premium p_F):

$$\lambda_1 dt: \quad 1 - p_F dt \qquad (D, ND)$$
$$\lambda_2 dt: \quad 1 - p_F dt \qquad (ND, D)$$
$$1 - (\lambda_1 + \lambda_2) dt: \quad -p_F dt \qquad (ND, ND)$$

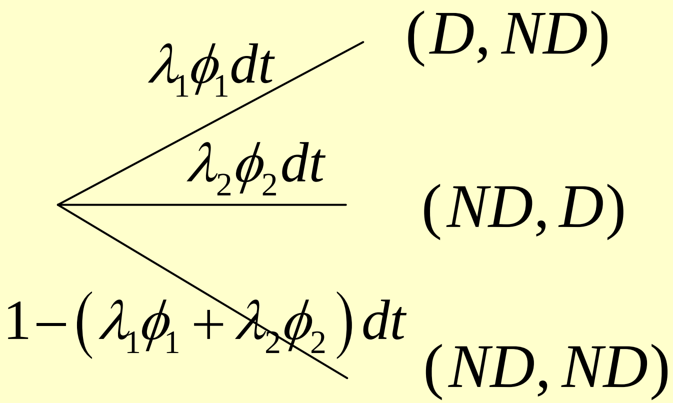
- Cash flows of holding CDS 1 + CDS 2:

$$\lambda_1 dt: \quad 1 - (\lambda_1 \phi_1 + \lambda_2 \phi_2) dt \qquad (D, ND)$$
$$\lambda_2 dt: \quad 1 - (\lambda_1 \phi_1 + \lambda_2 \phi_2) dt \qquad (ND, D)$$
$$1 - (\lambda_1 + \lambda_2) dt: \quad -(\lambda_1 \phi_1 + \lambda_2 \phi_2) dt \qquad (ND, ND)$$

- Absence of arbitrage opportunities imply:
  - $p_F = \lambda_1 \phi_1 + \lambda_2 \phi_2$
  - Perfect hedge of first to default swap by holding 1 CDS 1 + 1 CDS 2

Hedging First to default swaps: introduction

- Three possible states: (*D*,*ND*), (*ND*,*D*), (*ND*,*ND*)
- Three tradable assets: CDS1, CDS2, risk-free asset
    - ➢The market is still « complete »
- Risk-neutral probabilities
  - Used for computing prices
  - Consistent pricing of traded instruments
  - Uniquely determined from CDS premiums
  - $p_{(D,D)}=0$, $p_{(D,ND)}=\lambda_1\,\phi_1 dt$, $p_{(ND,D)}=\lambda_2\,\phi_2 dt$, $p_{(ND,ND)}=1-(\lambda_1\,\phi_1+\lambda_2\,\phi_2)\,dt$



Hedging First to default swaps: introduction

- *hedge ratios* for first to default swaps
- Consider a first to default swap associated with a basket of two defaultable loans.
  - Hedging portfolios based on standard underlying default swaps
  - Hedge ratios if:
    - simultaneous default events
    - *Jumps* of credit spreads at default times
- Simultaneous default events:
  - If counterparties default *altogether*, holding the *complete* set of default swaps is a conservative (and thus expensive) hedge.
  - In the *extreme* case where default *always* occur altogether, we only need a single default swap on the loan with largest nominal.
  - In other cases, holding a *fraction* of underlying default swaps does not hedge default risk (if *only one* counterparty defaults).

Hedging First to default swaps: introduction

- Default hedge ratios for first to default swaps and contagion
- What occurs if there is a *jump in the credit spread* of the second counterparty after default of the first ?
  - default of first counterparty means *bad news* for the second.
  - Contagion effects
- If hedging with short-term default swaps, no capital gain at default.
  - Since PV of short-term default swaps is not *sensitive* to credit spreads.
- This is not the case if hedging with long term default swaps.
  - If credit spreads jump, PV of long-term default swaps jumps.
- Then, the amount of hedging default swaps can be reduced.
  - This reduction is *model-dependent*.

Hedging First to default swaps: introduction

- Default hedge ratios for first to default swaps and stochastic credit spreads
- If one uses short maturity CDS to hedge the FTD?
  - Sell protection on FTD
  - Buy protection on underlying CDS
  - Short maturity CDS: no contagion
  - But, roll-over the hedge until first to default time
  - Negative exposure to an increase in CDS spreads
- If one uses long maturity CDS to hedge the FTD
  - unknown cost of unwinding the remain CDS
  - Credit spreads might have risen or decreased

Hedging First to default swaps: introduction

- Pricing at the cost of the hedge:
  - If some risk can be hedged, its price should be the cost of the hedge.
  - Think of a plain vanilla stock index call. Its replication price is 10% (say).
  - One given investor is ready to pay for 11% (He feels better of with such an option, then doing nothing). Should he really give this 1% to the market ?
- The feasibility of hedging
  - « *completeness* » of credit markets?
  - incomplete markets, multiplicity of risk-neutral measures.

Models for multivariate credit risk analysis

- Structural models
- Intensity models
  - Cox or doubly stochastic Poisson processes, conditionally independent defaults
- Contagion models
- Copula models
- Multivariate Poisson models
- Aggregate loss models

Models for multivariate credit risk analysis

- Structural models
  - modeling of firm's assets
  - First time passage below a critical threshold
  - Similar to ruin models in insurance theory
- CDS as a barrier option on asset value
  - CDS appear as deep out of the money options
- Log-normal or normal asset dynamics
  - Very similar to Gaussian copula
    - Hull, Pedrescu & White
- Model can improved by introducing stochastic volatility, jumps in asset values
  - Numerical issues, Monte Carlo simulation

Models for multivariate credit risk analysis

- Multivariate Poisson models
  - Shock models
  - Default indicators are driven by a multivariate Poisson model
    - ➢ Lindskog & McNeil, Elouerkhaoui, Duffie & Singleton
  - Common and idiosyncratic shocks
  - Common shocks can be fatal or non fatal
    - ➢ A name can survive a non fatal shock
  - There might be multiple defaults
    - ➢ This drives the dependence
  - High degree of incompleteness
    - ➢ Armageddon risk
    - ➢ possibly large values for senior tranches
  - Intensities are deterministic between two shocks

Models for multivariate credit risk analysis

- Intensity models
  - Default arrivals are no longer predictable
  - Model conditional local probabilities of default $\lambda(t)\, dt$
  - τ : default date, $\lambda(t)$ *risk intensity* or *hazard rate*

$$\lambda_i(t)dt = P\left[\tau_i \in \left[t, t+dt\right[\middle| \tau_i > t\right]$$

  - *Marginal default intensity*
- Multivariate case: no simultaneous defaults
  - Model starts from specifying default intensities
- Multivariate Cox processes
  - Credit spreads do not jump at default times
  - Duffie Singleton, Lando, …
- Contagion models (interacting intensities)
  - Jumps of credit spreads of survival names at default times
  - Jarrow & Yu, Yu, Frey & Backhaus

Models for multivariate credit risk analysis

- Copula models
  - Starting point : copula of default times
  - Copula specification states the dependence between default times
  - Marginal default time distributions are self-calibrated onto credit spread curves
  - Intensities in copula models
  - Related to partial derivatives of the copula
  - May be difficult to compute
  - Default intensities are deterministic between two default times
  - Jump at default times
  - Contagion effects in copula models

Models for multivariate credit risk analysis

- Structural models, multivariate Cox processes, contagion models (interacting intensities), copula models, multivariate Poisson models,…
- Name per name (individual) models
- The aggregate loss on a portfolio is obtained by summation of individual losses
  - Bottom-up approach
- Aggregate loss models (collective models)
  - Direct specification of loss dynamics
  - CDO tranches only involve European options on aggregate loss
  - Aggregate loss : Marked Point Process

Models for multivariate credit risk analysis

- Aggregate loss models (following)
  - Increasing Market Point Process
  - Aggregate loss intensity = sum of name default intensities
  - Magnitude of jumps = 1 – recovery of defaulted name
- Markovian models
  - SPA, Schonbucher
  - Markov chain (or more general) processes for the aggregate loss
- Non Markovian
  - Giesecke & Goldberg
  - Self-exciting processes, Hawkes, ACD type
  - Loss intensity only depends upon past losses
- Top-down approach ?
  - Individual intensities need to be equal for self-consistency
  - Homogeneous models
  - Assessment of credit spread homogeneity wrt to CDO tranche hedging

Hedging in different modeling framework

- Econometric approach to credit spread hedging
- Hedging liquid tranches with the index
  - iTraxx or CDX
  - Look for historical data on tranche premiums and index credit spread
  - Try to relate through some regression analysis changes in tranche premiums to changes in spreads
  - Check the hedging performance of different models
    - Houdain & Guegan
    - Similar ideas in equity derivatives markets
    - Baskhi, Cao & Chen

Hedging in different modeling framework

- Structural models
- Multiname credit derivatives can be perfectly hedged in a Black-Cox framework
- Defaults are predictable
- Only one kind of risk
  - Credit spread or asset price risk
  - Stock prices and credit spreads are perfectly correlated
  - Complete markets
- Equity derivatives type hedge

Hedging in different modeling framework

- Multivariate Poisson models
  - Possibility of simultaneous defaults
    - ➢Name 1 and 2 may default altogether
    - ➢Name 1 and 3 may default altogether
    - ➢Name 2 and 3 may default altogether
    - ➢Name 1, 2, and 3 may default altogether
  - 2^n states of the world
  - n hedging instruments (single name CDS)
  - High degree of default risk incompleteness
  - Intensities are deterministic between two shocks
    - ➢Not really any credit spread risk

Hedging in different modeling framework

- Copula and contagion models: theory
  - Default intensities are only related to past defaults
  - In other words, credit spread risk derives from default risk
- Smooth copula precludes simultaneous defaults
  - In previous models, perfect hedge of multiname credit derivatives with single name CDS
  - Complete markets
  - Representation theorems for multivariate point processes
  - Only default risk, no "true" credit spread risk
  - Work in progress
    - ➢Bielecki, Jeanblanc & Rutkowski

Hedging in different modeling framework

- Copula models: practice very different from theory
- Practical implementation of hedging strategies
- Focus on credit spread risk only
- Price of a CDO tranche depends upon marginal credit curves and the copula
- Compute CDS hedge ratio by bumping the marginal credit curves and compute the CDO price increment
- Local sensitivity analysis
  - Model dependent
  - No guarantee that local hedging leads to a correct global hedge
  - Does gamma effects offset theta effects?

Hedging in different modeling framework

- Copula models: gamma effects
- Homogeneous portfolio
  - Gamma matrix of a CDO tranche (wrt credit spreads)

$$\begin{pmatrix} I & B & B & B & B \\ B & I & B & B & B \\ B & B & I & B & B \\ B & B & B & I & B \\ B & B & B & B & I \end{pmatrix}$$

  - $(s_1, \ldots, s_n)$ change in credit spreads
    - ➢Assume credit delta hedging with CDS
    - ➢First order change in PV are equal to zero

Hedging in different modeling framework

- Copula models: gamma effects
  - Assume $s_2 = \cdots = s_n = 0$
    - ➢Change in PV $\frac{I}{2} s_1^2$ idiosyncratic gamma effect
  - Assume $s_1 = \cdots = s_n = s$
    - ➢Change in PV $\frac{n}{2}\left(I + (n-1)B\right) s^2$ parallel gamma
- Homogeneous portfolio
  - Credit spread covariance matrix

$$\sigma^2 \Delta t \begin{pmatrix} 1 & \rho & \rho & \rho & \rho \\ \rho & 1 & \rho & \rho & \rho \\ \rho & \rho & 1 & \rho & \rho \\ \rho & \rho & \rho & 1 & \rho \\ \rho & \rho & \rho & \rho & 1 \end{pmatrix}$$

$$\boxed{\text{expected gamma P\&L} = n \frac{\sigma^2 \Delta t}{2} \left((1-\rho) I + \rho (I + (n-1) B) \right),}$$

  - $(n-1)B\rho$ high spread correlation sensitivity

Hedging in different modeling framework

- Hedging CDO tranches in the base correlation approach
  - Tranchelets on standard indices
  - Bespoke portfolios
- Correlation depends upon the expected loss of the tranche
- Change in credit spreads changes the marginal credit curves and the implied correlation parameter
  - Sticky deltas
- Still main focus upon credit spread hedging
  - Still dispersion risk (idiosyncratic gamma) and parallel spread risk

Hedging in different modeling framework

- Hedging in aggregate loss models
  - No notion of idiosyncratic gamma
  - Individual credit spreads are perfectly correlated
  - Jumps in aggregate loss process (default risk)
  - Change in loss intensity: parallel Gamma
- Hedging on a name per name basis
- Or based upon the index: same hedge ratios for all names
  - Hedging equity tranche with an aggregate loss model can become problematic
  - High sensitivity to heterogeneity between credit spreads
  - Hedge ratios for riskier names are likely to be higher
  - Does not take into account idiosyncratic gamma

Hedging in different modeling framework

- **Hedging CDO tranches with liquid tranches**
  - Case of tranchelets on iTraxx or CDX
  - Not the same hedging instruments
- **Entropic calibration**
  - Perfect copula type approach
  - Start from some specification of conditional default probabilities
  - g_0 a priori density function of conditional default probabilities
  - Look for some a posteriori density function of cdp:

$$\min_g \int g(p) \ln \frac{g(p)}{g_0(p)} dp$$

  - consistency constraints with liquid tranches prices $\int_0^1 (p - k_i)^+ g(p) dp = \pi_i$

$$g(p) = g_0(p) \exp\left(\lambda + \sum_{i=0}^{I} \lambda_i (p - k_i)^+ \right)$$

  - Hedge ratios: compute partial derivatives of tranchelets wrt π_i

Hedging credit spread risk for large portfolios

- When dealing with the risk management of CDOs, traders concentrate upon credit spread and correlation risk
- What about default risk ?
  - For large indices, default of one name has only a small effect on the aggregate loss
- Model framework
  - Given probability Q such that:
    - ➢Defaultable bond prices are martingales
    - ➢Default times follow a multivariate Cox process
    - ➢Q equivalent to historical probability P
    - ➢Bounded risk premiums

Hedging credit spread risk for large portfolios

- No simultaneous defaults
- No contagion effects
  - credit spreads drive defaults but defaults do not drive credit spreads
  - For a large portfolio, default risk is perfectly diversified
  - Only remains credit spread risks: parallel & idiosyncratic
- Technical background
  - Projection of default indicators on the information generated by credit spreads
  - Smooth projection of the aggregate loss
  - No default risk in the market with incomplete information
  - Only credit spread risk

Hedging credit spread risk for large portfolios

- Example
  - Assume that credit spreads follow a multivariate CIR process
  - Framework similar to interest rate models
  - Step 1: approximate the CDO tranche payoff
    - ➢Replace actual aggregate loss by its smoothed projection on credit spreads
  - Step 2: consider some pseudo CDS
    - ➢Similar to well diversified portfolios
      - Björk & Naslund, de Donno
    - ➢Diversification of default risk of a CDS at the name level
  - Smoothed or shadow market only involves credit spread risk

Hedging credit spread risk for large portfolios

- Shadow market is complete and Markovian
  - Step 3: compute perfect hedge ratios
    - ➢With respect to pseudo CDS 1, … , *n*
    - ➢Technicalities are left aside
      - High dimensionality
      - Use of semi-analytical techniques
      - Not detailed in the paper
  - Step 4: apply the hedging strategy to the <u>true</u> CDS
- Main result
  - Bound on the hedging error following the previous hedging strategy
  - When hedging an actual CDO tranche with actual CDS
  - Hedging error decreases with the number of names
    - ➢Default risk diversification

Hedging credit spread risk for large portfolios

- Provides a hedging technique for CDO tranches
  - Known theoretical properties
  - Takes into account idiosyncratic and parallel gamma risks
  - Good theoretical properties rely on no simultaneous defaults, no contagion effects assumptions
  - Empirical work remains to be done
  - Comparison with standard base correlation approaches
- Thought provocative
  - To construct a practical hedging strategy, do not forget default risk
  - Equity tranche [0,3%]
  - iTraxx or CDX first losses cannot be considered as smooth

Hedging credit spread risk for large portfolios

- Linking pricing and hedging ?
- The black hole in CDO modeling ?
- Standard valuation approach in derivatives markets
    - ➢Complete markets
    - ➢Price = cost of the hedging/replicating portfolio
- Mixing of dynamic hedging strategies
    - – for credit spread risk
- And diversification/insurance techniques
    - – For default risk