# *Hedging Demand Deposits Interest Rate Margins*

**Risk Management and Financial Crisis Forum**
**March 19th 2009.**

Mohamed HOUKARI

mohamed.houkari@bnpparibas.com

Alexandre ADAM, BNP Paribas Asset and Liability Management

Mohamed HOUKARI, ISFA, Université de Lyon, Université Lyon 1 and BNP Paribas ALM

Jean-Paul LAURENT, ISFA, Université de Lyon, Université Lyon 1

# PRESENTATION OUTLOOK

- **Modeling Framework, Objective and Optimal Strategy**

- **Empirical Results**

- **Conclusions**

# Demand Deposit Interest Rate Margin – Definition

- **Demand Deposit Interest Rate Margin for a given quarter:**
  - *Income generated by the investment of Demand Deposit Amount on interbank markets while paying a deposit rate to customers*
- **Risks in Interest Rate Margins:**
  - Interest Rate Risk:
    - 1. Investment on interbank markets
    - 2. Paying an interest rate to customers (possibly correlated to market rates)
    - 3. Demand Deposit amount is subject to transfer effects from customers, due to market rate variations
  - Non hedgeable Risk Factors on the Deposit Amount:
    - **Business Risk**: Competition between banks, customer behavior independent from market conditions, etc.
    - **Model Risk**

## Setting the Objective

Interest Rate Margin $IRM_g(K_T, L_T) = K_T(L_T - g(L_T)) \cdot \Delta T$

- $K_T$: Deposit Amount at T
- $L_T$: Investment Market Rate during time interval [$T$,$T$+$\Delta T$]
- $g(L_T)$: Customer rate at T

- **Mean-variance framework:**
  - Including a return constraint – due to the interest rate risk premium

$$\min_S \mathbf{E}\left[IRM_g(K_T, L_T) - S\right]^2 \text{ under constraint } \mathbf{E}\left[IRM_g(K_T, L_T) - S\right] \geq r$$

## Dynamics for Market Rate $L_t = L(t, T, T + \Delta T)$

- **Libor Market Model for Investment Market Rate**

$$\frac{dL_t}{L_t} = \mu_L dt + \sigma_L dW_L(t)$$

Ex.: Brace, Gatarek, Musiela (1997)

$$\mu_L \neq 0$$

Long-Term Investment Risk Premium

- **Coefficient specification assumptions:**
  - **Our model:** $\mu_L, \sigma_L$ constant

*(and can be easily extended to time-dependent framework)*

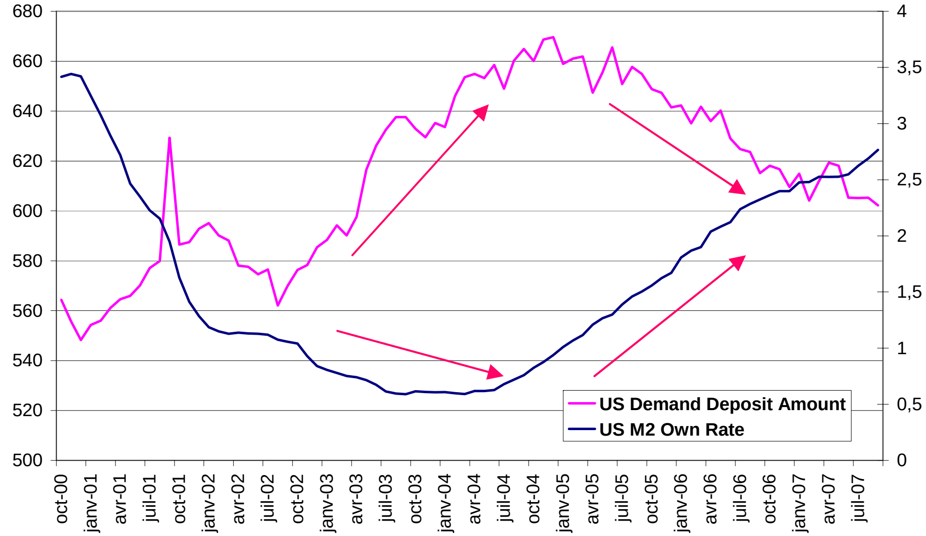
# Deposit Amount Dynamics

- <u>Diffusion process for Deposit Amount</u>

$$dK_t = K_t\left[\mu_K dt + \sigma_K d\overline{W_K}(t)\right]$$

- Sensitivity of deposit amount to market rates
  - Money transfers between deposits and other accounts
- Interest Rate partial contingence.
  - Business risk, …
  - **Incomplete market framework**

*(US marketplace)*

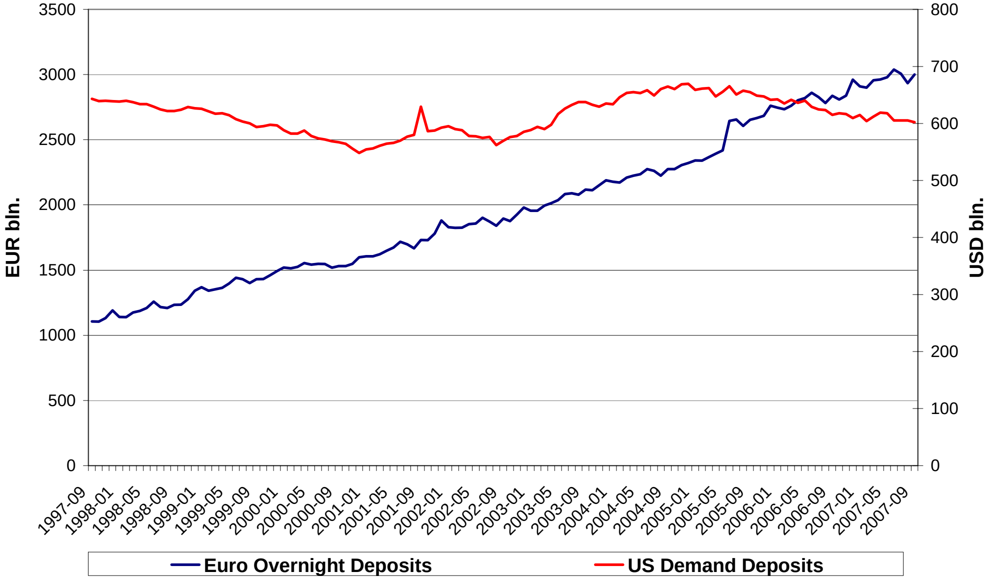



$$d\overline{W_K}(t) = \rho dW_L(t) + \sqrt{1-\rho^2}\, dW_K(t) \qquad \boxed{-1 < \rho < 0}$$

# Deposit Amount Dynamics – Examples

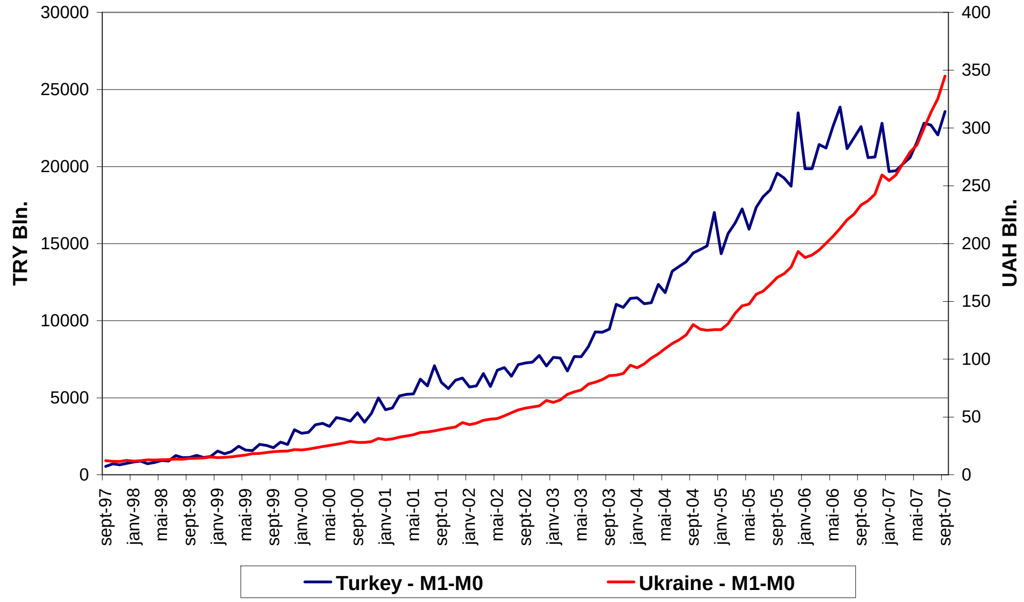
$$dK_t = K_t\left(\mu_K dt + \sigma_K d\overline{W}_K(t)\right)$$

US and Euro Zone




**EuroZone** $- \hat{\mu}_K = 10.19\%, \hat{\sigma}_K = 6.56\%$

Emerging Markets (Turkey, Ukraine)




**Turkey** $- \hat{\mu}_K = 51.74\%, \hat{\sigma}_K = 37.38\%$

# Modeling Deposit Rate – Examples

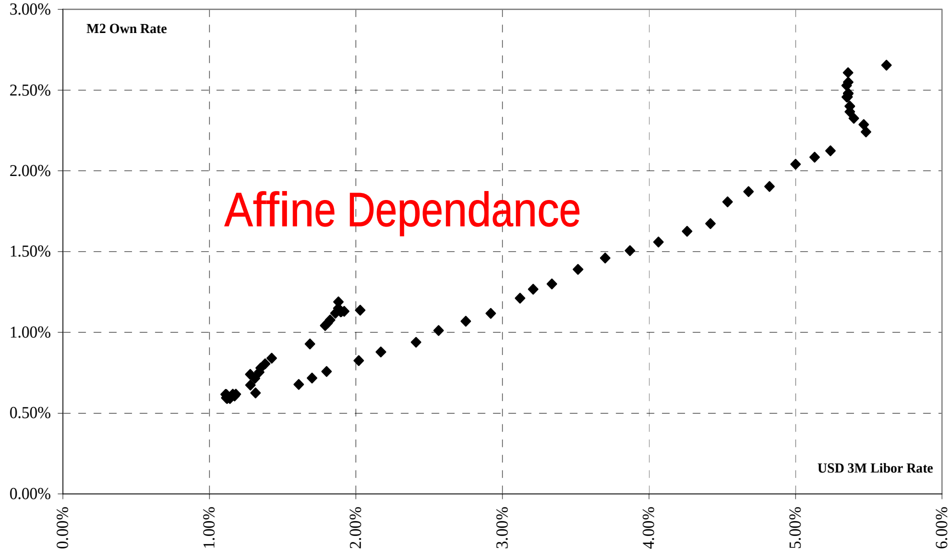
- **We assume the customer rate to be a function of the market rate.**
  - **Affine in general (US) / Sometimes more complex (Japan)**

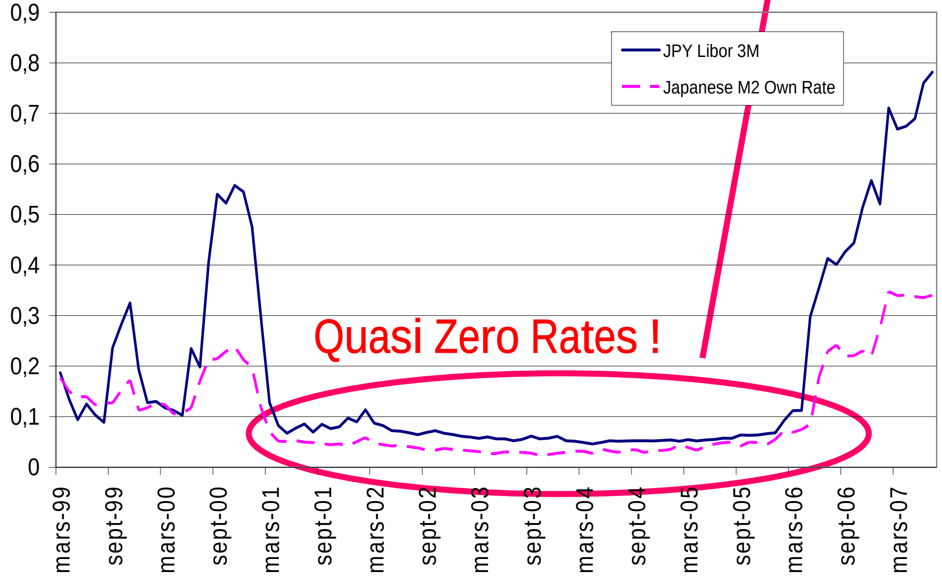
$$g(L_T) = \alpha + \beta \cdot L_T$$

United States

$$g(L_T) = (\alpha + \beta \cdot L_T) \cdot \mathbf{1}\{L_T \geq R\}$$

Japan



# Sets of Hedging Strategies

- 1st case: Investment in FRAs contracted at *t=0*

$$H_{S1} = \left\{ S = \theta \left( L_T - L_0 \right) \; ; \; \theta \in \mathbf{R} \right\}$$

*… is contained in …*

- 2nd case: Dynamic self-financed strategies taking into account the evolution of market rates only

$$H_{S2} = \left\{ S = \int_0^T \theta_t^L dL_t \; ; \; \theta^L \in \Theta^L \right\}$$

*Set of admissible investment strategies adapted to* $F^{W_L}$

*… is contained in …*

- 3rd case: Dynamic strategies taking into account the evolution of the deposit amount

$$H_D = \left\{ S = \int_0^T \theta_t dL_t \; ; \; \theta \in \Theta \right\}$$

*Set of admissible investment strategies adapted to* $F^{W_L} \vee F^{W_K}$

- *'Admissible strategies' are such that each of the sets above are closed*

## Variance-Minimal Measure

- ***Martingale Minimal Measure / Variance Minimal Measure***
  - Martingale Minimal Measure: $\frac{d\overline{\mathbf{P}}}{d\mathbf{P}} = \exp\left( -\frac{1}{2}\int_0^T \lambda^2 dt - \int_0^T \lambda dW_L(t) \right)$
    - Föllmer, Schweizer (1990)
  - In *'almost complete models'*, it coincides with the variance minimal measure: $$\overline{\mathbf{P}} \in \mathbf{Arg} \min_{\mathbf{Q}\in\Pi_{RN}} \mathbf{E}^{\mathbf{P}}\left[\frac{d\mathbf{Q}}{d\mathbf{P}}\right]^2$$
    - Delbaen, Schachermayer (1996)
  - N.B.: In our case, the Variance Minimal Measure density is a power function of the Libor rate. $$\frac{d\overline{\mathbf{P}}}{d\mathbf{P}} = \left(\frac{L_T}{L_0}\right)^{-\frac{\lambda}{\sigma_L}} \exp\left(\frac{1}{2}\left(\lambda^2 - \lambda\sigma_L\right)T\right)$$

# Optimal Dynamic Hedging Strategy – Case #2

- **In Case #2, we determine:** $$\min_{\theta \in \Theta^L} \mathbf{E}^{\mathbf{P}}\left[ IRM_g(K_T, L_T) - \int_0^T \theta_t dL_t \right]^2$$

- **The projection theorem applies**
  - *Delbaen, Monat, Schachermayer, Schweizer, Stricker (1997)*
  - **In case #2, the solution consists in replicating** $\varphi^{S2}(L_T)$

**where** $\varphi^{S2}(x) = \mathbf{E}^{\mathbf{P}}\left[ IRM_g(K_T, L_T) \middle| L_T = x \right] - \mathbf{E}^{\overline{\mathbf{P}}}\left[ IRM_g(K_T, L_T) \right]$

  - *This payoff can be replicated on interest rate markets.*
    - *This is a function of* $L_T$

# Optimal Dynamic Hedging Strategy – Case #3

- **We recall the related problem:** $\min_{\theta\in\Theta} \mathbf{E}^{\mathbf{P}}\left[ IRM_g(K_T, L_T) - \int_0^T \theta_t dL_t \right]^2$
- **The solution is dynamically determined as follows:**

$$\theta_t^{**} = \underbrace{\frac{\partial \mathbf{E}_t^{\overline{\mathbf{P}}}\left[IRM(K_T, L_T)\right]}{\partial L_t}}_{\text{Delta term}} + \underbrace{\frac{\lambda}{\sigma_L L_t}}_{\text{Hedging Numéraire}} \times \underbrace{\left[\mathbf{E}_t^{\overline{\mathbf{P}}}\left[IRM_g(K_T, L_T)\right] - V_t\left(x^{**}, \theta^{**}\right)\right]}_{\text{Feedback term}}$$

Delta term + *Hedging Numéraire* × Feedback term

- Shift between the RN anticipation of the *margin* and the present value of the hedging portfolio

Investment in some *Elementary Portfolio* which verifies

*This portfolio aims at some fixed return while minimizing the final quadratic dispersion.*

$$\mathbf{E}^{\mathbf{P}}\left[\int_0^T \frac{\lambda}{\sigma_L L_t} dL_t - (-1)\right]^2 = \min_{\theta\in\Theta} \mathbf{E}^{\mathbf{P}}\left[\int_0^T \theta_t dL_t - (-1)\right]^2$$

## Optimal Dynamic Hedging Strategy – Some Remarks

- **Case of No Deposit Rate:** $g\left(L_T\right)=0$
  - **Explicit solution (Duffie and Richardson (1991)):**

$$\mathbf{E}_t^{\overline{\mathbf{P}}}\left[IRM_g\left(K_T,L_T\right)\right]=K_t L_t \exp\left[\left(T-t\right)\left(\mu_K-\rho\sigma_K\lambda+\rho\sigma_K\sigma_L\right)\right]$$

$$\frac{\partial \mathbf{E}_t^{\overline{\mathbf{P}}}\left[IRM_g\left(K_T,L_T\right)\right]}{\partial L_t}=\left(1+\frac{\rho\sigma_K}{\sigma_L}\right)K_t \exp\left[\left(T-t\right)\left(\mu_K-\rho\sigma_K\lambda+\rho\sigma_K\sigma_L\right)\right]$$

- **The model works for 'almost complete models'**
  - The Hedging Numéraire remains the following:

$$HN_t=1+\int_0^t \frac{\lambda}{\sigma_L L_t}dL_t$$

or

$$\mathbf{E}^{\mathbf{P}}\left[\int_0^T \frac{\lambda}{\sigma_L L_t}dL_t-\left(-1\right)\right]^2=\min_{\theta\in\Theta}\mathbf{E}^{\mathbf{P}}\left[\int_0^T \theta_t dL_t-\left(-1\right)\right]^2$$

# PRESENTATION OUTLOOK

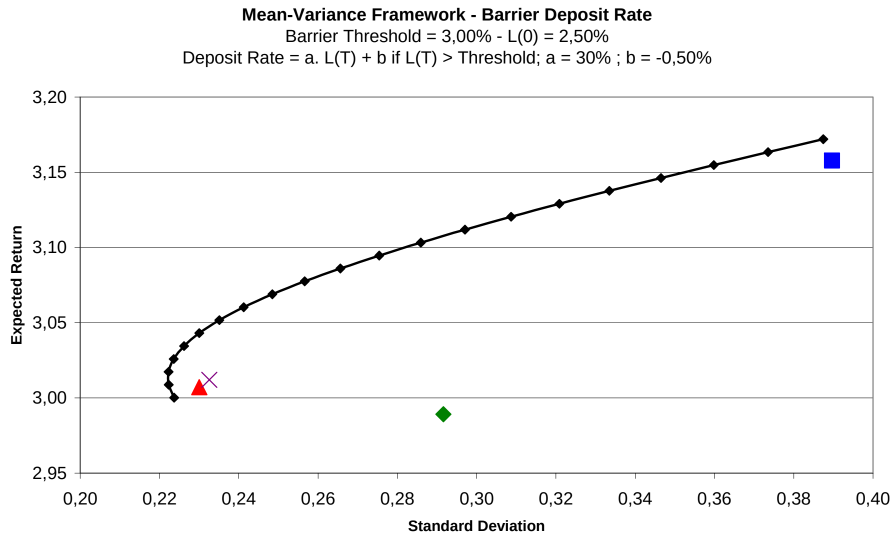
- Modeling Framework, Objective and Optimal Strategy

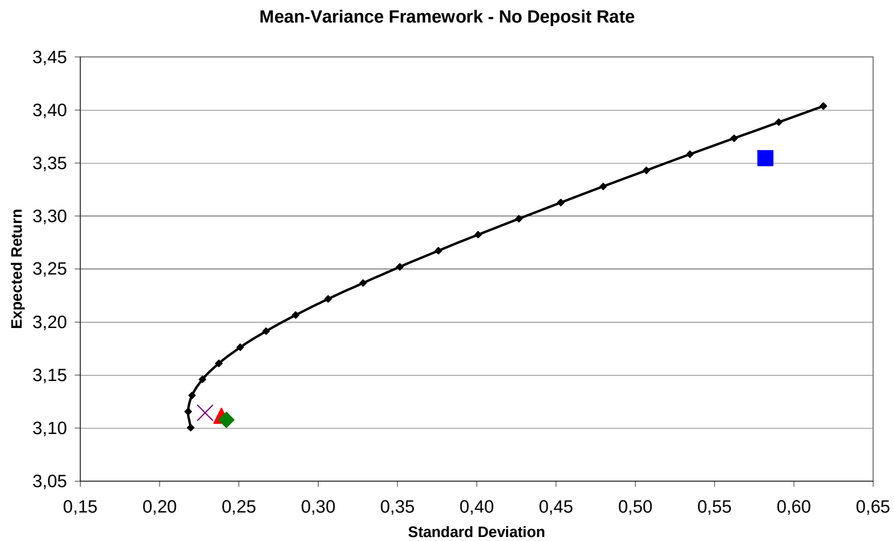
- **Empirical Results**

- Conclusions

# Comparing Strategies in Mean-Variance Framework

- **Efficient Frontiers**
  - Dynamic Efficient Frontier vs. Other Strategies at minimum variance point
  - More discrepancies between strategies when the deposit rate escapes from linearity




Blue: Unhedged Margin

Red: Optimal Dynamic Strategy following only market rates

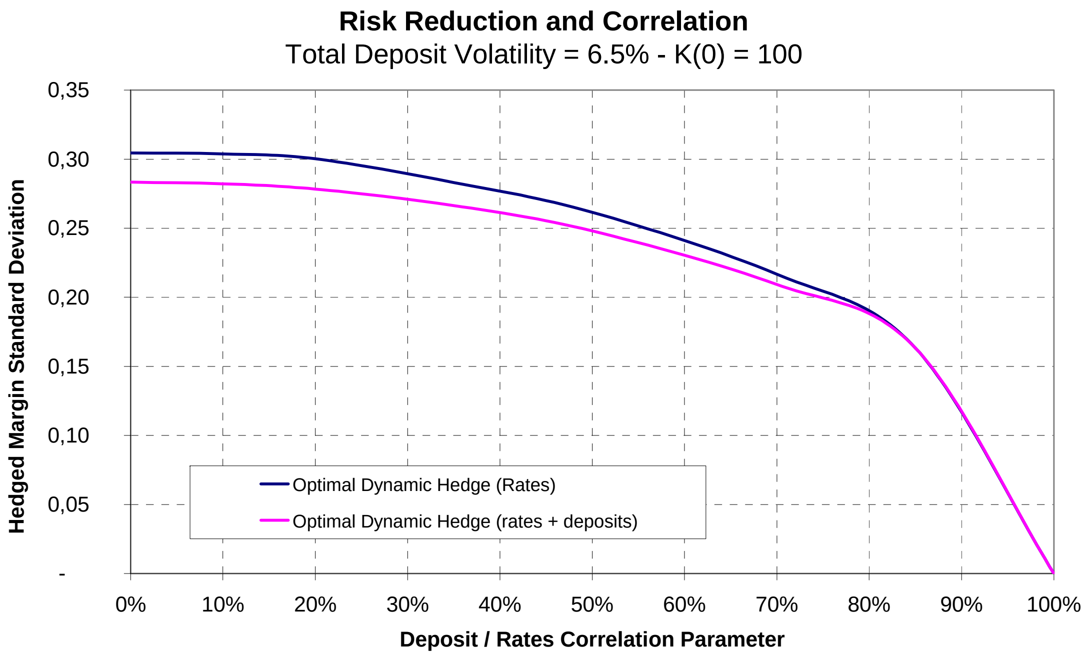
Green: Delta-Hedging at *t=0* only

Purple: Dynamic Delta-Hedging

- **The performances of other hedging strategies strongly depend upon the specification of the deposit rate.**

# Dealing with Deposits' 'Specific' Risk

- **Comparing the optimal dynamic strategy following only market rates (blue) and the optimal dynamic strategy following both rates and deposits (pink):**
  - At minimum variance point (*risk minimization*)
- As expected, the deposits' 'specific' risk is better assessed using a dynamic strategy following both rates and the deposit amount

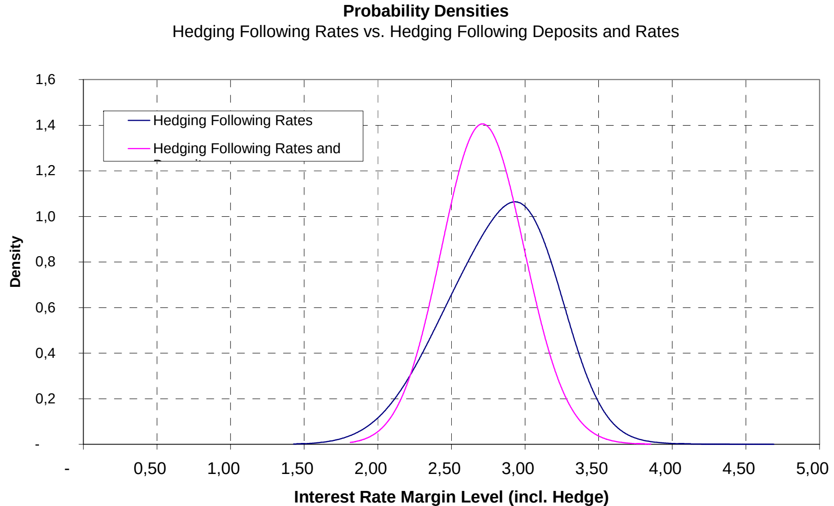


# Robustness towards Risk Criterion

- **The mean-variance optimal dynamic strategy (following deposits and rates) behaves quite well under other risk criteria**
  - Example of Expected Shortfall (99.5%) and VaR (99.95%).

| *Barrier Deposit Rate* | **Expected Return** | **Standard Deviation** | | **ES** (99.5%) | | **VaR** (99.95%) | |
|---|---|---|---|---|---|---|---|
| | | Level | *Risk Reduction* | Level | *Risk Reduction* | Level | *Risk Reduction* |
| **Unhedged Margin** | 3.16 | 0.39 | | -2.02 | | -1.90 | |
| **Static Hedge Case 1** | 3.04 | 0.28 | *-0.11* | -2.34 | *-0.32* | -2.26 | *-0.36* |
| **Static Hedge Case 2** | 3.01 | 0.23 | *-0.16* | -2.26 | *-0.24* | -2.04 | *-0.14* |
| **Jarrow and van Deventer** | 3.01 | 0.24 | *-0.15* | -2.35 | *-0.33* | -2.25 | *-0.35* |
| **Optimal Dynamic Hedge** | 3.01 | 0.22 | ***-0.17*** | -2.38 | ***-0.36*** | -2.29 | ***-0.39*** |

- **The optimal dynamic strategy features better tail distribution than for other strategies**
  - Blue: Optimal Dynamic Strategy (following rates)
  - Pink: Optimal Dynamic Strategy (following both deposits and rates)



## Dealing with Massive Bank Run

- **Introducing a Poisson Jump component in the deposit amount:**

$$dK_t = K_t\left[\mu_K dt + \sigma_K d\overline{W_K}(t) - dN(t)\right]$$

$\left(N(t)\right)_{0\le t\le T}$ is assumed to be independent from $W_K$ and $W_L$

- **Then, we have:** $\theta_t^{**} = \frac{\partial \mathbf{E}_t^{\overline{\mathbf{P}}}\left[IRM\left(K_T, L_T\right)\right]}{\partial L_t} + \frac{\lambda}{\sigma_L L_t}\left[\mathbf{E}_t^{\overline{\mathbf{P}}}\left[IRM_g\left(K_T, L_T\right)\right] - V_t\left(x^{**}, \theta^{**}\right)\right]$

$$\mathbf{E}_t^{\overline{\mathbf{P}}}\left[IRM_g\left(K_T, L_T\right)\right] = e^{-\gamma(T-t)} \times \text{ (\textit{Previous conditional expectation term})}$$

  - Due to independence, the jump element can be put out the conditional expectations

- **N.B.: *When a bank run occurs, the manager keeps investing the current hedging portfolio's value in the Hedging Numéraire***

# PRESENTATION OUTLOOK

- Modeling Framework, Objective and Optimal Strategy

- Empirical Results

- Conclusions

# Conclusions (1)

- **A dynamic strategy to assess risk in mean-variance framework**
  - Results about Mean-variance hedging in incomplete markets yield explicit dynamic hedging strategies

- **Practical Conclusions:**
  - Better assessment of deposits' 'specific' risk with a dynamic strategy taking into account both deposits and rates;
  - Lack of stability for other strategies towards the deposit rate's specification;
  - Robustness towards risk criterion
  - No negative consequences as for tail distribution
  - Additivity of Optimal Dynamic Strategies
    - Applicable to **various balance sheet items**

# Conclusions (2)

- **We use some mathematical finance concepts:**
  - **For Financial Engineering problems**
  - **with the aim of providing applicable strategies**
  - **And improve risk management processes**

# Technical References


- Duffie, D., Richardson, H. R., 1991. *Mean-variance hedging in continuous time*. Annals of Applied Probability 1(1).
- Gouriéroux, C., Laurent, J.-P., Pham, H., 1998. *Mean-variance hedging and numéraire*. Mathematical Finance 8(3).
- Hutchison, D., Pennacchi, G., 1996. Measuring Rents and Interest Rate Risk in Imperfect Financial Markets : The Case of Retail Bank Deposits. Journal of Financial and Quantitative Analysis 31(3).
- Jarrow, R., van Deventer, D., 1998. The arbitrage-free valuation and hedging of demand deposits and credit card loans. Journal of Banking and Finance 22.
- O'Brien, J., 2000. Estimating the value and interest risk of interest-bearing transactions deposits. Division of Research and Statistics / Board of Governors / Federal Reserve System.