Hedging Demand Deposits Interest Rate Margins

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PRESENTATION OUTLOOK

- Modeling Framework, Objective and Optimal Strategy

- Empirical Results

- Conclusions
Demand Deposit Interest Rate Margin – Definition

- Demand Deposit Interest Rate Margin for a given quarter:
  - *Income generated by the investment of Demand Deposit Amount on interbank markets while paying a deposit rate to customers*

- Risks in Interest Rate Margins:
  - Interest Rate Risk:
    - 1. Investment on interbank markets
    - 2. Paying an interest rate to customers (possibly correlated to market rates)
    - 3. Demand Deposit amount is subject to transfer effects from customers, due to market rate variations
  - Non hedgeable Risk Factors on the Deposit Amount:
    - **Business Risk**: Competition between banks, customer behavior independent from market conditions, etc.
    - **Model Risk**
Setting the Objective

Interest Rate Margin \( IRM_g(K_T, L_T) = K_T(L_T - g(L_T)) \cdot \Delta T \)

- Deposit Amount at \( T \)
- Investment Market Rate during time interval \([T, T+\Delta T]\)
- Customer rate at \( T \)

**Mean-variance framework:**
- Including a return constraint – due to the interest rate risk premium

\[
\min_S \mathbb{E}\left[ IRM_g(K_T, L_T) - S \right]^2 \quad \text{under constraint} \quad \mathbb{E}\left[ IRM_g(K_T, L_T) - S \right] \geq r
\]
Dynamics for Market Rate \( L_t = L(t, T, T + \Delta T) \)

- Libor Market Model for Investment Market Rate

\[
\frac{dL_t}{L_t} = \mu_L dt + \sigma_L dW_L(t) \quad \text{Ex.: Brace, Gatarek, Musiela (1997)}
\]

\( \mu_L \neq 0 \) Long-Term Investment Risk Premium

- Coefficient specification assumptions:
  - **Our model:** \( \mu_L, \sigma_L \) constant

  *(and can be easily extended to time-dependent framework)*
Deposit Amount Dynamics

- **Diffusion process for Deposit Amount**
  \[ dK_t = K_t \left[ \mu_K dt + \sigma_K dW_K(t) \right] \]

- **Sensitivity of deposit amount to market rates**
  - Money transfers between deposits and other accounts

- **Interest Rate partial contingency.**
  - Business risk, …
  - **Incomplete market framework**

\[ dW_K(t) = \rho dW_L(t) + \sqrt{1 - \rho^2} dW_K(t) \quad -1 < \rho < 0 \]
Deposit Amount Dynamics – Examples

\[ dK_t = K_t \left( \mu_K dt + \sigma_K dW_K(t) \right) \]

US and Euro Zone

Emerging Markets (Turkey, Ukraine)

EuroZone – \( \hat{\mu}_K = 10.19\% , \hat{\sigma}_K = 6.56\% \)

Turkey – \( \hat{\mu}_K = 51.74\% , \hat{\sigma}_K = 37.38\% \)
Modeling Deposit Rate – Examples

- We assume the customer rate to be a function of the market rate.
  - Affine in general (US) / Sometimes more complex (Japan)

\[
g(L_T) = \alpha + \beta \cdot L_T
\]

United States

\[
g(L_T) = (\alpha + \beta \cdot L_T) \cdot 1\{L_T \geq R\}
\]

Japan

USD 3M Libor Rate

M2 Own Rate

Affine Dependence

Quasi Zero Rates!
**Sets of Hedging Strategies**

- **1st case:** Investment in FRAs contracted at $t=0$
  
  $$H_{S1} = \left\{ S = \theta (L_T - L_0) ; \theta \in \mathbb{R} \right\}$$

- **2nd case:** Dynamic self-financed strategies taking into account the evolution of market rates only
  
  $$H_{S2} = \left\{ S = \int_0^T \theta^L_t \, dL_t ; \theta^L \in \Theta^L \right\}$$

- **3rd case:** Dynamic strategies taking into account the evolution of the deposit amount
  
  $$H_D = \left\{ S = \int_0^T \theta_t \, dL_t ; \theta \in \Theta \right\}$$

- ‘Admissible strategies’ are such that each of the sets above are closed
Variance-Minimal Measure

- **Martingale Minimal Measure / Variance Minimal Measure**
  - Martingale Minimal Measure: \[ \frac{d\bar{P}}{dP} = \exp\left( -\frac{1}{2} \int_0^T \lambda^2 dt - \int_0^T \lambda dW_L(t) \right) \]
    - Föllmer, Schweizer (1990)
  - In *almost complete models*, it coincides with the variance minimal measure:
    \[ \bar{P} \in \text{Arg min}_{Q \in \Pi_{RN}} E^P \left[ \frac{dQ}{dP} \right]^2 \]
    - Delbaen, Schachermayer (1996)
  - N.B.: In our case, the Variance Minimal Measure density is a power function of the Libor rate.
    \[ \frac{d\bar{P}}{dP} = \left( \frac{L_T}{L_0} \right)^{-\frac{\lambda}{\sigma_L}} \exp\left( \frac{1}{2} \left( \lambda^2 - \lambda \sigma_L \right) T \right) \]
Optimal Dynamic Hedging Strategy – Case #2

- In Case #2, we determine:
  \[
  \min_{\theta \in \Theta_T} \mathbb{E}^\mathbb{P} \left[ IRM_g(K_T, L_T) - \int_0^T \theta_t dL_t \right]^2
  \]

- The projection theorem applies
  - *Delbaen, Monat, Schachermayer, Schweizer, Stricker (1997)*
  - In case #2, the solution consists in replicating \( \varphi^{S2}(L_T) \)
    
    where \( \varphi^{S2}(x) = \mathbb{E}^\mathbb{P}[IRM_g(K_T, L_T)|L_T = x] - \mathbb{E}^\mathbb{P}[IRM_g(K_T, L_T)] \)

- This payoff can be replicated on interest rate markets.
  - This is a function of \( L_T \)
Optimal Dynamic Hedging Strategy – Case #3

- We recall the related problem: 
  \[ \min_{\theta \in \Theta} \mathbb{E}^P \left[ IRM_g(K_T, L_T) - \int_0^T \theta_t dL_t \right] \]

- The solution is dynamically determined as follows:

\[ \theta^{**}_t = \frac{\partial \mathbb{E}^P_t [IRM(K_T, L_T)]}{\partial L_t} + \frac{\lambda}{\sigma_L L_t} \left[ \mathbb{E}^P_t [IRM_g(K_T, L_T)] - V_t(x^{**}, \theta^{**}) \right] \]

Delta term + Hedging Numéraire × Feedback term

- Shift between the RN anticipation of the margin and the present value of the hedging portfolio

Investment in some Elementary Portfolio which verifies

This portfolio aims at some fixed return while minimizing the final quadratic dispersion.

\[ \mathbb{E}^P \left[ \int_0^T \frac{\lambda}{\sigma_L L_t} dL_t - (-1) \right]^2 = \min_{\theta \in \Theta} \mathbb{E}^P \left[ \int_0^T \theta_t dL_t - (-1) \right]^2 \]
Optimal Dynamic Hedging Strategy – Some Remarks

- **Case of No Deposit Rate:** \( g(L_T) = 0 \)

- **Explicit solution (Duffie and Richardson (1991)):**

\[
\mathbb{E}_t^P \left[ IRM_g (K_T, L_T) \right] = K_t L_t \exp \left[ (T-t) (\mu_K - \rho \sigma_K \lambda + \rho \sigma_K \sigma_L) \right]
\]

\[
\frac{\partial \mathbb{E}_t^P \left[ IRM_g (K_T, L_T) \right]}{\partial L_t} = \left( 1 + \frac{\rho \sigma_K}{\sigma_L} \right) K_t \exp \left[ (T-t) (\mu_K - \rho \sigma_K \lambda + \rho \sigma_K \sigma_L) \right]
\]

- **The model works for ‘almost complete models’**

  - The Hedging Numéraire remains the following:

\[
HN_t = 1 + \int_0^t \frac{\lambda}{\sigma_L L_t} dL_t
\]

or

\[
\mathbb{E}^P \left[ \int_0^T \frac{\lambda}{\sigma_L L_t} dL_t - (-1) \right]^2 = \min_{\theta \in \Theta} \mathbb{E}^P \left[ \int_0^T \theta dL_t - (-1) \right]^2
\]
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Comparing Strategies in Mean-Variance Framework

- **Efficient Frontiers**
  - Dynamic Efficient Frontier vs. Other Strategies at minimum variance point
  - More discrepancies between strategies when the deposit rate escapes from linearity

The performances of other hedging strategies strongly depend upon the specification of the deposit rate.
Dealing with Deposits’ ‘Specific’ Risk

- Comparing the optimal dynamic strategy following only market rates (blue) and the optimal dynamic strategy following both rates and deposits (pink):
  - At minimum variance point (*risk minimization*)

- As expected, the deposits’ ‘specific’ risk is better assessed using a dynamic strategy following both rates and the deposit amount

![Graph showing risk reduction and correlation](image)

**Risk Reduction and Correlation**
Total Deposit Volatility = 6.5% - K(0) = 100
Robustness towards Risk Criterion

- The mean-variance optimal dynamic strategy (following deposits and rates) behaves quite well under other risk criteria
  - Example of Expected Shortfall (99.5%) and VaR (99.95%).

<table>
<thead>
<tr>
<th>Barrier Deposit Rate</th>
<th>Expected Return</th>
<th>Standard Deviation</th>
<th>ES (99.5%)</th>
<th>VaR (99.95%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unhedged Margin</td>
<td>3.16</td>
<td>0.39</td>
<td>-2.02</td>
<td>-1.90</td>
</tr>
<tr>
<td>Static Hedge Case 1</td>
<td>3.04</td>
<td>0.28</td>
<td>-0.11</td>
<td>-2.34</td>
</tr>
<tr>
<td>Static Hedge Case 2</td>
<td>3.01</td>
<td>0.23</td>
<td>-0.16</td>
<td>-2.26</td>
</tr>
<tr>
<td>Jarrow and van Deventer</td>
<td>3.01</td>
<td>0.24</td>
<td>-0.15</td>
<td>-2.35</td>
</tr>
<tr>
<td>Optimal Dynamic Hedge</td>
<td>3.01</td>
<td>0.22</td>
<td>-0.17</td>
<td>-2.38</td>
</tr>
</tbody>
</table>

- The optimal dynamic strategy features better tail distribution than for other strategies
  - Blue: Optimal Dynamic Strategy (following rates)
  - Pink: Optimal Dynamic Strategy (following both deposits and rates)
Dealing with *Massive Bank Run*

- Introducing a Poisson Jump component in the deposit amount:
  \[
  dK_t = K_t \left[ \mu_K dt + \sigma_K d\overline{W}_K(t) - dN(t) \right]
  \]
  
  \( (N(t))_{0 \leq t \leq T} \) is assumed to be independent from \( W_K \) and \( W_L \)

- Then, we have:
  \[
  \theta_t^{**} = \frac{\partial E_t^P[IRM(K_T, L_T)]}{\partial L_t} + \frac{\lambda}{\sigma_L L_t} \left[ E_t^P[IRM_g(K_T, L_T)] - V_t(x^{**}, \theta^{**}) \right]
  \]

  \[
  E_t^P \left[ IRM_g \left( K_T, L_T \right) \right] = e^{-\gamma(T-t)} \times \text{(Previous conditional expectation term)}
  \]

  - Due to independence, the jump element can be put out the conditional expectations

- **N.B.:** *When a bank run occurs, the manager keeps investing the current hedging portfolio’s value in the Hedging Numéraire*
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Conclusions (1)

- A **dynamic** strategy to assess risk in mean-variance framework
  - Results about Mean-variance hedging in incomplete markets yield explicit dynamic hedging strategies

- **Practical Conclusions:**
  - Better assessment of deposits’ ‘specific’ risk with a dynamic strategy taking into account both deposits and rates;
  - Lack of stability for other strategies towards the deposit rate’s specification;
  - Robustness towards risk criterion
  - No negative consequences as for tail distribution
  - Additivity of Optimal Dynamic Strategies
    - Applicable to various balance sheet items
Conclusions (2)

- We use some mathematical finance concepts:
  - For Financial Engineering problems
  - with the aim of providing applicable strategies
  - And improve risk management processes
Technical References


