Hedging Demand Deposits Interest Rate Margins

Jean-Paul LAURENT, Mohamed HOUKARI

laurent.jeanpaul@univ-lyon1.fr ; mohamed.houkari@bnpparibas.com

Alexandre ADAM, BNP Paribas Asset and Liability Management

Mohamed HOUKARI, ISFA, Université de Lyon, Université Lyon 1 and BNP Paribas ALM

Jean-Paul LAURENT, ISFA, Université de Lyon, Université Lyon 1

PRESENTATION OUTLOOK

- Overview and Context
- Modeling Framework, Objective and Optimal Strategy
- Empirical Results
- Conclusions

Demand Deposits in Bank Balance Sheet

- Demand Deposits involve huge amounts
  - **(Bank of America Annual Report – Dec. 2007; Source: SEC)**

	Average Balance	
(Dollars in millions)	**2007**	2006
Assets		
Federal funds sold and securities purchased under agreements to resell	$ **155,828**	$ 175,334
Trading account assets	**187,287**	145,321
Debt securities	**186,466**	225,219
Loans and leases, net of allowance for loan and lease losses	**766,329**	643,259
All other assets	**306,163**	277,548
Total assets	$ **1,602,073**	$ 1,466,681
Liabilities		
Deposits	$ **717,182**	$ 672,995
Federal funds purchased and securities sold under agreements to repurchase	**253,481**	286,903
Trading account liabilities	**82,721**	64,689
Commercial paper and other short-term borrowings	**171,333**	124,229
Long-term debt	**169,855**	130,124
All other liabilities	**70,839**	57,278
Total liabilities	**1,465,411**	1,336,218
Shareholders' equity	**136,662**	130,463
Total liabilities and shareholders' equity	$ **1,602,073**	$ 1,466,681

- ***US Banks are monitored by the SEC as for Interest Rate Risk***

Demand Deposit Interest Rate Margin – Definition

- **Demand Deposit Interest Rate Margin for a given quarter:**
  - *Income generated by the investment of Demand Deposit Amount on interbank markets while paying a deposit rate to customers*
- **Risks in Interest Rate Margins:**
  - Interest Rate Risk:
    - 1. Investment on interbank markets
    - 2. Paying an interest rate to customers (possibly correlated to market rates)
    - 3. Demand Deposit amount is subject to transfer effects from customers, due to market rate variations
  - Non hedgeable Risk Factors on the Deposit Amount:
    - **Business Risk**: Competition between banks, customer behavior independent from market conditions, etc.
    - **Model Risk**

We need to focus on Interest Rate Margins…

- … according to the IFRS (International accounting standards) :
  - The IFRS recommend the accounting of non maturing assets and liabilities at Amortized Cost / Historical Cost
- Being studied: Recognition of related hedging strategies from the accounting viewpoint
  - *Interest Margin Hedge* (IMH).

Why do not we use the Demand Deposit *Fair Value*?

- **The *fair value* of Demand Deposits:**
  - is computed by *Discounting* future interest rate margins on the DD activity
  - *Risk-neutral expectation* of the related sum
- ***Demand Deposits are a complex financial product!***
  - **The fair-value involves some pricing of non-hedgeable risks**
    - Business risk, customers' behaviour, etc.
  - *Which risk-neutral measure should we use?*
- **Practical concern for banking establishments**
  - Fair Value-based hedging strategies lack of robustness as for model specification.

Risk Mitigation within Interest Rate Margins

- Hedging Demand Deposit Interest Rate Margins:
  - We mitigate risk using Interest Rate Derivatives such as Interest Rate Swaps
  - We include a risk premium on interest rate markets
    - Investing in ***long-term*** assets financed by ***short-term*** liabilities is rewarding.

- Return-Risk Tradeoff between:
  - Risk Reduction:
    - Using Interest Rate Swaps
  - Return Opportunities:
    - Taking advantage of long term investment risk premium.

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Setting the Objective

Interest Rate Margin $IRM_g(K_T, L_T) = K_T(L_T - g(L_T)) \cdot \Delta T$

- K_T: Deposit Amount at T
- L_T: Investment Market Rate during time interval [T,T+ΔT]
- $g(L_T)$: Customer rate at T

- **Mean-variance framework:**
  - Including a return constraint – due to the interest rate risk premium

$$\min_S \mathbf{E}\left[IRM_g(K_T, L_T) - S\right]^2 \text{ under constraint } \mathbf{E}\left[IRM_g(K_T, L_T) - S\right] \geq r$$

Dynamics for Market Rate $L_t = L(t, T, T + \Delta T)$

- **Libor Market Model for Investment Market Rate**

$$\frac{dL_t}{L_t} = \mu_L dt + \sigma_L dW_L(t)$$

Ex.: Brace, Gatarek, Musiela (1997)

$\mu_L \neq 0$ Long-Term Investment Risk Premium

- **Coefficient specification assumptions:**
  - **Our model:** μ_L, σ_L constant

    (and can be easily extended to time-dependent framework)
  - *'Almost Complete'* framework
    - H. Pagès (1987), Pham, Rheinländer, Schweizer (1998), Laurent, Pham (1998)

      μ_L, σ_L bounded and adapted to F^{W_L}

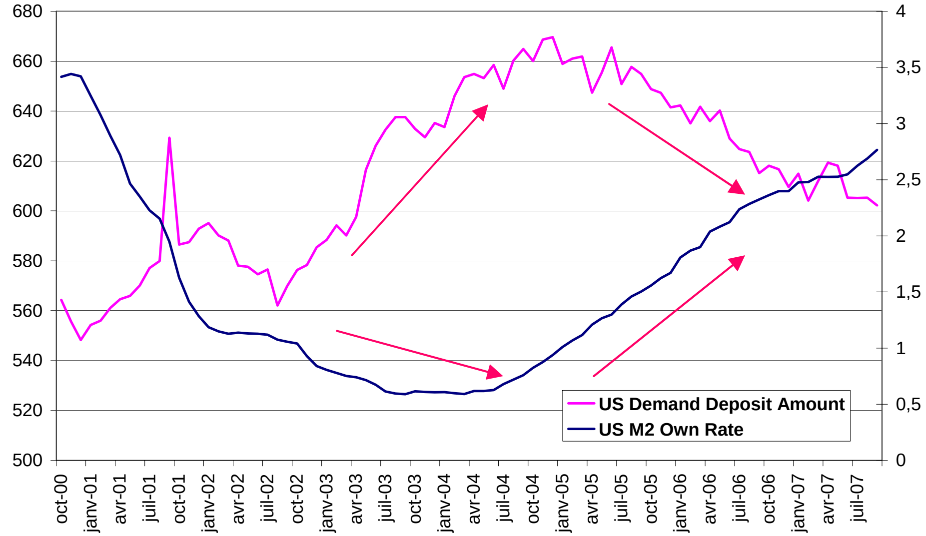
Deposit Amount Dynamics

- Diffusion process for Deposit Amount

$$dK_t = K_t\left[\mu_K dt + \sigma_K d\overline{W_K}(t)\right]$$

- Sensitivity of deposit amount to market rates
  - Money transfers between deposits and other accounts
- Interest Rate partial contingence.
  - Business risk, …
  - **Incomplete market framework**

(US marketplace)



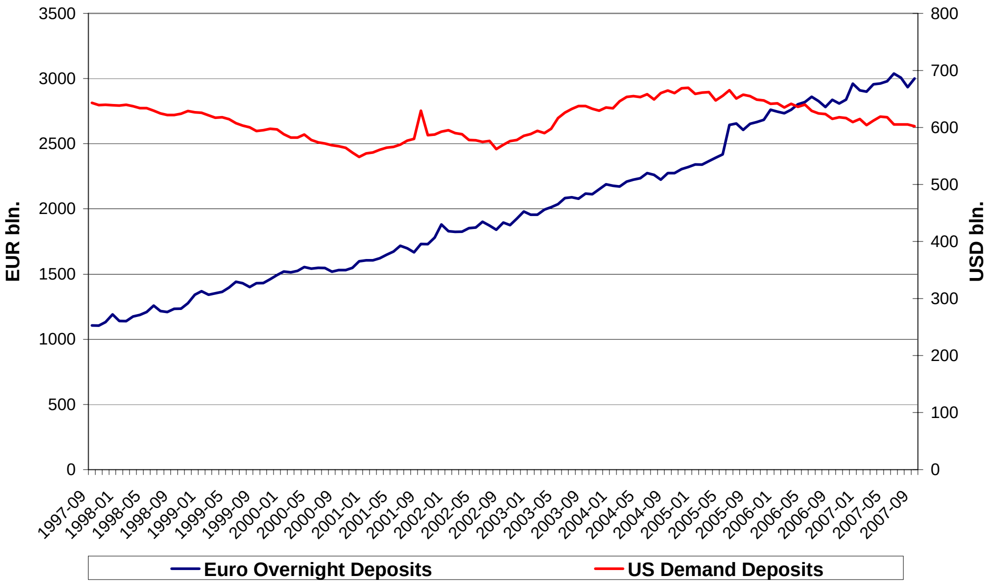
$$d\overline{W_K}(t) = \rho dW_L(t) + \sqrt{1-\rho^2}\, dW_K(t)$$

$$-1 < \rho < 0$$

Deposit Amount Dynamics – Examples

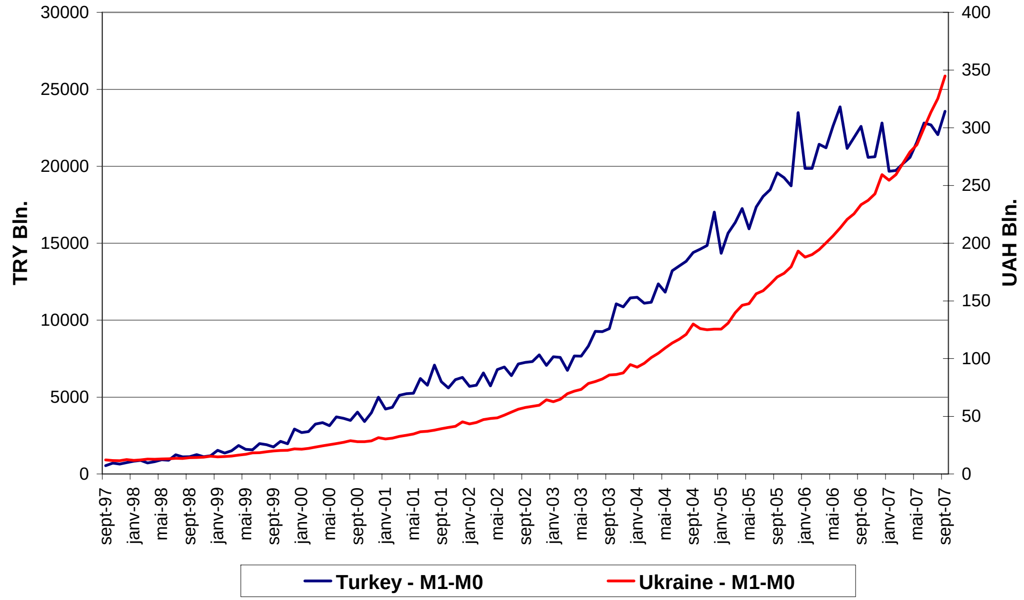
$$dK_t = K_t\left(\mu_K dt + \sigma_K d\overline{W}_K(t)\right)$$

US and Euro Zone



EuroZone $- \hat{\mu}_K = 10.19\%, \hat{\sigma}_K = 6.56\%$

Emerging Markets (Turkey, Ukraine)



Turkey $- \hat{\mu}_K = 51.74\%, \hat{\sigma}_K = 37.38\%$

Modeling Deposit Rate – Examples

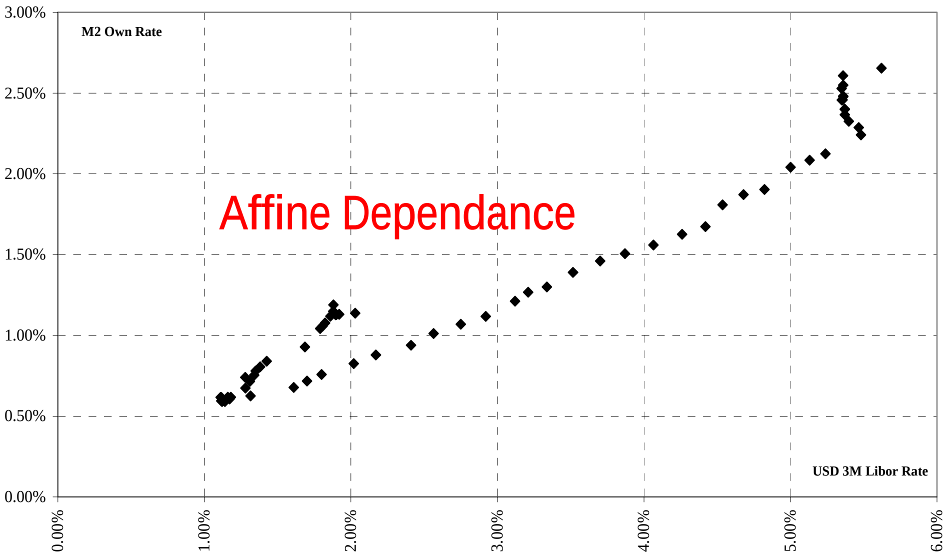
- **We assume the customer rate to be a function of the market rate.**
  - **Affine in general (US) / Sometimes more complex (Japan)**

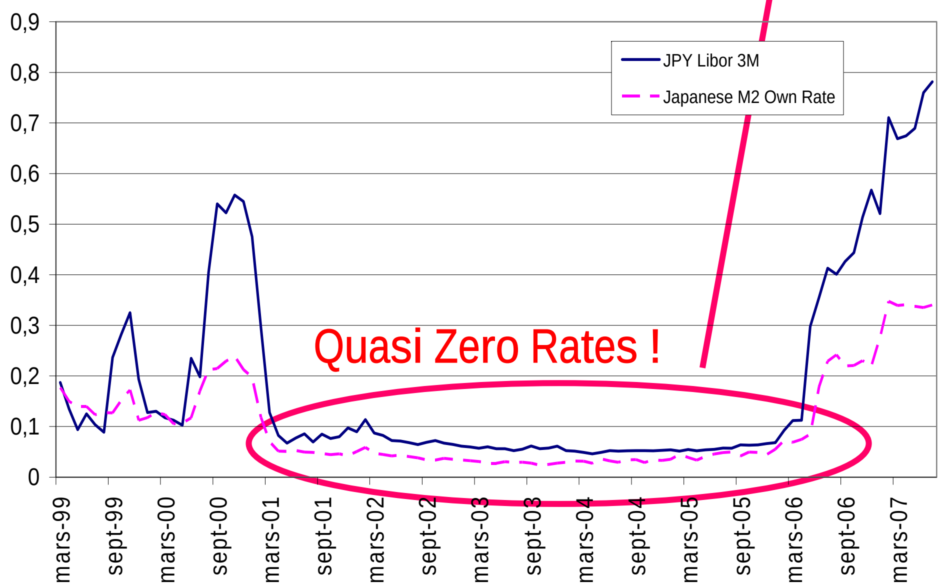
$$g(L_T) = \alpha + \beta \cdot L_T$$

United States

$$g(L_T) = (\alpha + \beta \cdot L_T) \cdot \mathbf{1}\{L_T \geq R\}$$

Japan





Sets of Hedging Strategies

… is contained in …

… is contained in …

- 1st case: Investment in FRAs contracted at *t=0*

$$H_{S1} = \left\{ S = \theta\left(L_T - L_0\right) \,;\, \theta \in \mathbf{R} \right\}$$

- 2nd case: Dynamic self-financed strategies taking into account the evolution of market rates only

$$H_{S2} = \left\{ S = \int_0^T \theta_t^L dL_t \,;\, \theta^L \in \Theta^L \right\}$$

Set of admissible investment strategies adapted to F^{W_L}

- 3rd case: Dynamic strategies taking into account the evolution of the deposit amount

$$H_D = \left\{ S = \int_0^T \theta_t dL_t \,;\, \theta \in \Theta \right\}$$

Set of admissible investment strategies adapted to $F^{W_L} \vee F^{W_K}$

- *'Admissible strategies' are such that each of the sets above are closed*

Variance-Minimal Measure

- ***Martingale Minimal Measure / Variance Minimal Measure***
  - Martingale Minimal Measure: $\frac{d\overline{\mathbf{P}}}{d\mathbf{P}} = \exp\left(-\frac{1}{2}\int_0^T \lambda^2 dt - \int_0^T \lambda dW_L(t)\right)$
    - Föllmer, Schweizer (1990)
  - In *'almost complete models'*, it coincides with the variance minimal measure: $$\overline{\mathbf{P}} \in \mathbf{Arg}\min_{\mathbf{Q}\in\Pi_{RN}} \mathbf{E}^{\mathbf{P}}\left[\frac{d\mathbf{Q}}{d\mathbf{P}}\right]^2$$
    - Delbaen, Schachermayer (1996)
  - N.B.: In our case, the Variance Minimal Measure density is a power function of the Libor rate. $$\frac{d\overline{\mathbf{P}}}{d\mathbf{P}} = \left(\frac{L_T}{L_0}\right)^{-\frac{\lambda}{\sigma_L}} \exp\left(\frac{1}{2}\left(\lambda^2 - \lambda\sigma_L\right)T\right)$$

Optimal Dynamic Hedging Strategy – Case #2

- **In Case #2, we determine:** $$\min_{\theta \in \Theta^L} \mathbf{E}^{\mathbf{P}}\left[IRM_g(K_T, L_T) - \int_0^T \theta_t \, dL_t \right]^2$$

- **The projection theorem applies**
  - *Delbaen, Monat, Schachermayer, Schweizer, Stricker (1997)*
  - **In case #2, the solution consists in replicating** $\varphi^{S2}(L_T)$

where $\varphi^{S2}(x) = \mathbf{E}^{\mathbf{P}}\left[IRM_g(K_T, L_T) \middle| L_T = x \right] - \mathbf{E}^{\overline{\mathbf{P}}}\left[IRM_g(K_T, L_T) \right]$

  - *Under the "almost complete" assumption, this payoff can be replicated on interest rate markets.*
    - *N.B.: The latter payoff is a function of* L_T

Optimal Dynamic Hedging Strategy – Case #3

- **We recall the related problem:** $$\min_{\theta\in\Theta} \mathbf{E}^{\mathbf{P}}\left[IRM_g(K_T, L_T) - \int_0^T \theta_t dL_t \right]^2$$
- **The solution is dynamically determined as follows:**

$$\theta_t^{**} = \underbrace{\frac{\partial \mathbf{E}_t^{\overline{\mathbf{P}}}\left[IRM(K_T, L_T)\right]}{\partial L_t}}_{\text{Delta term}} + \underbrace{\frac{\lambda}{\sigma_L L_t}}_{\text{Hedging Numéraire}} \times \underbrace{\left[\mathbf{E}_t^{\overline{\mathbf{P}}}\left[IRM_g(K_T, L_T)\right] - V_t\left(x^{**}, \theta^{**}\right)\right]}_{\text{Feedback term}}$$

Delta term + Hedging Numéraire × Feedback term

Feedback term
- Shift between the RN anticipation of the *margin* and the present value of the hedging portfolio

Hedging Numéraire → Investment in some *Elementary Portfolio* which verifies

This portfolio aims at some fixed return while minimizing the final quadratic dispersion.

$$\mathbf{E}^{\mathbf{P}}\left[\int_0^T \frac{\lambda}{\sigma_L L_t} dL_t - (-1)\right]^2 = \min_{\theta\in\Theta} \mathbf{E}^{\mathbf{P}}\left[\int_0^T \theta_t dL_t - (-1)\right]^2$$

Optimal Dynamic Hedging Strategy – Some Remarks

- **Case of No Deposit Rate:** $g\left(L_T\right)=0$
  - **Explicit solution (Duffie and Richardson (1991)):**

$$\mathbf{E}_t^{\overline{\mathbf{P}}}\left[IRM_g\left(K_T, L_T\right)\right]=K_t L_t \exp\left[\left(T-t\right)\left(\mu_K-\rho\sigma_K\lambda+\rho\sigma_K\sigma_L\right)\right]$$

$$\frac{\partial\mathbf{E}_t^{\overline{\mathbf{P}}}\left[IRM_g\left(K_T, L_T\right)\right]}{\partial L_t}=\left(1+\frac{\rho\sigma_K}{\sigma_L}\right)K_t \exp\left[\left(T-t\right)\left(\mu_K-\rho\sigma_K\lambda+\rho\sigma_K\sigma_L\right)\right]$$

- **The model works for 'almost complete models'**
  - The Hedging Numéraire remains the following:

$$HN_t=1+\int_0^t \frac{\lambda}{\sigma_L L_t}dL_t$$

or

$$\mathbf{E}^{\mathbf{P}}\left[\int_0^T \frac{\lambda}{\sigma_L L_t}dL_t-\left(-1\right)\right]^2=\min_{\theta\in\Theta}\mathbf{E}^{\mathbf{P}}\left[\int_0^T \theta_t dL_t-\left(-1\right)\right]^2$$

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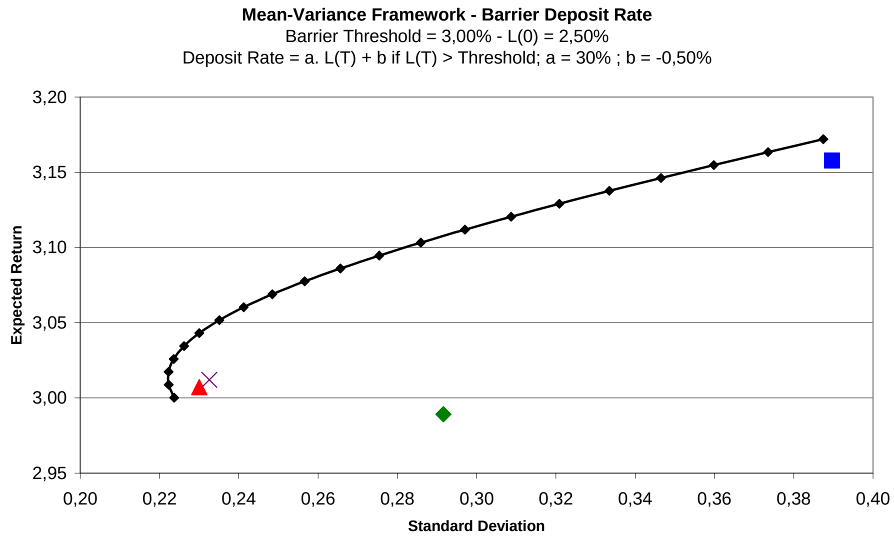
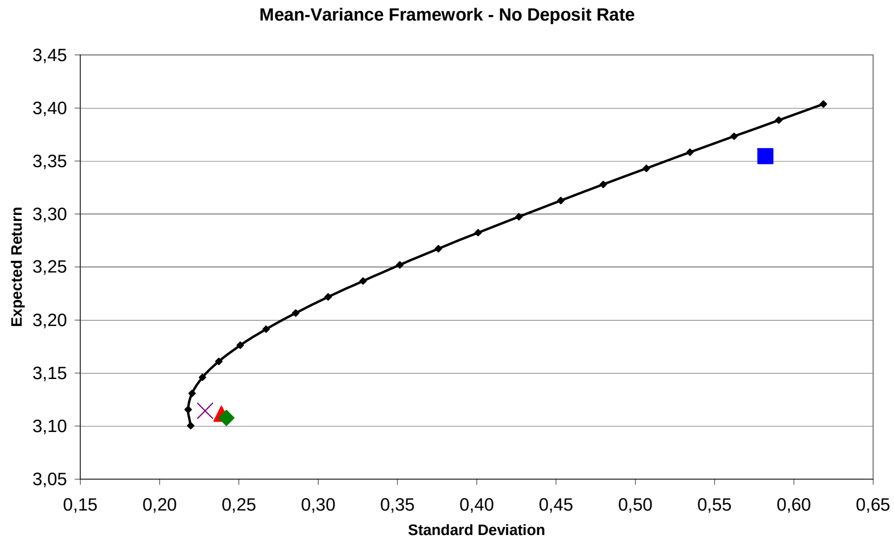
Comparing Strategies in Mean-Variance Framework

- **Efficient Frontiers**
  - Dynamic Efficient Frontier vs. Other Strategies at minimum variance point
  - More discrepancies between strategies when the deposit rate escapes from linearity





Blue: Unhedged Margin

Red: Optimal Dynamic Strategy following only market rates

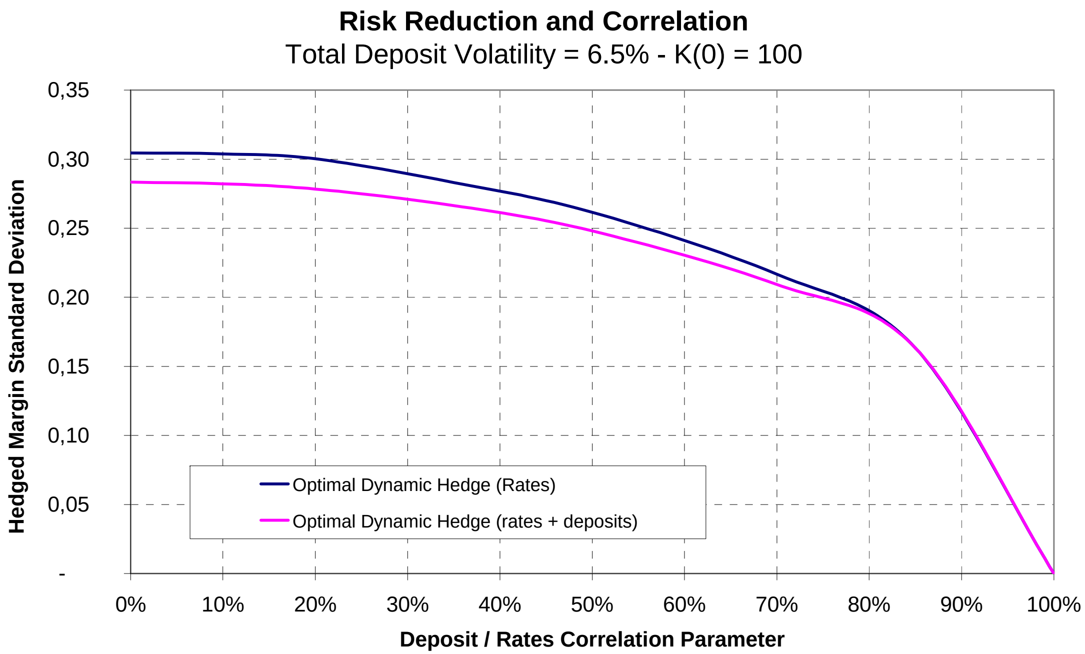
Green: Delta-Hedging at *t=0* only

Purple: Dynamic Delta-Hedging

- **The performances of other hedging strategies strongly depend upon the specification of the deposit rate.**

Dealing with Deposits’ ‘Specific’ Risk

- **Comparing the optimal dynamic strategy following only market rates (blue) and the optimal dynamic strategy following both rates and deposits (pink):**
  - At minimum variance point (*risk minimization*)
- As expected, the deposits’ ‘specific’ risk is better assessed using a dynamic strategy following both rates and the deposit amount

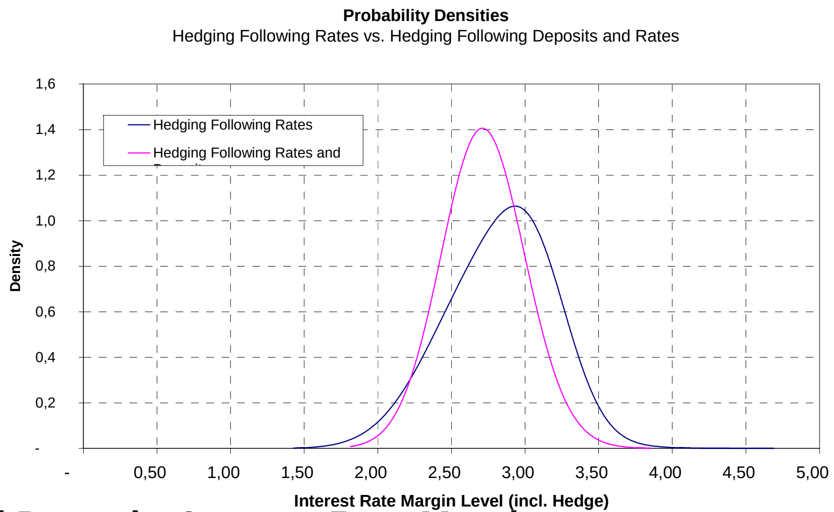


Robustness towards Risk Criterion

- **The mean-variance optimal dynamic strategy (following deposits and rates) behaves quite well under other risk criteria**
  - Example of Expected Shortfall (99.5%) and VaR (99.95%).

Barrier Deposit Rate	**Expected Return**	**Standard Deviation**		**ES** (99.5%)		**VaR** (99.95%)	
		Level	*Risk Reduction*	Level	*Risk Reduction*	Level	*Risk Reduction*
Unhedged Margin	3.16	0.39		-2.02		-1.90	
Static Hedge Case 1	3.04	0.28	*-0.11*	-2.34	*-0.32*	-2.26	*-0.36*
Static Hedge Case 2	3.01	0.23	*-0.16*	-2.26	*-0.24*	-2.04	*-0.14*
Jarrow and van Deventer	3.01	0.24	*-0.15*	-2.35	*-0.33*	-2.25	*-0.35*
Optimal Dynamic Hedge	3.01	0.22	***-0.17***	-2.38	***-0.36***	-2.29	***-0.39***

- **The optimal dynamic strategy features better tail distribution than for other strategies**
  - Blue: Optimal Dynamic Strategy (following rates)
  - Pink: Optimal Dynamic Strategy (following both deposits and rates)



Dealing with Massive Bank Run

- **Introducing a Poisson Jump component in the deposit amount:**

$$dK_t = K_t\left[\mu_K dt + \sigma_K d\overline{W_K}(t) - dN(t)\right]$$

$\left(N(t)\right)_{0\le t\le T}$ is assumed to be independent from W_K and W_L

- **Then, we have:** $\theta_t^{**} = \dfrac{\partial \mathbf{E}_t^{\overline{\mathbf{P}}}\left[IRM\left(K_T, L_T\right)\right]}{\partial L_t} + \dfrac{\lambda}{\sigma_L L_t}\left[\mathbf{E}_t^{\overline{\mathbf{P}}}\left[IRM_g\left(K_T, L_T\right)\right] - V_t\left(x^{**}, \theta^{**}\right)\right]$

$$\mathbf{E}_t^{\overline{\mathbf{P}}}\left[IRM_g\left(K_T, L_T\right)\right] = e^{-\gamma(T-t)} \times \text{ (\textit{Previous conditional expectation term})}$$

  - Due to independence, the jump element can be put out the conditional expectations

- **N.B.:** ***When a bank run occurs, the manager keeps investing the current hedging portfolio's value in the Hedging Numéraire***

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Conclusions (1)

- **A dynamic strategy to assess risk in mean-variance framework**
  - Results about Mean-variance hedging in incomplete markets yield explicit dynamic hedging strategies

- **Practical Conclusions:**
  - Better assessment of deposits' 'specific' risk with a dynamic strategy taking into account both deposits and rates;
  - Lack of stability for other strategies towards the deposit rate's specification;
  - Robustness towards risk criterion
  - No negative consequences as for tail distribution
  - Additivity of Optimal Dynamic Strategies
    - Applicable to **various balance sheet items**

Conclusions (2)

- **We use some mathematical finance concepts:**
  - **For Financial Engineering problems**
  - **with the aim of providing applicable strategies**
  - **And improve risk management processes**

Technical References

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