Credit Correlation Modelling
Comparative analysis of CDO pricing models

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Joint work with X. Burtschell and J. Gregory (BNP-Paribas)

A comparative analysis of CDO pricing models available on www.defaultrisk.com

Comparative analysis of CDO pricing models

Purpose of the presentation

Some insights about current issues in CDO modelling
Gaussian copula
One factor Gaussian copula
Ordering of risks
Correlation sensitivities and Gaussian extensions
Model dependence/Choice of copula
Student $t$, double $t$, Clayton, Marshall-Olkin, Stochastic correlation
Calibration issues
Distribution of conditional default probabilities
Matching the correlation skew
Further issues

Agenda

Conditional default probabilities and pricing of CDOs
One factor Gaussian copula
Dependence to the correlation parameter
Gaussian extensions, correlation sensitivities
Model dependence/Choice of copula
Student $t$, double $t$, Clayton, Marshall-Olkin, Stochastic correlation
Calibration
Empirical results
Matching the correlation skew

Semi explicit pricing, conditional default probabilities

Semi-explicit pricing for CDO tranches
Laurent & Gregory [2003]
Default payments are based on the accumulated losses on the pool of credits:

$L(t) = \sum_{i=1}^{n} LGD_i (1 - \delta_i)$
$LGD_i = N_i (1 - \delta_i)$

Tranche premiums only involves call options on the accumulated losses

$E\left[ (L(t) - K)_+ \right]$

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$E\left[ (L(t) - K)_+ \right]$
Semi explicit pricing, conditional default probabilities

One factor Gaussian copula:
- $V_i, V_j, i = 1, \ldots, n$ independent Gaussian,
- $V_i = \rho V + \sqrt{1 - \rho^2} V_j$
- Default times: $\tau_i = F_i^{-1}(\Phi(V_i))$
- $F_i$ marginal distribution function of default times
- Conditional default probabilities:
  $p_i = \Phi\left( -\rho V + \Phi^{-1}(F_i(0)) \right)$

One factor Gaussian copula

CDO margins (bps pa)
- With respect to correlation
- Gaussian copula
- Attachment points: 3%, 10%
- 100 names
- Unit nominal
- Credit spreads 100 bp
- 5 years maturity

One factor Gaussian copula

Equity tranche premiums are decreasing wrt $\rho$
- General result ?
- Supermodular function $f$ is such that:
  $f : \mathbb{R}^n \to \mathbb{R}$
  $\Delta f(x) = f(x + rt) - f(x)$
  $\forall x \in \mathbb{R}^n, \forall r, \delta > 0$
  $\Delta \Delta f(x) \geq 0$
- Supermodular order (increase in dependence)
  $X = (X_1, \ldots, X_n)$
  $Y = (Y_1, \ldots, Y_n)$
  $X \leq_{sl} Y \iff E[f(X)] \leq E[f(Y)]$, \forall supermodular

One factor Gaussian copula

« Supermodular » order of Gaussian vectors
- Let $X$ and $Y$ be Gaussian vectors with zero mean
  $\Sigma = \begin{pmatrix} 1 & \sigma_{12}^e & \sigma_{13}^e & \cdots & \sigma_{1n}^e \\ \sigma_{21}^e & 1 & \sigma_{23}^e & \cdots & \sigma_{2n}^e \\ \sigma_{31}^e & \sigma_{32}^e & 1 & \cdots & \sigma_{3n}^e \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1}^e & \sigma_{n2}^e & \sigma_{n3}^e & \cdots & 1 \end{pmatrix}$
  $\Sigma' = \begin{pmatrix} 1 & \sigma_{11}' & \sigma_{12}' & \cdots & \sigma_{1n}' \\ \sigma_{21}' & 1 & \sigma_{22}' & \cdots & \sigma_{2n}' \\ \sigma_{31}' & \sigma_{32}' & 1 & \cdots & \sigma_{3n}' \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1}' & \sigma_{n2}' & \sigma_{n3}' & \cdots & 1 \end{pmatrix}$
  $\Sigma' \leq \Sigma$ \iff $\sigma_{ij}' \leq \sigma_{ij}^e$, \forall $i, j$
- Müller & Scarsini (2000), Müller (2001)

One factor Gaussian copula

« Stop-Loss » order
- Accumulated losses: $L(t), L'(t)$
  $L(t) \leq_{sl} L'(t) \iff E[(L(t) - K)^+] \leq E[(L'(t) - K)^+], \forall K \geq 0$
- Supermodular order of latent variables implies stop-loss order of accumulated losses
- Thus, equity tranche premium is always decreasing with correlation
- Guarantees uniqueness of « base correlation »
- Monotonicity properties extend to Student $t$, Clayton (Wei & Hu [2002]) and Marshall-Olkin copulas

One factor Gaussian copula

Second issue
- Equity tranche premium decrease with correlation
- Does $\rho = 100\%$ correspond to some lower bound?
- $\rho = 100\%$ corresponds to « comonotonic » default dates:
  $(\tau_1, \ldots, \tau_n) \text{ comonotonic} \iff (F_1(U), \ldots, F_n(U))$
  where $U$ is uniform
  $F_\ldots F_n(U) \leq_{com} F_\ldots F_n(U)$
- Tchen (1980)
- $\rho = 100\%$ is a model free lower bound for the equity tranche premium
One factor Gaussian copula

Third issue

- Does $\rho = 0\%$ corresponds to the higher bound on the equity tranche premium?
- $\rho = 0\%$ corresponds to the independence case between default dates
- The answer is no, negative dependence can occur
- Base correlation does not always exists
  - Even in Gaussian copula models
  - Factor models are usually associated with positive dependence
  - i.e. independent default dates are smaller with respect to supermodular order

Gaussian extensions

Pairwise correlation sensitivities

Intra and intersector correlations

In the core of correlation, Gregory & Laurent, Risk october 2004

Model dependence / choice of copula

Stochastic correlation copula

- $V_i, V_j, i, j = 1, \ldots, n$ independent Gaussian variables
- $B_i = 1$ correlation $\rho$, $B_j = 0$ correlation $\beta$

$$
V_i = B_i \left( \rho^3 + \sqrt{1 - \rho^2} \rho_3 \right) + (1 - B_i) \left( \beta^3 + \sqrt{1 - \beta^2} \beta_3 \right)
$$

$$
V_j = (B_j \rho + (1 - B_j) \beta) V_i + \sqrt{1 - (B_j \rho + (1 - B_j) \beta)^2} \beta_3
$$

$$
\tau_{ij} = E^{-1} \left( \Phi(V_i) \right)
$$

$$
\rho^\nu = \rho \frac{\phi \left( -\rho^3 + \Phi^{-1}(F_i) \right)}{\sqrt{1 - \rho^2}} + (1 - \rho) \phi \frac{-\beta^3 + \Phi^{-1}(F_j)}{\sqrt{1 - \beta^2}}
$$

Clayton copula

- Schönbucher & Schubert, Rogge & Schönbucher, Friend & Rogge, Madan et al

$$
V_i = \phi \left( \ln U_i \right)
$$

$$
\tau_{ij} = E^{-1} \left( H_i \left( \Phi(V_i) \right) \right)
$$

- Marshall-Olkin construction of archimedean copulas
  - $V_i$ Gamma distribution with parameter $\theta$
  - $U_1, \ldots, U_n$ independent uniform variables
  - Conditional default probabilities (one factor model)

$$
\rho^\theta = \exp \left( 1 - F_i^{-1} \right)
$$

Student $t$ copula

- Embrechts, Lindskog & McNeil, Greenberg et al, Mashal et al, O’Kane & Schloegl, Gilkes & Jobst

$$
V_i, F_i \text{ independent Gaussian variables}
$$

$$
V_i \text{ follows a } X^2 \text{ distribution}
$$

$$
\text{Conditional default probabilities (two factor model)}
$$

$$
\rho^\nu = \Phi \left( -\rho^3 + \frac{1}{\nu^2} + \sqrt{1 - \rho^2} \frac{\nu - 2}{\nu} \frac{\nu^2 - 2}{\nu^2} \Phi^{-1}(F_i) \right)
$$

Double $t$ model (Hull & White)

- $V_i = \beta \left( \frac{V_i - 2}{\nu^2} \right)^{1/2} V_i + \sqrt{1 - \rho^2} \left( \frac{\nu - 2}{\nu} \right)^{1/2} \beta_i

- V_i, \beta_i \text{ are independent Student } t \text{ variables}

- with $\nu$ and $\beta$ degrees of freedom

$$
\tau_{ij} = E^{-1} \left( H_i \left( \Phi(V_i) \right) \right)
$$

where $H_i$ is the distribution function of $V_i$

$$
\rho^\nu = \nu^2 \left( \frac{\nu - 2}{\nu} \right) \Phi^{-1}(F_i) \rho \left( \frac{\nu - 2}{\nu} \right)^{1/2} \frac{\nu^2 - 2}{\nu^2}
$$
Model dependence / choice of copula

- Shock models (multivariate exponential copulas)
  - Duffie & Singleton, Giesecke, Elouerkhaoui, Lindskog & McNeil, Wong

- Modelling of default dates: $V_i = \min\{\tau_i, \tau'_i\}$
  - $V, \tau_i$ exponential with parameters $\alpha_i, 1 - \alpha_i$

- Default dates $S_i = S'_i \left( \exp\left( \min\{\tau_i, \tau'_i\} \right) \right)$
  - $S_i$ marginal survival function

- Conditionally on $V, \tau_i$ are independent.

- Conditional default probabilities
  $$q_{iV}^j = (1 - \exp(1 - q_{iV}))^{-\alpha_i}$$

Model dependence / choice of copula

- Calibration issues
  - One parameter copulas
    - Well suited for homogeneous portfolios
    - Dependence is « monotonic » in the parameter

- Calibration procedure
  - Fit Clayton, Student $t$, double $t$, Marshall Olkin parameters onto CDO equity tranches
    - Computed under one factor Gaussian model
  - Or given market quotes on equity tranches
    - Reprice mezzanine and senior CDO tranches
    - Given the previous parameters

Model dependence / choice of copula

- CDO margins (bps pa)
  - With respect to correlation
    - Gaussian copula
    - Attachment points: 3%, 10%
    - 100 names
    - Unit nominal
    - Credit spreads 100 bp
    - 5 years maturity

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>0%</th>
<th>10%</th>
<th>30%</th>
<th>50%</th>
<th>70%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>0</td>
<td>0.02</td>
<td>0.18</td>
<td>0.36</td>
<td>0.66</td>
<td>=∞</td>
</tr>
<tr>
<td>$\rho_{iV}$</td>
<td>0.14</td>
<td>1.4</td>
<td>3.9</td>
<td>6.3</td>
<td>9.6</td>
<td>100%</td>
</tr>
<tr>
<td>$\rho_{iV}'$</td>
<td>-2.2</td>
<td>-4.3</td>
<td>-6.7</td>
<td>-9.4</td>
<td>-11.9</td>
<td>100%</td>
</tr>
<tr>
<td>$\rho_{iV}(4)-\rho_{iV}(3)$</td>
<td>0</td>
<td>12%</td>
<td>24%</td>
<td>35%</td>
<td>43%</td>
<td>50%</td>
</tr>
<tr>
<td>$\rho_{iV}(4)-\rho_{iV}(5)$</td>
<td>0</td>
<td>12%</td>
<td>34%</td>
<td>55%</td>
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<tr>
<td>$\rho_{iV}(4)-\rho_{iV}(6)$</td>
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<tr>
<td>$\rho_{iV}(3)-\rho_{iV}(4)$</td>
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</tr>
</tbody>
</table>

Table 5: correspondence between parameters

- CDO margins (bps pa)
  - With respect to correlation
    - Gaussian copula
    - Attachment points: 3%, 10%
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    - Credit spreads 100 bp
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</tr>
<tr>
<td>$\rho_{iV}(3)-\rho_{iV}(4)$</td>
<td>0</td>
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<td>54%</td>
<td>73%</td>
<td>100%</td>
</tr>
</tbody>
</table>

Table 6: mezzanine tranche (bps pa)
### Table 7: senior tranche (bps pa)

<table>
<thead>
<tr>
<th>Model dependence / choice of copula</th>
<th>0%</th>
<th>10%</th>
<th>30%</th>
<th>50%</th>
<th>70%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>100%</td>
</tr>
<tr>
<td>Clayton</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>100%</td>
</tr>
<tr>
<td>Student (6)</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>100%</td>
</tr>
<tr>
<td>Student (12)</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>100%</td>
</tr>
</tbody>
</table>

Gaussian, Clayton and Student \( t \) CDO premiums are close.

### Table 8: coefficient of lower tail dependence (%)

<table>
<thead>
<tr>
<th>Model dependence / choice of copula</th>
<th>0%</th>
<th>10%</th>
<th>30%</th>
<th>50%</th>
<th>70%</th>
<th>100%</th>
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</thead>
<tbody>
<tr>
<td>Gaussian</td>
<td>0%</td>
<td>1%</td>
<td>6%</td>
<td>16%</td>
<td>33%</td>
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</tr>
<tr>
<td>Clayton</td>
<td>0%</td>
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<td>8%</td>
<td>15%</td>
<td>25%</td>
<td>100%</td>
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<tr>
<td>Student (6)</td>
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<td>25%</td>
<td>44%</td>
<td>100%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student (12)</td>
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<td>30%</td>
<td>47%</td>
<td>100%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MO</td>
<td>0%</td>
<td>28%</td>
<td>53%</td>
<td>69%</td>
<td>80%</td>
<td>100%</td>
</tr>
</tbody>
</table>

Tail dependence is poorly related to CDO tranche premiums.

### Table 9: Kendall's \( \tau \) (%)

<table>
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<tr>
<th>Model dependence / choice of copula</th>
<th>0%</th>
<th>10%</th>
<th>30%</th>
<th>50%</th>
<th>70%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian</td>
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<td>0.91%</td>
<td>1.54%</td>
<td>2.41%</td>
<td>3.59%</td>
<td>8.1%</td>
</tr>
<tr>
<td>Clayton</td>
<td>0.66%</td>
<td>0.88%</td>
<td>1.45%</td>
<td>2.24%</td>
<td>3.31%</td>
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</tr>
<tr>
<td>Student (6)</td>
<td>1.41%</td>
<td>2.31%</td>
<td>3.52%</td>
<td>8.1%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student (12)</td>
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<td>2.36%</td>
<td>3.56%</td>
<td>8.1%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MO</td>
<td>0.66%</td>
<td>2.63%</td>
<td>4.53%</td>
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<td>8.1%</td>
<td></td>
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</table>

Kendall's tau is poorly related to CDO tranche premiums.

### Table 13: bivariate default probabilities (5 year time horizon)

<table>
<thead>
<tr>
<th>Model dependence / choice of copula</th>
<th>0%</th>
<th>10%</th>
<th>30%</th>
<th>50%</th>
<th>70%</th>
<th>100%</th>
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</thead>
<tbody>
<tr>
<td>Gaussian</td>
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<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>100%</td>
</tr>
<tr>
<td>Clayton</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>100%</td>
</tr>
<tr>
<td>Student (6)</td>
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<td>0%</td>
<td>0%</td>
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<td>100%</td>
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<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>100%</td>
</tr>
<tr>
<td>MO</td>
<td>0%</td>
<td>28%</td>
<td>53%</td>
<td>69%</td>
<td>80%</td>
<td>100%</td>
</tr>
</tbody>
</table>

Bivariate default probabilities are well related to tranche premiums.

---

**Why Clayton and Gaussian copulas provide same SL premiums?**

- Loss distributions only depend on the distribution of conditional default probabilities

\[
p^3 = \exp(\{1-F(t)^3\})
\]

\[
p^3 = \phi(\frac{3 \rho + \Phi^{-1}(F(t))}{\sqrt{1-\rho^2}})
\]

- Distribution functions of conditional default probabilities

---

### Distribution of conditional default probabilities

[Graph showing the distribution of conditional default probabilities for different copulas.]
Matching the correlation skew

<table>
<thead>
<tr>
<th>Tranches</th>
<th>Market</th>
<th>Gaussian</th>
<th>Clayton</th>
<th>Student (12)</th>
<th>$t(4)\times t(4)$ Stoch.</th>
<th>MO</th>
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<td>916</td>
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<td>163</td>
<td>164</td>
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<td>6-9%</td>
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<td>48</td>
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<td>53</td>
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<td>9-12%</td>
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<td>17</td>
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<tr>
<td>12-22%</td>
<td>9</td>
<td>3</td>
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<td>2</td>
<td>13</td>
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</tr>
</tbody>
</table>

Table 17: CDO tranche premiums (bps pa)

<table>
<thead>
<tr>
<th>Tranches</th>
<th>Market</th>
<th>Gaussian</th>
<th>Clayton</th>
<th>Student (12)</th>
<th>$t(4)\times t(4)$ Stoch.</th>
<th>MO</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-3%</td>
<td>916</td>
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<td>916</td>
</tr>
<tr>
<td>3-6%</td>
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<td>503</td>
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<td>504</td>
<td>456</td>
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<tr>
<td>6-9%</td>
<td>311</td>
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<td>254</td>
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Table 18: “equity tranche” CDO tranche premiums (bps pa)

Matching the correlation skew

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<tr>
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Matching the correlation skew

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Table 18: “equity tranche” CDO tranche premiums (bps pa)

Conclusion

- Matching the skew with second generation models
  - RFL, stochastic correlation, double $t$
  - Conditional default probability distributions are the drivers
- Pricing bespoke portfolios, CDO squared with a consistent model
- Not yet fully satisfactory
  - Matching 5Y/10Y with same set of parameters
  - Stability of parameters through time
  - Dynamics of the correlation skew / risk management
  - Calibration of multiple parameters, possibly name dependent
- Still more work on the quant agenda.