On the Edge of Completeness

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Jean-Paul LAURENT
Professor, ISFA Actuarial School, University of Lyon,
Scientific Advisor, BNP Paribas

Correspondence
laurent.jeanpaul@online.fr
On the Edge of Completeness: Purpose and main ideas

• Purpose:
  − risk-analysis of exotic credit derivatives:
    ➢credit contingent contracts, basket default swaps.
  − pricing and hedging exotic credit derivatives.

• Main ideas:
  − distinguish between credit spread volatility and default risk.
  − dynamic hedge of exotic default swaps with standard default swaps.

• **Trading credit risk**: closing the gap between supply and demand

• **Modelling credit derivatives**: the state of the art

• **A new approach to credit derivatives modelling**:
  - closing the gap between pricing and hedging
  - disentangling default risk and credit spread risk
Trading credit risk: 
Closing the gap between supply and demand

- From stone age to the new millennium:
  - Technical innovations in credit derivatives are driven by economic forces.
  - Transferring risk from commercial banks to institutional investors:
    - Securitization.
    - Default Swaps: portfolio and hedging issues.
    - Credit Contingent Contracts, Basket Credit Derivatives.
  - The previous means tend to be more integrated.
Trading credit risk: Closing the gap between supply and demand

• Securitization of credit risk:

  - Credit risk seller
  - Credits
    - SPV
      - Senior debt
      - Junior debt
  - Investor 1
  - Investor 2

• simplified scheme:
  - No residual risk remains within SPV.
  - All credit trades are simultaneous.
Trading Credit Risk: Closing the gap between supply and demand

• Financial intermediaries provide structuring and arrangement advice.
  – Credit risk seller can transfer loans to SPV or instead use default swaps

• good news: low capital at risk for investment banks

• Good times for modelling credit derivatives
  – No need of hedging models
  – credit pricing models are used to ease risk transfer
  – need to assess the risks of various tranches
Trading Credit Risk: Closing the gap between supply and demand

• There is room for financial intermediation of credit risk
  – The transfers of credit risk between commercial banks and investors may not be simultaneous.
  – Since at one point in time, demand and offer of credit risk may not match.
    ➢ Meanwhile, credit risk remains within the balance sheet of the financial intermediary.
  – It is not further required to find customers with exact opposite interest at every new deal.
    ➢ Residual risks remain within the balance sheet of the financial intermediary.
Credit risk management without hedging default risk

- Emphasis on:
  - portfolio effects: correlation between default events
  - posting collateral
  - computation of capital at risk, risk assessment

- Main issues:
  - capital at risk can be high
  - what is the competitive advantage of investment banks

Credit risk seller

Credit derivatives trading book

bank

Investor 1

Investor 2

Default swap

Default swap

Default swap
Trading against other dealers enhances ability to transfer credit risk by lowering capital at risk.

- Credit risk seller
- Default swap
- Credit derivatives trading book
- Default swaps
- Credit derivatives dealer
- Bank
- Default swaps
- Bond dealer
- Default swap
- Investor 1
- Default swap
- Investor 2
- Repos
**New ways to transfer credit risk: credit contingent contracts**

- **Anatomy of a general credit contingent contract**
  - A credit contingent contract is like a standard default swap but with variable nominal (or exposure).
  - However, the periodic premium paid for the credit protection remains fixed.
  - The protection payment arises at default of one given single risky counterparty.

- **Examples**
  - cancellable swaps
  - quanto default swaps
  - credit protection of vulnerable swaps, OTC options (stand-alone basis)
  - credit protection of a portfolio of contracts (full protection, excess of loss insurance, partial collateralization)
New ways to transfer credit risk: 
**Basket default derivatives**

- Consider a basket of $M$ risky bonds
  - multiple counterparties
- First to default swaps
  - protection against the first default
- $N$ out of $M$ default swaps ($N < M$)
  - protection against the first $N$ defaults
- **Hedging and valuation of basket default derivatives**
  - involves the joint (multivariate) modelling of default arrivals of issuers in the basket of bonds.
  - Modelling accurately the dependence between default times is a critical issue.
Modelling credit derivatives: the state of the art

- Modelling credit derivatives: Where do we stand?
- Financial industry approaches
  - Plain default swaps and risky bonds
  - Credit risk management approaches

- The Noah’s arch of credit risk models
  - “firm-value” models
  - Risk-intensity based models
  - Looking desperately for a hedging based approach to pricing.
Modelling credit derivatives: Where do we stand?

Plain default swaps

- Static arbitrage of plain default swaps with short selling underlying bond
  - plain default swaps hedged using underlying risky bond
  - “bond strippers”: allow to compute prices of risky zero-coupon bonds
  - repo risk, squeeze risk, liquidity risk, recovery rate assumptions

- Computation of the P&L of a book of default swaps
  - Involves the computation of a P&L of a book of default swaps
  - The P&L is driven by changes in the credit spread curve and by the occurrence of default.
Assessing the varieties of risks involved in credit derivatives

- Specific risk or credit spread risk
  - *prior to default*, the P&L of a book of credit derivatives is driven by changes in credit spreads.

- Default risk
  - *in case of default, if unhedged*,
Modelling credit derivatives: Where do we stand? The Noah’s arch of credit risk models

• “firm-value” models:
  – Modelling of firm’s assets
  – First time passage below a critical threshold

• risk-intensity based models
  – Default arrivals are no longer predictable
  – Model conditional local probabilities of default $\lambda(t) \, dt$
  – $\tau$: default date, $\lambda(t)$ risk intensity or hazard rate

\[
\lambda(t) \, dt = P[\tau \in [t, t+dt]|\tau > t]
\]

• Lack of a hedging based approach to pricing.
  – Misunderstanding of hedging against default risk and credit spread risk
A new approach to credit derivatives modelling based on an hedging point of view

- **Rolling over the hedge:**
  - Short term default swaps v.s. long-term default swaps
  - Credit spread transformation risk

- **Credit contingent contracts, basket default swaps**
  - Hedging default risk through dynamics holdings in standard default swaps
  - Hedging credit spread risk by choosing appropriate default swap maturities
  - Closing the gap between pricing and hedging

- **Practical hedging issues**
  - Uncertainty at default time
  - Managing net residual premiums
Purpose:
- Introduction to dynamic trading of default swaps
- Illustrates how default and credit spread risk arise

Arbitrage between long and short term default swap
- sell one long-term default swap
- buy a series of short-term default swaps

Example:
- default swaps on a FRN issued by BBB counterparty
- 5 years default swap premium: 50bp, recovery rate = 60%

Credit derivatives dealer

If default, 60%

Until default, 50 bp

Client
Long-term Default Swaps v.s. Short-term Default Swaps
Rolling over the hedge

• Rolling over short-term default swap
  – at inception, one year default swap premium: 33bp
  – cash-flows after one year:
    - 33 bp
    - 60% if default

• Buy a one year default swap at the end of every yearly period, if no default:
  – Dynamic strategy,
  – future premiums depend on future credit quality
  – future premiums are unknown

Credit derivatives dealer → 33 bp → Market
60% if default

Credit derivatives dealer → ?? bp → Market
60% if default
Long-term Default Swaps v.s. Short-term Default Swaps
Rolling over the hedge

- **Risk analysis** of rolling over short term against long term default swaps

Credit derivatives dealer \( \rightarrow \) ?? bp \( \rightarrow \) Market + Client

\( \arrows \quad 50 \text{ bp} \)

- **Exchanged cash-flows:**
  - Dealer receives 5 years (fixed) credit spread,
  - Dealer pays 1 year (variable) credit spread.

- **Full one to one protection at default time**
  - the previous strategy has **eliminated** one source of risk, that is default risk
Long-term Default Swaps v.s. Short-term Default Swaps
Rolling over the hedge

- negative exposure to an increase in short-term default swap premiums
  - if short-term premiums increase from 33bp to 70bp
  - reflecting a lower (short-term) credit quality
  - and no default occurs before the fifth year

- Loss due to negative carry
  - long position in long term credit spreads
  - short position in short term credit spreads
Consider a portfolio of homogeneous loans

- same unit nominal, non-amortising
- \( \tau_i \): default time of counterparty \( i \)
- same default time distribution (same hazard rate \( \lambda(t) \)):
  \[
P[\tau_i \in [t, t + dt] | \tau_i > t] = \lambda(t) dt
\]
- \( F_t \): available information at time \( t \)
- Conditional independence between default events \( \{\tau_i \in [t, t + dt]\} \)
  \[
P[\tau_i, \tau_j \in [t, t + dt] | F_t] = P[\tau_i \in [t, t + dt] | F_t] \times P[\tau_j \in [t, t + dt] | F_t]
\]
- equal to zero or to \( \lambda^2(t)(dt)^2 \), i.e., no simultaneous defaults.
- Remark that indicator default variables \( 1_{\{\tau_i \in [t, t + dt]\}} \) are (conditionally) independent and equally distributed.
Denote by $N(t)$ the outstanding amount of the portfolio (i.e. the number of non defaulted loans) at time $t$.

By law of large numbers,
\[
\frac{1}{N(t)} \sum 1\{\tau_i \in [t, t+dt]\} \rightarrow \lambda(t) dt
\]

Since
\[
N(t + dt) - N(t) = - \sum 1\{\tau_i \in [t, t+dt]\}
\]
we get,
\[
\frac{N(t + dt) - N(t)}{N(t)} = -\lambda(t) dt
\]

The outstanding nominal decays as
\[
N(t) = N(0) \exp\left(-\int_0^t \lambda(s) ds\right)
\]

Assume zero recovery; Total default loss $t$ and $t+dt$: $N(t) - N(t+dt)$

Cost of default per outstanding loan:
\[
\frac{N(t) - N(t + dt)}{N(t)} = \lambda(t) dt
\]
Rolling over the hedge: portfolio of homogeneous loans

- Cost of default per outstanding loan $= \lambda(t)dt$ is known at time $t$.
- **Insurance** diversification approach holds
- *Fair premium* for a short term insurance contract on a single loan (i.e. a short term default swap) has to be equal to $\lambda(t)dt$.
- Relates *hazard rate* and *short term default swap premiums*.

**Expanding on rolling over the hedge**
- Let us be short in 5 years (say) default swaps written on all individual loans.
  - $p_{5Y} dt$, periodic premium per loan.
- Let us buy the short term default swaps on the outstanding loans.
  - Corresponding premium per loan: $\lambda(t)dt$.
- Cash-flows related to default events $N(t) - N(t+dt)$ perfectly offset
Rolling over the hedge: portfolio of homogeneous loans

- **Net (premium) cash-flows** between $t$ and $t+dt$: 
  \[ N(t)(p_{5Y} - \lambda(t))dt \]

- **Where** 
  \[ N(t) = N(0)\exp{-\int_0^t \lambda(s)ds} \]

  ➢ Payoff similar to an “index amortising swap”.

- **At inception**, $p_{5Y}$ must be such that the risk-neutral expectation of the discounted net premiums equals zero:

- **Pricing equation for the long-term default swap premium $p_{5Y}$**:

  \[
  E\left[ \int_0^T \left( \exp{-\int_0^t r(s)ds} \right) \times N(t)(p_{5Y} - \lambda(t))dt \right] = 0
  \]

  ➢ where $r(t)$ is the short rate at time $t$.

- **Premiums received when selling long-term default swaps**: 
  \[ N(t)p_{5Y}dt \]

- **Premiums paid on “hedging portfolio”**: 
  \[ N(t)\lambda(t)dt \]
Rolling over the hedge: portfolio of homogeneous loans

- Convexity effects and the cost of the hedge
  - Net premiums paid \( N(t)[p_{5Y} - \lambda(t)]dt \)

- What happens if short term premiums \( \lambda(t) \) become more volatile?
  - Net premiums become negative when \( \lambda(t) \) is high.
  - Meanwhile, the outstanding amount \( N(t) \) tends to be small, mitigating the losses.
  - Contrarily when \( \lambda(t) \) is small, the dealer experiments positive cash-flows \( p_{5Y} - \lambda(t) \) on a larger amount \( N(t) \).

- The more volatile \( \lambda(t) \), the smaller the average cost of the hedge and thus the long term premium \( p_{5Y} \).
Hedging exotic default swaps: main features

- Exotic credit derivatives can be hedged against default:
  - Constrains the amount of underlying standard default swaps.
  - Variable amount of standard default swaps.
  - Full protection at default time by construction of the hedge.
  - No more discontinuity in the P&L at default time.
  - “Safety-first” criteria: main source of risk can be hedged.
  - Model-free approach.

- Credit spread exposure has to be hedged by other means:
  - Appropriate choice of maturity of underlying default swap
  - Computation of sensitivities with respect to changes in credit spreads are model dependent.
**Hedging Default Risk in Credit Contingent Contracts**

- **Credit contingent contracts**
  - client pays to dealer a periodic premium $p_T(C)$ until default time $\tau$, or maturity of the contract $T$.
  - dealer pays $C(\tau)$ to client at default time $\tau$, if $\tau \leq T$.

**Credit derivatives**

- **Hedging side:**
  - **Dynamic** strategy based on **standard** default swaps:
  - At time $t$, hold an amount $C(t)$ of standard default swaps
  - $\lambda(t)$ denotes the periodic premium at time $t$ for a short-term default swap
Hedging Default Risk in Credit Contingent Contracts

- **Hedging side:**
  - Amount of standard default swaps equals the (variable) credit exposure on the credit contingent contract.

- **Net position is a “basis swap”:**

- **The client transfers credit spread risk to the credit derivatives dealer**
What is the cost of hedging default risk?

Discounted value of hedging default swap premiums:

\[ E \left[ \int_0^T \exp - \int_0^t (r + \lambda(s)) ds \right] \lambda(t) C(t) dt \]

Discounting term

Premium paid at time t on protection portfolio

Equals the discounted value of premiums received by the seller:

\[ E \left[ \int_0^T \exp - \int_0^t (r + \lambda(s)) ds \right] p_T dt \]
Case study: defaultable interest rate swap

Consider a defaultable interest rate swap (with unit nominal)
- We are default-free, our counterparty is defaultable (default intensity $\lambda(t)$).
- We consider a (fixed-rate) receiver swap on a standalone basis.

Recovery assumption, payments in case of default.
- if default at time $\tau$, compute the default-free value of the swap: $PV_\tau$
- and get: $\delta(PV_\tau)^+ + (PV_\tau)^- = PV_\tau - (1-\delta)(PV_\tau)^+$
- $0 \leq \delta \leq 1$ recovery rate, $(PV_\tau)^+ = \text{Max}(PV_\tau, 0)$, $(PV_\tau)^- = \text{Min}(PV_\tau, 0)$
- In case of default,
  - we receive default-free value $PV_\tau$
  - minus
  - loss equal to $(1-\delta)(PV_\tau)^+$. 
Case study: defaultable interest rate swap

- Defaultable and default-free swap
  - Present value of defaultable swap = Present value of default-free swap (with same fixed rate) – Present value of the loss.
  - To compensate for default, fixed rate of defaultable swap (with given market value) is greater than fixed rate of default-free swap (with same market value).
  - Let us remark, that default immediately after negotiating a defaultable swap results in a positive jump in the P&L, because recovery is based on default-free value.

- To account for the possibility of default, we may constitute a credit reserve.
  - Amount of credit reserve equals expected Present Value of the loss
  - This accounts for the expected loss but does not hedge against realized loss.
Case study: defaultable interest rate swap

- Using a hedging instrument rather than a credit reserve
  - Consider a credit contingent contract that pays \((1-\delta)(PV_{\tau})^+\) at default time \(\tau\) (if \(\tau \leq T\)), where \(PV_{\tau}\) is the present value of a default-free swap with same fixed rate than defaultable swap.
  - Such a credit contract + a defaultable swap synthesises a default-free swap (at a fixed rate equal to the initial fixed rate):
  - At default, we receive \((1-\delta)(PV_{\tau})^+ + PV_{\tau} - (1-\delta)(PV_{\tau})^+ = PV_{\tau}\)
  - The upfront premium for this credit protection is equal to the Present Value of the loss \((1-\delta)(PV_{\tau})^+\) given default:

\[
E\left[\int_0^T \left( \exp - \int_0^t (r + \lambda)(u)\,du \right) \lambda(t)(1-\delta)(PV_t)^+ \, dt \right]
\]
Case study: defaultable interest rate swap
Interpreting the cost of the hedge

- Average cost of default on a large portfolio of swaps
  - Large number of homogeneous defaultable receiver swaps:
    - Same fixed rate and maturity; initial nominal value \( N(0) = 1 \)
    - Independent default dates and same default intensity \( \lambda(t) \).
  - Outstanding nominal amount: \( N(t) = \exp\left(\int_0^t \lambda(s) ds\right) \)
  - Nominal amount defaulted in \([t, t+dt] \): \( N(t) - N(t+dt) = \lambda(t) dt \exp\left(\int_0^t \lambda(s) ds\right) \)
  - Cost of default in \([t, t+dt] \): \( \left(N(t) - N(t+dt)\right) \left(1 - \delta\right) (PV_t)^+ \)
  - Where \( PV_t \): present value of receiver swap with unit nominal.
  - Aggregate cash-flows do not depend on default risk.
  - Aggregate cash-flows are those of an index amortising swap
  - Standard discounting provides previous slide pricing equation
Case study: defaultable interest rate swap
Interpreting the cost of the hedge

• Randomly exercised swaption:
  – Assume for simplicity no recovery ($\delta=0$).
  – Interpret default time as a random time $\tau$ with intensity $\lambda(t)$.
  – At that time, defaulted counterparty “exercises” a swaption, i.e. decides whether to cancel the swap according to its present value.
  – PV of default-losses equals price of that randomly exercised swaption

• American Swaption
  – PV of American swaption equals the supremum over all possible stopping times of randomly exercised swaptions.
    - The upper bound can be reached for special default arrival dates:
    - $\lambda(t)=0$ above exercise boundary and $\lambda(t)=\infty$ on exercise boundary
• Previous hedge leads to (small) jumps in the P&L:
  – Consider a 5.1% fixed rate defaultable receiver swap with PV=3%.
  – Buy previous credit contingent contract at market price.
    ➢ Due to credit protection, we hold a synthetic default-free 5.1% swap.
    ➢ Total PV remains equal to 3%.
  – Assume that default immediate default: \( \tau = 0^+ \).
  – Clearly a 5.1% default free swap has PV>3%, thus occurring a positive jump in P&L.

• Jumps in the P&L due to extra default insurance:
  – To hedge the previous credit contingent contract:
  – At time 0, we hold an amount of short term default swap that is equal to the Present Value of a default-free 5.1% swap
  – This amount is greater than 3%, the current Present Value.
Case study: defaultable interest rate swap

- **Alternative hedging approach:**
  - Fixed rate of default-free swap with 3% PV = 5% (say)
  - Consider a credit contingent contract that pays *at default time*:
  - Present value of a default free 5% swap minus recovered value on the 5,1% defaultable swap.
  - *at default time*, holder of defaultable swap + credit contract receives:
    - recovery value on 5,1% defaultable swap + PV of default free 5% swap - recovered value on 5,1% defaultable swap
    - $\text{PV of default free 5\% swap} - \text{PV of default free 5\% swap}
  - Assume credit contract has a periodic annual premium denoted by $p$.
  - Prior to default time, defaultable swap + credit contract pays:
    - Default-free swap cash-flows with fixed rate = 5,1%-p
  - $p$ must be equal to 10bp = 5,1%-5%, otherwise arbitrage with 5% default-free swap.
Case study: defaultable interest rate swap

- Credit contingent contract transforms 5.1% defaultable swap into a 5% default free swap with the same PV.
  - If default occurs immediately, *no jump* in the hedged P&L.
  - To hedge the default payment on the credit contingent contract, we must hold default swaps providing payments of:

  - PV of default free 5% swap - recovery on 5.1% defaultable swap:
    
    $$ PV_\tau(5\%) - \delta PV_\tau(5.1\%)^+ - PV_\tau(5.1\%)^- $$

    - $PV_\tau(5.1\%)$ is *close* to $PV_\tau(5\%)$ (here 3\%=PV of defaultable swap).
    - Required payment on hedging default swap *close* to $(1- \delta) PV_\tau(5.1\%)^+$

    $$ \Rightarrow \text{Plain default swap pays } 1- \delta \text{ at default time.} $$

- *Nominal amount* of hedging default swap *almost* equal to $PV_\tau(5.1\%)^+$
Hedging Default risk and credit spread risk in
Credit Contingent Contracts

• Purpose: joint hedge of default risk and credit spread risk
• Hedging default risk only constrains the amount of underlying standard default swap.
  – Maturity of underlying default swap is arbitrary.
• Choose maturity to be protected against credit spread risk
  – PV of credit contingent contracts and standard default swaps are sensitive to the level of credit spreads
  – Sensitivity of standard default swaps to a shift in credit spreads increases with maturity
  – Choose maturity of underlying default swap in order to equate sensitivities.
• Example:
  - dependence of simple default swaps on defaultable forward rates.
  - Consider a $T$-maturity default swap with continuously paid premium $p$.
    Assume zero-recovery (digital default swap).
  - PV (at time 0) of a long position provided by:
    \[
    PV = E \left[ \int_0^T \left( \exp - \int_0^t (r + \lambda)(s) ds \right) \times (\lambda(t) - p) dt \right]
    \]
  - where $r(t)$ is the short rate and $\lambda(t)$ the default intensity.
  - Assume that $r(.)$ and $\lambda(.)$ are independent.
  - $B(0,t)$: price at time 0 of a $t$-maturity default-free discount bond
  - $f(0,t)$: corresponding forward rate
    \[
    B(0,t) = E \left[ \exp - \int_0^t r(u) du \right] = \exp - \int_0^t f(0,u) du
    \]
Hedging credit spread risk

− Let $\overline{B}(0, t)$ be the defaultable discount bond price and $\bar{f}(0, t)$ the corresponding instantaneous forward rate:

$$\overline{B}(0, t) = E\left[\exp\left(-\int_{0}^{t}(r + \lambda(u))du\right)\right] = \exp\left(-\int_{0}^{t}\bar{f}(0, u)du\right)$$

− Simple expression for the PV of the $T$-maturity default swap:

$$PV(T) = \int_{0}^{T} \overline{B}(0, t)\left(\bar{f}(0, t) - f(0, t) - p\right)dt$$

− The derivative of default swap present value with respect to a shift of defaultable forward rate $\bar{f}(0, t)$ is provided by:

$$\frac{\partial PV}{\partial \bar{f}}(t) = PV(t) - PV(T) + \overline{B}(0, t)$$

➢ $PV(t) - PV(T)$ is usually small compared with $\overline{B}(0, t)$. 
Similarly, we can compute the sensitivities of plain default swaps with respect to default-free forward curves $f(0,t)$.

And thus to credit spreads.

Same approach can be conducted with the credit contingent contract to be hedged.

All the computations are model dependent.

Several maturities of underlying default swaps can be used to match sensitivities.

For example, in the case of defaultable interest rate swap, the nominal amount of default swaps $(\text{PV}_\tau)^+$ is usually small.

Single default swap with nominal $(\text{PV}_\tau)^+$ has a smaller sensitivity to credit spreads than defaultable interest rate swap, even for long maturities.

Short and long positions in default swaps are required to hedge credit spread risk.
Explaining theta effects with and without hedging

- **Different aspects** of “carrying” credit contracts through time.
  - Assume “historical” and “risk-neutral” intensities are equal.
- Consider a short position in a credit contingent contract.
- Present value of the deal provided by:
  \[
  PV(u) = E_u \left[ \int_u^T \left( \exp - \int_u^t (r + \lambda(s)) ds \right) \times (p_T - \lambda(t)C(t)) dt \right]
  \]
- (after computations) **Net expected capital gain**:
  \[
  E_u \left[ PV(u + du) - PV(u) \right] = \left( r(u) + \lambda(u) \right) PV(u) du + (\lambda(u)C(u) - p_T) du
  \]
- **Accrued cash-flows** (received premiums): \( p_T du \)
  - By summation, Incremental P&L (if no default between \( u \) and \( u + du \)):
    \[
    r(u)PV(u) du + \lambda(u)\left( C(u) + PV(u) \right) du
    \]
Explaining theta effects with and without hedging

- **Apparent extra return effect**: \( \lambda(u)(C(u) + PV(u))du \)
  - But, probability of default between \( u \) and \( u + du \): \( \lambda(u)du \).
  - Losses in case of default:
    - Commitment to pay: \( C(u) \)
    - Loss of PV of the credit contract: \( PV(u) \)
    - \( PV(u) \) consists in *unrealised* capital gains or losses in the credit derivatives book that “disappear” in case of default.
  - Expected loss charge: \( \lambda(u)(C(u) + PV(u))du \)

- **Hedging aspects**:
  - If we hold \( C(u) + PV(u) \) short-term digital default swaps, we are protected at default-time (no jump in the P&L).
  - Premiums to be paid: \( \lambda(u)(C(u) + PV(u))du \)
  - Same average rate of return, but smoother variations of the P&L.
Hedging Default Risk in Basket Default Swaps

• Example: first to default swap from a basket of two risky bonds.
  – If the first default time occurs before maturity,
  – The seller of the first to default swap pays the non recovered fraction of the defaulted bond.
  – Prior to that, he receives a periodic premium.
• Assume that the two bonds cannot default simultaneously
  – We moreover assume that default on one bond has no effect on the credit spread of the remaining bond.
• How can the seller be protected at default time?
  – The only way to be protected at default time is to hold two default swaps with the same nominal than the nominal of the bonds.
  – The maturity of underlying default swaps does not matter.
Real World hedging and risk-management issues

- uncertainty at default time
  - illiquid default swaps
  - recovery risk
  - simultaneous default events

- Managing net premiums
  - Maturity of underlying default swaps
  - Lines of credit
  - Management of the carry
  - Finite maturity and discrete premiums
  - Correlation between hedging cash-flows and financial variables
Consider a first to default swap associated with a basket of two defaultable loans.

- Hedging portfolios based on standard underlying default swaps
- Uncertain hedge ratios if:
  - simultaneous default events
  - Jumps of credit spreads at default times

**Simultaneous default events:**
- If counterparties default altogether, holding the complete set of default swaps is a conservative (and thus expensive) hedge.
- In the extreme case where default always occur altogether, we only need a single default swap on the loan with largest nominal.
- In other cases, holding a fraction of underlying default swaps does not hedge default risk (if only one counterparty defaults).
What occurs if there is a \textit{jump in the credit spread} of the second counterparty after \textit{default} of the first?

- default of first counterparty means \textit{bad news} for the second.

If hedging with short-term default swaps, \textit{no capital gain} at default.

- Since PV of short-term default swaps is not \textit{sensitive} to credit spreads.

This is not the case if hedging with long term default swaps.

- If credit spreads \textit{jump}, PV of long-term default swaps \textit{jumps}.

Then, the amount of hedging default swaps can be \textit{reduced}.

- This reduction is \textit{model-dependent}.
On the edge of completeness?

• **Firm-value** structural default models:
  – Stock prices follow a diffusion processes (no jumps).
  – Default occurs at first time the stock value hits a barrier

• *In this modelling*, default credit derivatives can be **completely** hedged by trading the stocks:
  – “*Complete*” pricing and hedging model:

• **Unrealistic features** for hedging *basket default swaps*:
  – Because default times are predictable, *hedge ratios are close to zero* except for the counterparty with the smallest “distance to default”.
On the edge of completeness?

**hazard rate based models**

- In **hazard rate** based models:
  - default is a sudden, *non predictable* event,
  - that causes a sharp *jump* in defaultable bond prices.
  - Most credit contingent contracts and basket default derivatives have payoffs that are *linear* in the prices of defaultable bonds.
  - Thus, good news: default risk can be *hedged*.
  - Credit spread risk can be *substantially reduced* but not completely eliminated.
  - More *realistic* approach to default.
  - *Hedge ratios* are *robust* with respect to default risk.
Looking for a better understanding of credit derivatives
- payments in case of default,
- volatility of credit spreads.

Bridge between risk-neutral valuation and the cost of the hedge approach.

**dynamic** hedging strategy based on *standard default swaps*.
- hedge ratios in order to get protection at default time.
- hedging default risk is *model-independent*.
- importance of quantitative models for a better management of the P&L and the residual premiums.