On the Edge of Completeness

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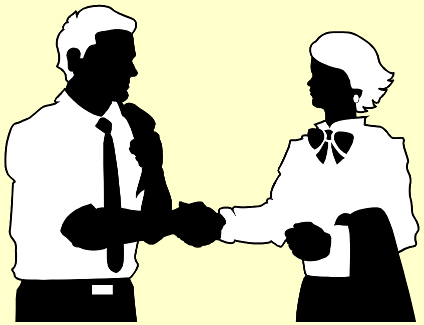
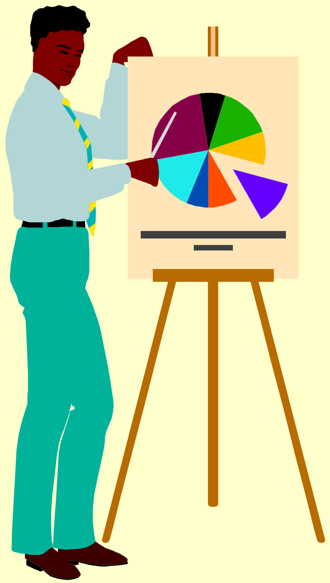
On the Edge of Completeness: Purpose and main ideas

- **Purpose:**
  - **risk-analysis of exotic credit derivatives:**
    - credit contingent contracts, basket default swaps.
  - **pricing and hedging exotic credit derivatives.**
- **Main ideas:**
  - **distinguish between credit spread volatility and default risk.**
  - **dynamic hedge of exotic default swaps with standard default swaps.**
- **Reference paper: “On the edge of completeness”, with Angelo Arvanitis, RISK, October 1999.**

On the Edge of completeness: Overview

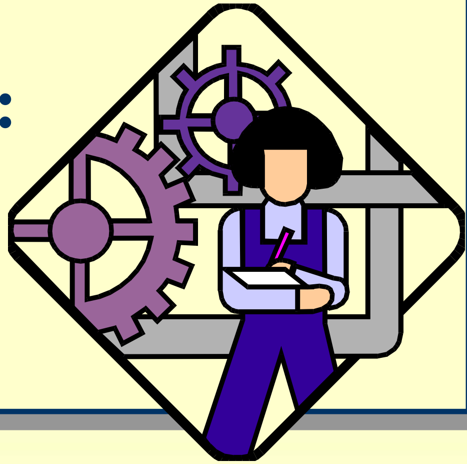
- **Trading credit risk : closing the gap between supply and demand**





- **Modelling credit derivatives: the state of the art**

- **A new approach to credit derivatives modelling:**
  - **closing the gap between pricing and hedging**
  - **disentangling default risk and credit spread risk**

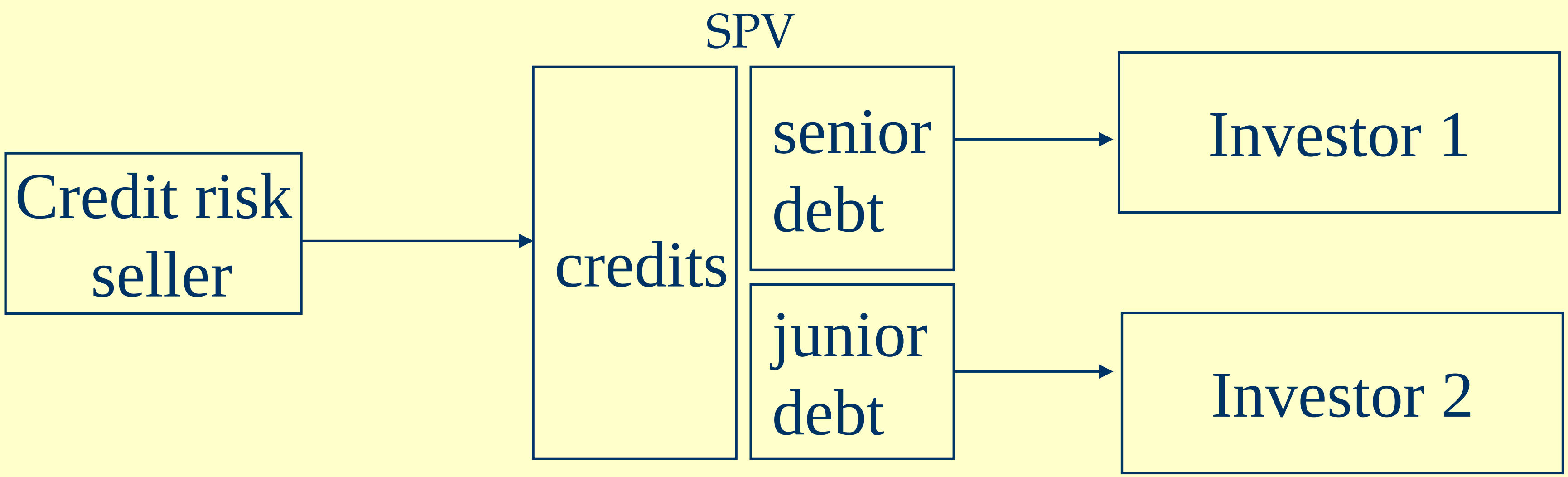


Trading credit risk: Closing the gap between supply and demand

- **From stone age to the new millennium:**
  - **Technical innovations in credit derivatives are driven by economic forces.**
  - **Transferring risk from commercial banks to institutional investors:**
    - Securitization.
    - Default Swaps : portfolio and hedging issues.
    - Credit Contingent Contracts, Basket Credit Derivatives.
  - **The previous means tend to be more integrated.**

Trading credit risk: Closing the gap between supply and demand

- **Securitization of credit risk:**



- **simplified scheme:**
  - **No residual risk remains within SPV.**
  - **All credit trades are simultaneous.**

Trading Credit Risk: Closing the gap between supply and demand

- **Financial intermediaries provide structuring and arrangement advice.**
  - **Credit risk seller can transfer loans to SPV or instead use default swaps**
- **good news : low capital at risk for investment banks**

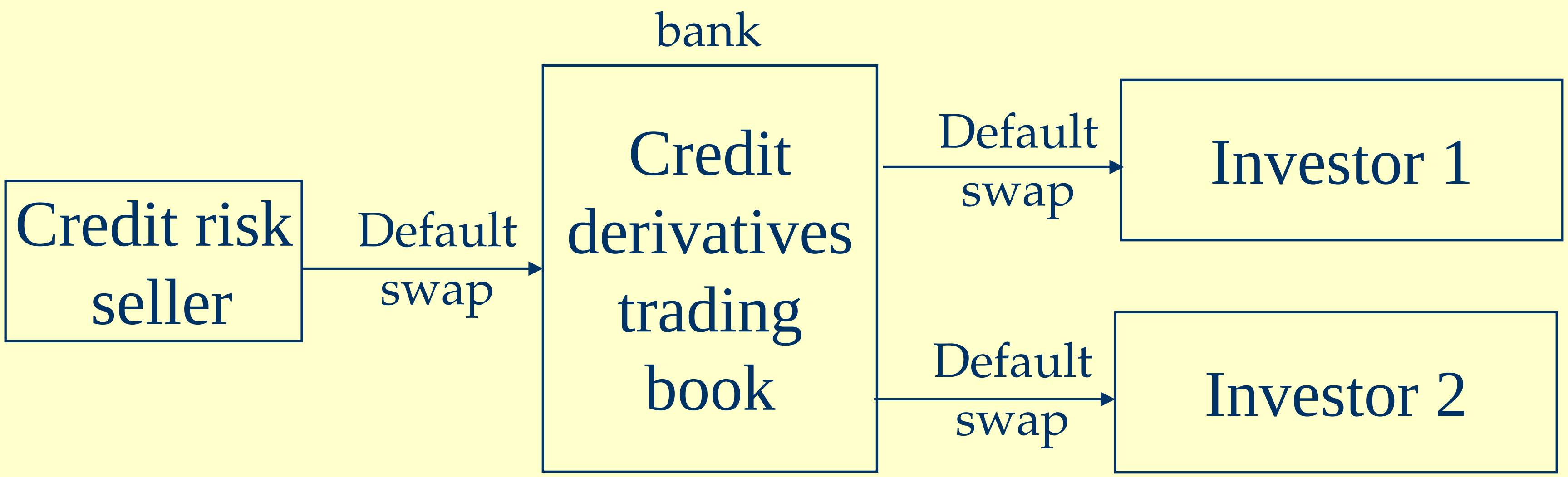
- **Good times for modelling credit derivatives**
  - **No need of <u>hedging</u> models**
  - **credit pricing models are used to ease risk transfer**
  - **need to assess the risks of various tranches**

Trading Credit Risk: Closing the gap between supply and demand

- **There is room for financial intermediation of credit risk**
  - **The transfers of credit risk between commercial banks and investors may not be simultaneous.**
  - **Since at one point in time, demand and offer of credit risk may not match.**
    - ➢Meanwhile, credit risk remains within the balance sheet of the financial intermediary.
  - **It is not further required to find customers with exact opposite interest at every new deal.**
    - ➢Residual risks remain within the balance sheet of the financial intermediary.

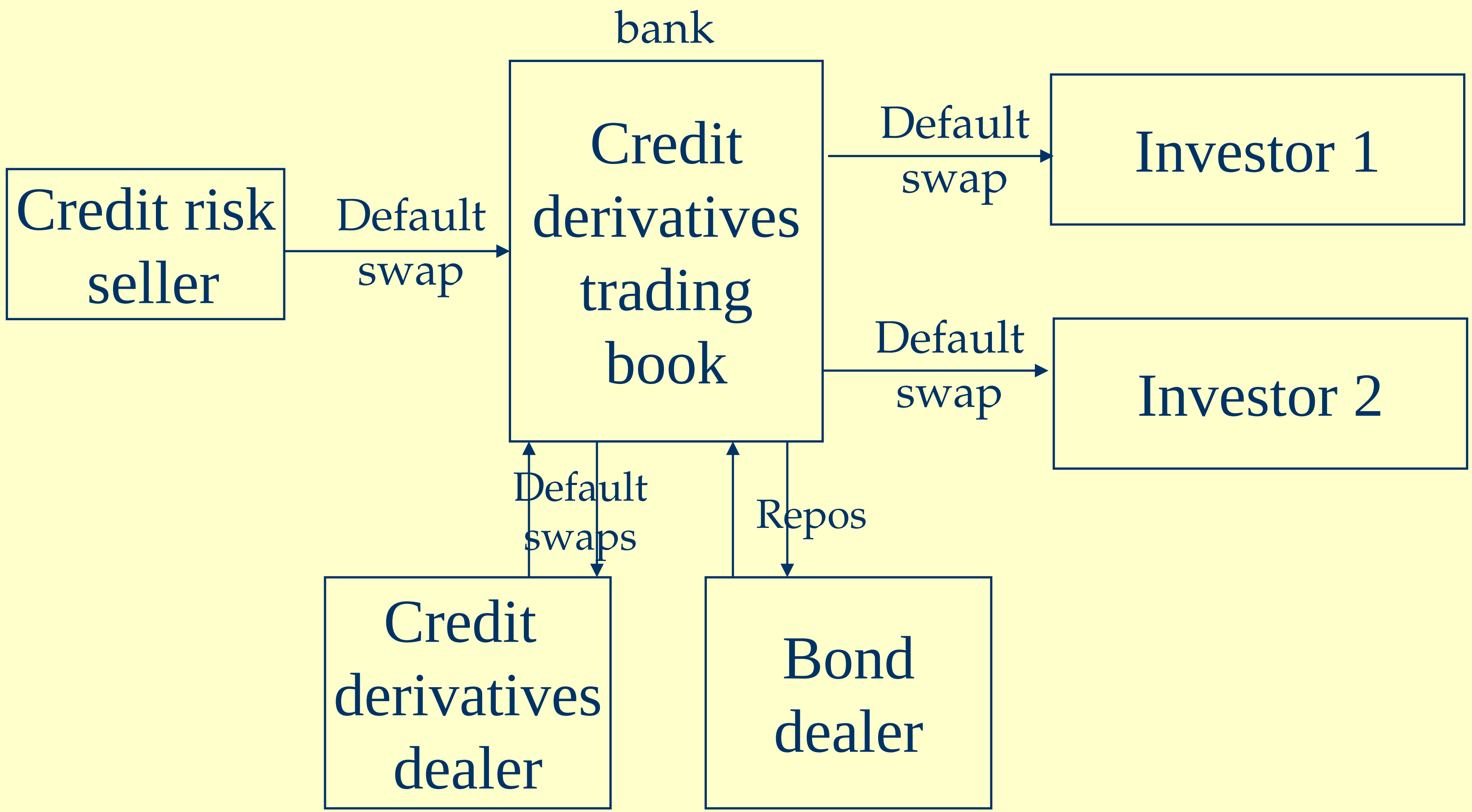
Credit risk management without hedging default risk

- **Emphasis on:**
  - **portfolio effects: correlation between default events**
  - **posting collateral**
  - **computation of capital at risk, risk assessment**
- **Main issues:**
  - **capital at risk can be high**
  - **what is the competitive advantage of investment banks**



Credit risk management with hedging default risk

- **Trading against other dealers enhances ability to transfer credit risk by lowering capital at risk**



New ways to transfer credit risk :
credit contingent contracts

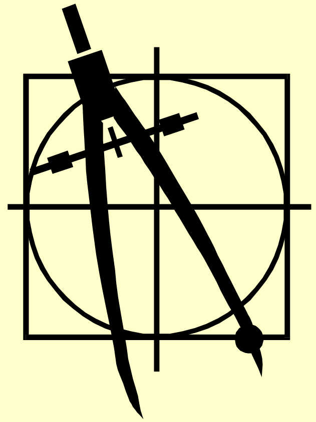
- **Anatomy of a general credit contingent contract**
  - **A credit contingent contract is like a standard default swap but with variable nominal (or exposure)**
  - **However the periodic premium paid for the credit protection remains fixed.**
  - **The protection payment arises at default of one given single risky counterparty.**
- **Examples**
  - ➢ **cancellable swaps**
  - ➢ quanto default swaps
  - ➢ credit protection of vulnerable swaps, OTC options (stand-alone basis)
  - ➢ credit protection of a portfolio of contracts (full protection, excess of loss insurance, partial collateralization)

New ways to transfer credit risk :
Basket default derivatives

- **Consider a basket of *M* risky bonds**
  - **multiple counterparties**
- **First to default swaps**
  - **protection against the first default**
- ***N* out of *M* default swaps (*N* < *M*)**
  - **protection against the first *N* defaults**
- **Hedging and valuation of basket default derivatives**
  - **involves the joint (multivariate) modelling of default arrivals of issuers in the basket of bonds.**
  - **Modelling accurately the dependence between default times is a critical issue.**

Modelling credit derivatives: the state of the art

- **Modelling credit derivatives : Where do we stand ?**
- **Financial industry approaches**
  - **Plain default swaps and risky bonds**
  - **credit risk management approaches**

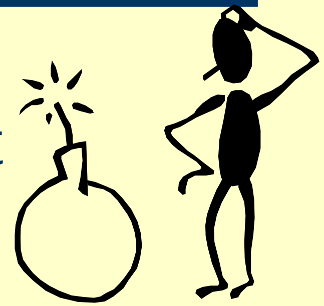


- **The Noah's arch of credit risk models**
  - **"firm-value" models**
  - **risk-intensity based models**
  - **Looking desperately for a hedging based approach to pricing.**

Modelling credit derivatives : Where do we stand ?
Plain default swaps

- **Static arbitrage of plain default swaps with short selling underlying bond**
  - **plain default swaps hedged using underlying risky bond**
  - **“bond strippers” : allow to compute prices of risky zero-coupon bonds**
  - **repo risk, squeeze risk, liquidity risk, recovery rate assumptions**
- **Computation of the P&L of a book of default swaps**
  - **Involves the computation of a P&L of a book of default swaps**
  - **The P&L is driven by changes in the credit spread curve and by the occurrence of default.**

Modelling credit derivatives: Where do we stand ?
Credit risk management

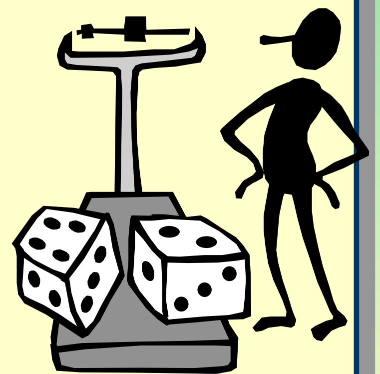


- **Assessing the varieties of risks involved in credit derivatives**
  - **Specific risk or credit spread risk**
    - *prior to default*, the P&L of a book of credit derivatives is driven by changes in credit spreads.
  - **Default risk**
    - *in case of default*, if unhedged,
    - dramatic jumps in the P&L of a book of credit derivatives.

Modelling credit derivatives: Where do we stand ? The Noah's arch of credit risk models

- **"firm-value" models :**
  - Modelling of firm's assets
  - First time passage below a critical threshold

- **risk-intensity based models**
  - Default arrivals are no longer <u>predictable</u>
  - Model conditional local probabilities of default $\lambda(t)\,dt$
  - τ : default date, $\lambda(t)$ *risk intensity* or *hazard rate*



$$\lambda(t)dt = P\left[\tau \in [t, t+dt[\,\middle|\, \tau > t\right]$$

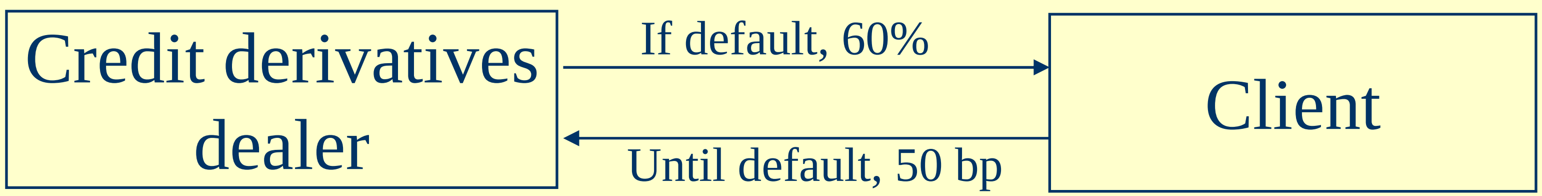
- **Lack of a <u>hedging based approach</u> to pricing.**
  - Misunderstanding of hedging against default risk and credit spread risk

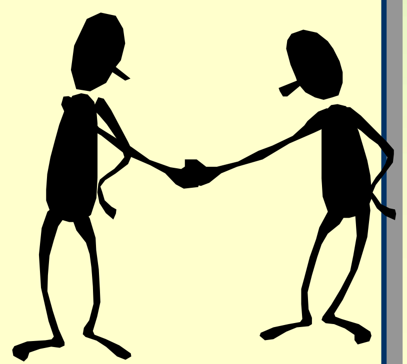
A new approach to credit derivatives modelling based on an hedging point of view

- *Rolling over* the hedge:
  - Short term default swaps v.s. long-term default swaps
  - Credit spread transformation risk
- Credit contingent contracts, basket default swaps
  - Hedging default risk through dynamics holdings in standard default swaps
  - Hedging credit spread risk by choosing appropriate default swap maturities
  - Closing the gap between pricing and hedging
- Practical hedging issues
  - Uncertainty at default time
  - Managing net residual premiums

Long-term Default Swaps v.s. Short-term Default Swaps Rolling over the hedge

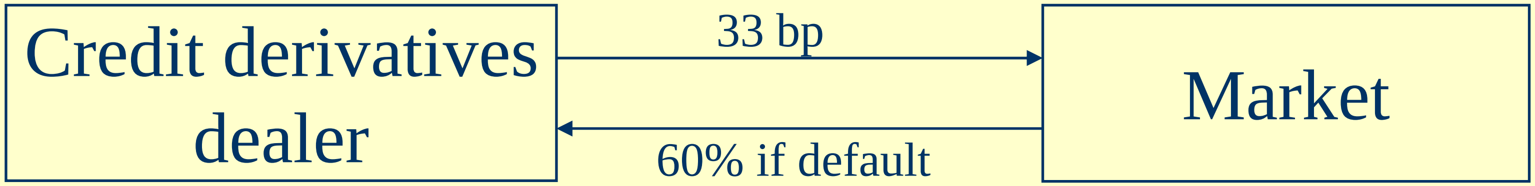
- **Purpose:**
  - **Introduction to dynamic trading of default swaps**
  - **Illustrates how default and credit spread risk arise**
- **Arbitrage between long and short term default swap**
  - **sell one long-term default swap**
  - **buy a series of short-term default swaps**
- **Example:**
  - **default swaps on a FRN issued by BBB counterparty**
  - **5 years default swap premium : 50bp, recovery rate = 60%**



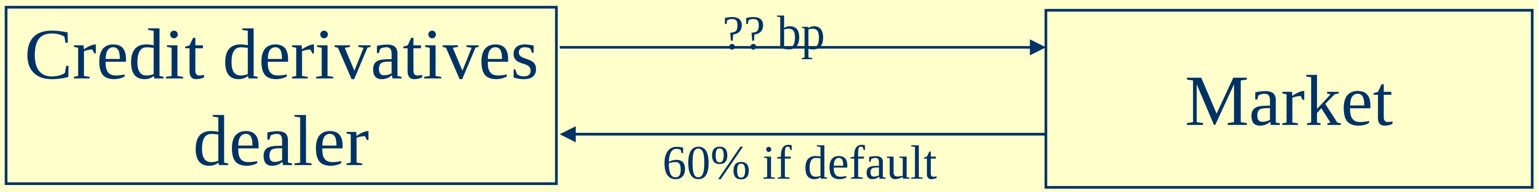


Long-term Default Swaps v.s. Short-term Default Swaps
Rolling over the hedge

- **Rolling over short-term default swap**
  - **at inception, one year default swap premium : 33bp**
  - **cash-flows after one year:**

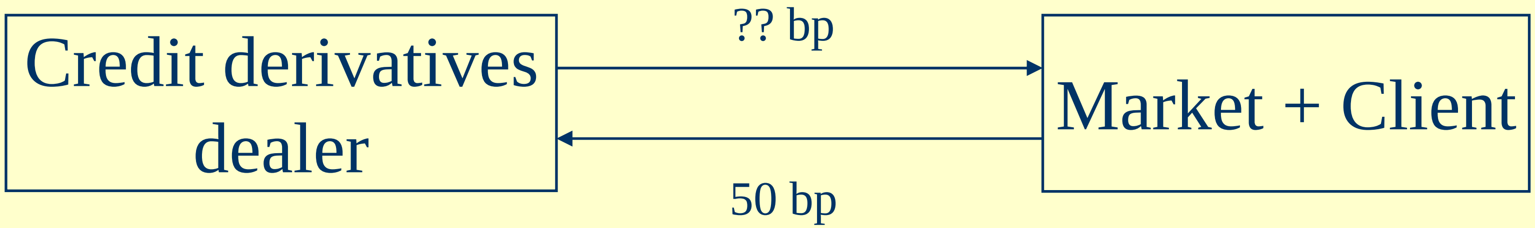


- **Buy a one year default swap at the end of every yearly period, if no default:**
  - **Dynamic strategy,**
  - **<u>future</u> premiums depend on <u>future</u> credit quality**
  - **future premiums are <u>unknown</u>**



Long-term Default Swaps v.s. Short-term Default Swaps
Rolling over the hedge

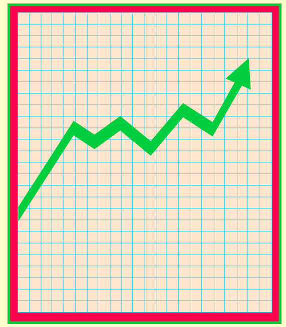
- ***Risk analysis*** **of rolling over short term against long term default swaps**

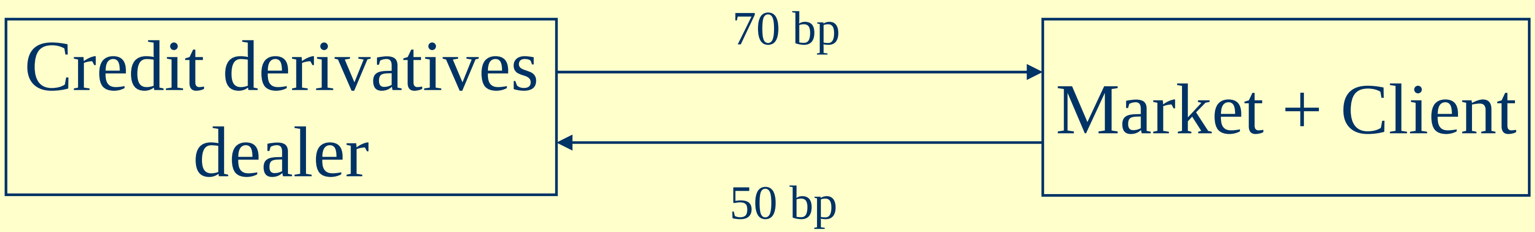


- **Exchanged cash-flows :**
  - **Dealer receives 5 years (fixed) credit spread,**
  - **Dealer pays 1 year (variable) credit spread.**
- **Full one to one protection at default time**
  - **the previous strategy has eliminated one source of risk, that is default risk**

Long-term Default Swaps v.s. Short-term Default Swaps
Rolling over the hedge

- **negative exposure to an increase in short-term default swap premiums**
  - **if short-term premiums increase from 33bp to 70bp**
  - **reflecting a lower (short-term) credit quality**
  - **and no default occurs before the fifth year**





- **Loss due to negative carry**
  - **long position in long term credit spreads**
  - **short position in short term credit spreads**

Rolling over the hedge : portfolio of homogeneous loans

- **Consider a portfolio of of <u>homogeneous</u> loans**
  - **same unit nominal, non amortising**
  - **τ_i : default time of counterparty i**
  - **same default time distribution (same hazard rate $\lambda(t)$):**

$$P\left[\tau_i \in [t, t+dt[\,\middle|\, \tau_i > t\right] = \lambda(t)dt$$

  - **F_t : available information at time t**
  - **Conditional independence between default events $\{\tau_i \in [t, t+dt[\}$**

$$P\left[\tau_i, \tau_j \in [t, t+dt[\,\middle|\, F_t\right] = P\left[\tau_i \in [t, t+dt[\,\middle|\, F_t\right] \times P\left[\tau_j \in [t, t+dt[\,\middle|\, F_t\right]$$

  - **equal to zero or to $\lambda^2(t)(dt)^2$, i.e no simultaneous defaults.**
  - **Remark that indicator default variables $1_{\{\tau_i \in [t, t+dt[\}}$ are (conditionally) independent and equally distributed.**

Rolling over the hedge : portfolio of homogeneous loans

- **Denote by *N*(*t*) the outstanding amount of the portfolio (i.e. the number of non defaulted loans) at time *t*.**
- **By law of large numbers,** $\frac{1}{N(t)}\sum 1_{\{\tau_i \in [t,t+dt[\}} \to \lambda(t)dt$
- **Since** $N(t+dt)-N(t) = -\sum 1_{\{\tau_i \in [t,t+dt[\}}$
- **we get,** $\frac{N(t+dt)-N(t)}{N(t)} = -\lambda(t)dt$
- **The outstanding nominal decays as** $N(t) = N(0)\exp-\int_0^t \lambda(s)ds$
- **Assume zero recovery; Total default loss *t* and *t+dt*: *N*(*t*)-*N*(*t+dt*)**
- **Cost of default per outstanding loan:** $\frac{N(t)-N(t+dt)}{N(t)} = \lambda(t)dt$

Rolling over the hedge : portfolio of homogeneous loans

- **Cost of default per outstanding loan = $\lambda(t)dt$ is known at time t.**
- **<u>Insurance</u> diversification approach holds**
- ***Fair premium* for a short term insurance contract on a single loan (i.e. a short term default swap) has to be equal to $\lambda(t)dt$.**
- **Relates *hazard rate* and *short term default swap premiums*.**

- **Expanding on rolling over the hedge**
  - **Let us be short in 5 years (say) default swaps written on all individual loans.**
    - $p_{5Y}\,dt$, periodic premium per loan.
  - **Let us buy the short term default swaps on the outstanding loans.**
    - Corresponding premium per loan: $\lambda(t)dt$.
  - **Cash-flows related to default events $N(t)$-$N(t+dt)$ perfectly offset**

Rolling over the hedge : portfolio of homogeneous loans

- **Net (premium) cash-flows between *t* and *t+dt*:** $N(t)\left[p_{5Y} - \lambda(t)\right]dt$
- **Where** $N(t) = N(0)\exp - \int_0^t \lambda(s)ds$
  - ➢Payoff similar to an "***index amortising swap***".
- **At inception, p_{5Y} must be such that the risk-neutral expectation of the discounted net premiums equals zero:**
- **Pricing equation for the long-term default swap premium p_{5Y} :**

$$E\left[\int_0^T \left(\exp - \int_0^t r(s)ds\right) \times N(t)\left(p_{5Y} - \lambda(t)\right)dt\right] = 0$$

  - ➢where $r(t)$ is the short rate at time t.
- **Premiums received when selling long-term default swaps:** $N(t)p_{5Y}dt$
- **Premiums paid on "hedging portfolio":** $N(t)\lambda(t)dt$

Rolling over the hedge : portfolio of homogeneous loans

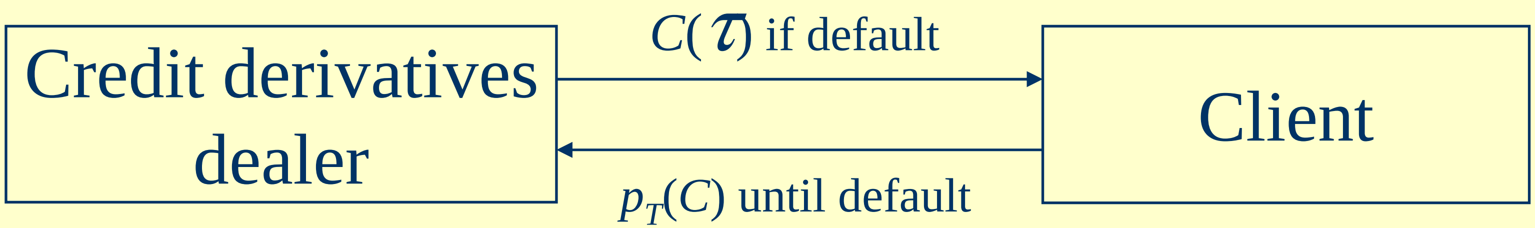
- **Convexity effects and the cost of the hedge**
  - **Net premiums paid** $N(t)\left[p_{5Y} - \lambda(t)\right]dt$
- **What happens if short term premiums $\lambda(t)$ become more volatile?**
  - *Net premiums* become negative when $\lambda(t)$ is high.
  - Meanwhile, the outstanding amount $N(t)$ tends to be small, mitigating the losses.
  - contrarily when $\lambda(t)$ is small, the dealer experiments positive cash-flows $p_{5Y} - \lambda(t)$ on a larger amount $N(t)$.
- **The more volatile $\lambda(t)$, the smaller the average cost of the hedge and thus the long term premium p_{5Y} .**

Hedging exotic default swaps : main features

- **Exotic credit derivatives can be *hedged* against <u>default</u>:**
  - **Constrains the <u>amount</u> of underlying <u>standard</u> default swaps.**
  - **<u>Variable</u> amount of standard default swaps.**
  - **<u>Full protection</u> at default time by construction of the hedge.**
  - **No more *<u>discontinuity</u>* in the P&L at default time.**
  - **"Safety-first" criteria: *main source of risk* can be hedged.**
  - **<u>Model-free</u> <u>approach.</u>**
- **<u>Credit spread exposure</u> has to be hedged by *other means*:**
  - **Appropriate *choice of maturity* of underlying default swap**
  - **Computation of sensitivities with respect to changes in credit spreads are <u>model</u> <u>dependent</u>.**

Hedging Default Risk in Credit Contingent Contracts

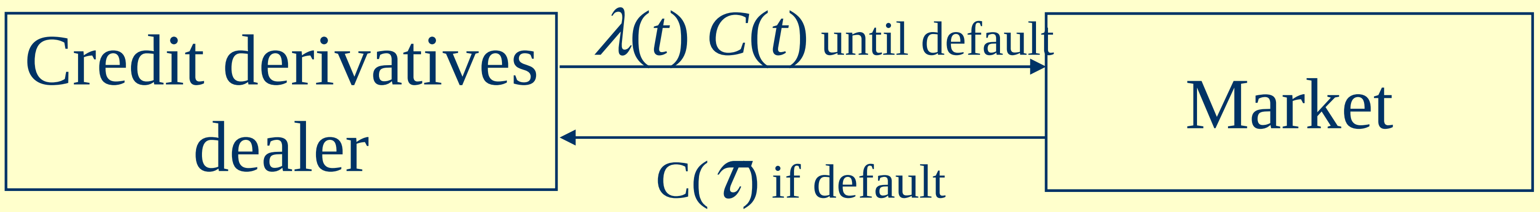
- **Credit contingent contracts**
  - **client pays to dealer a periodic premium $p_T(C)$ until default time τ, or maturity of the contract T.**
  - **dealer pays $C(\tau)$ to client at default time τ, if $\tau \leq T$.**



- **Hedging side:**
  - **Dynamic strategy based on standard default swaps:**
  - **At time t, hold an amount $C(t)$ of standard default swaps**
  - **$\lambda(t)$ denotes the periodic premium at time t for a short-term default swap**

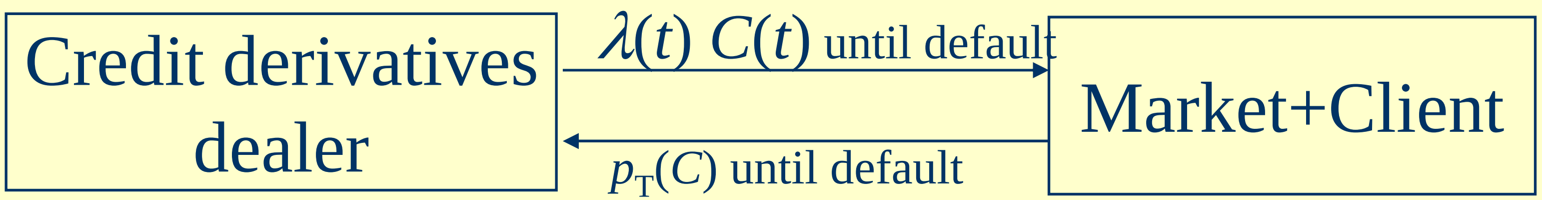
Hedging Default Risk in Credit Contingent Contracts

- **Hedging side:**



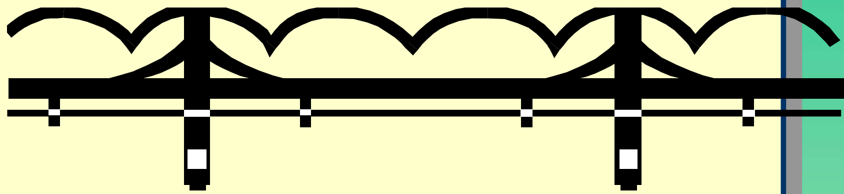
  - **Amount of standard default swaps equals the (variable) <u>credit exposure</u> on the credit contingent contract.**

- **Net position is a "*basis swap*":**



- **The client transfers credit spread risk to the credit derivatives dealer**

Closing the gap between pricing and hedging



- **What is the cost of hedging default risk ?**
- **Discounted value of hedging default swap premiums:**

$$E\left[\int_0^T \left(\exp - \int_0^t (r+\lambda)(s)\,ds\right)\lambda(t)C(t)\,dt\right]$$

Discounting term

Premium paid at time t on protection portfolio

- **Equals the discounted value of premiums received by the seller:**

$$E\left[\int_0^T \left(\exp - \int_0^t (r+\lambda)(s)\,ds\right)p_T\,dt\right]$$

Case study: defaultable interest rate swap

- **Consider a defaultable interest rate swap (with unit nominal)**
  - **We are <u>default-free</u>, our counterparty is <u>defaultable</u> (default intensity $\lambda(t)$).**
  - **We consider a (fixed-rate) *receiver* swap on a <u>standalone</u> basis.**
- **Recovery assumption, payments in case of default.**
  - **if default at time τ, compute the <u>default-free</u> value of the swap: PV_τ**
  - **and get:** $\delta\left(PV_\tau\right)^+ + \left(PV_\tau\right)^- = PV_\tau - \left(1-\delta\right)\left(PV_\tau\right)^+$
  - **$0 \leq \delta \leq 1$ recovery rate, $(PV_\tau)^+ = Max(PV_\tau, 0)$, $(PV_\tau)^- = Min(PV_\tau, 0)$**
  - ***In case of default,***
    - we <u>receive</u> default-free value PV_τ
    - ***minus***
    - <u>loss</u> equal to $(1-\delta)(PV_\tau)^+$.

Case study: defaultable interest rate swap

- **Defaultable and default-free swap**
  - **Present value of <u>defaultable</u> swap = Present value of <u>default-free</u> swap (with <u>same fixed rate</u>) – Present value of the loss.**
  - **To compensate for default, fixed rate of defaultable swap (with given market value) is *greater* than fixed rate of default-free swap (<u>with same market value</u>).**
  - **Let us remark, that default immediately after negotiating a defaultable swap results in a <u>positive</u> jump in the P&L, because recovery is based on default-free value.**
- **To account for the possibility of default, we may constitute a *credit reserve*.**
  - **Amount of credit reserve equals expected Present Value of the loss**
  - **This accounts for the *expected* loss but does not hedge against realized loss.**

Case study: defaultable interest rate swap

- **Using a hedging instrument rather than a credit reserve**
  - **Consider a <u>credit contingent contract</u> that pays $(1-\delta)(PV_\tau)^+$ at default time τ (if $\tau \leq T$), where PV_τ is the present value of a default-free swap with *same fixed rate* than defaultable swap.**
  - **Such a credit contract + a defaultable swap <u>*synthesises*</u> a *default-free* swap (at a fixed rate equal to the <u>initial</u> fixed rate):**
  - **At default, we receive $(1-\delta)(PV_\tau)^+ + PV_\tau - (1-\delta)(PV_\tau)^+ = PV_\tau$**
  - **The <u>upfront</u> premium for this credit protection is equal to the Present Value of the <u>loss</u> $(1-\delta)(PV_\tau)^+$ given default:**

$$E\left[\int_0^T \left(\exp - \int_0^t (r+\lambda)(u)\,du\right)\lambda(t)(1-\delta)(PV_t)^+ dt\right]$$

Case study: defaultable interest rate swap
Interpreting the cost of the hedge

- **<u>Average</u> cost of default on a large *portfolio* of swaps**
  - **Large number of *homogeneous* <u>defaultable</u> receiver swaps:**
    - ➢ Same fixed rate and maturity; initial nominal value $N(0)=1$
    - ➢ independent default dates and same default intensity $\lambda(t)$.
  - **Outstanding nominal amount:** $N(t) = \exp - \int_0^t \lambda(s)ds$
  - **Nominal amount defaulted in [*t*, *t+dt* [:** $N(t) - N(t+dt) = \lambda(t)dt \exp - \int_0^t \lambda(s)ds$
  - ***Cost of default* in [*t*, *t+dt* [:** $\left(N(t)\text{-}N(t+dt)\right)(1-\delta)(PV_t)^+$
  - **Where PV_t: present value of receiver swap with unit nominal.**
  - **<u>Aggregate</u> cash-flows do <u>not</u> depend on *default risk*.**
  - **Aggregate cash-flows are those of an <u>index amortising swap</u>**
  - **Standard discounting provides previous slide pricing equation**

Case study: defaultable interest rate swap
Interpreting the cost of the hedge

- **Randomly exercised swaption:**
  - **Assume for simplicity no recovery (δ=0).**
  - **Interpret default time as a *random time* τ with *intensity* $\lambda(t)$.**
  - **At that time, defaulted counterparty "exercises" a swaption, i.e. decides whether to cancel the swap according to its present value.**
  - **PV of default-losses equals price of that *randomly exercised swaption***
- **American Swaption**
  - **PV of American swaption equals the supremum over *all possible stopping times* of *randomly exercised swaptions*.**
    - ➢ The upper bound can be reached for *special default arrival dates*:
    - ➢ $\lambda(t)=0$ above exercise boundary and $\lambda(t)=\infty$ on exercise boundary

Case study: defaultable interest rate swap

- **Previous hedge leads to (small) jumps in the P&L:**
  - **Consider a 5,1% fixed rate defaultable receiver swap with PV=3%.**
  - **Buy previous credit contingent contract at market price.**
    - Due to credit protection, we hold a synthetic default-free 5,1% swap.
    - Total PV remains equal to 3%.
  - **Assume that default immediate default: $\tau=0^+$.**
  - **Clearly a 5,1% default free swap has PV>3%, thus occurring a positive jump in P&L.**
- **Jumps in the P&L due to *extra default insurance*:**
  - **To hedge the previous credit contingent contract:**
  - **At time 0, we hold an amount of short term default swap that is equal to the Present Value of a default-free 5,1% swap**
  - **This amount is greater than 3%, the *current Present Value*.**

Case study: defaultable interest rate swap

- **Alternative hedging approach:**
  - ➢Fixed rate of default-free swap with 3% PV = 5% (say)
  - **Consider a credit contingent contract that pays *at default time*:**
  - **Present value of a default free 5% swap minus *recovered value* on the 5,1% defaultable swap.**
  - ***at default time*, holder of defaultable swap + credit contract receives:**
    - ➢recovery value on 5,1% defaultable swap + PV of default free 5% swap - recovered value on 5,1% defaultable swap
    - ➢ = PV of default free 5% swap
  - **Assume credit contract has a periodic annual premium denoted by *p*.**
  - **Prior to default time, defaultable swap + credit contract pays:**
    - ➢ Default-free swap cash-flows with fixed rate = 5,1%-*p*
  - ***p* must be equal to 10bp = 5,1%-5%, otherwise arbitrage with 5% default-free swap.**

Case study: defaultable interest rate swap

- **Credit contingent contract transforms 5,1% defaultable swap into a 5% default free swap with the same PV.**
  - **If default occurs immediately, *no jump* in the hedged P&L.**
  - **To hedge the default payment on the credit contingent contract, we must hold default swaps providing payments of:**
  - **PV of default free 5% swap - recovery on 5,1% defaultable swap:**
    - $PV_\tau(5\%) - \delta PV_\tau(5.1\%)^+ - PV_\tau(5.1\%)^-$
  - **$PV_\tau(5.1\%)$ is *close* to $PV_\tau(5\%)$ (here 3%=PV of defaultable swap).**
  - **Required payment on hedging default swap *close* to $(1-\delta)\, PV_\tau(5.1\%)^+$**
    - Plain default swap pays $1-\delta$ at default time.
- ***Nominal amount* of hedging default swap *almost* equal to $PV_\tau(5.1\%)^+$**

Hedging Default risk ***and*** *credit spread risk in Credit Contingent Contracts*

- **Purpose : joint hedge of default risk and credit spread risk**
- **Hedging *default risk* only constrains the amount of underlying standard default swap.**
  - **Maturity of underlying default swap is arbitrary.**
- **Choose maturity to be protected against credit spread risk**
  - **PV of credit contingent contracts and standard default swaps are sensitive to the level of credit spreads**
  - **Sensitivity of standard default swaps to a shift in credit spreads increases with maturity**
  - **Choose maturity of underlying default swap in order to equate sensitivities.**

Hedging credit spread risk

- **Example:**
  - **dependence of simple default swaps on defaultable forward rates.**
  - **Consider a *T*-maturity default swap with continuously paid premium *p*. Assume zero-recovery (digital default swap).**
  - **PV (at time 0) of a long position provided by:**

$$PV = E\left[\int_0^T \left(\exp - \int_0^t (r+\lambda)(s)ds\right) \times (\lambda(t) - p)dt\right]$$

  - **where *r*(*t*) is the short rate and λ(*t*) the default intensity.**
  - **Assume that *r*(.) and λ(.) are independent.**
  - ***B*(0,*t*): price at time 0 of a *t*-maturity default-free discount bond**
  - ***f*(0,*t*): corresponding forward rate**

$$B(0,t) = E\left[\exp - \int_0^t r(u)du\right] = \exp - \int_0^t f(0,u)du$$

Hedging credit spread risk

- **Let $\overline{B}(0,t)$ be the *defaultable discount bond price* and $\overline{f}(0,t)$ the corresponding instantaneous forward rate:**

$$\overline{B}(0,t) = E\left[\exp - \int_0^t (r+\lambda)(u)\,du\right] = \exp - \int_0^t \overline{f}(0,u)\,du$$

- **Simple expression for the PV of the *T*-maturity default swap:**

$$PV(T) = \int_0^T \overline{B}(0,t)\left(\overline{f}(0,t) - f(0,t) - p\right)dt$$

- **The derivative of default swap present value with respect to a shift of defaultable forward rate $\overline{f}(0,t)$ is provided by:**

$$\frac{\partial PV}{\partial \overline{f}}(t) = PV(t) - PV(T) + \overline{B}(0,t)$$

  - PV(t)-PV(T) is usually small compared with $\overline{B}(0,t)$.

Hedging credit spread risk

- **Similarly, we can compute the sensitivities of plain default swaps with respect to *default-free forward curves* $f(0,t)$.**
- **And thus to <u>credit</u> <u>spreads</u>.**
- **Same approach can be conducted with the *credit contingent contract* to be hedged.**
  - All the <u>computations</u> are ***model dependent***.
- ***Several maturities* of underlying default swaps can be used to match sensitivities.**
  - For example, in the case of **defaultable** interest rate swap, the nominal amount of default swaps $(PV_\tau)^+$ is usually small.
  - ***Single*** default swap with nominal $(PV_\tau)^+$ has a ***smaller sensitivity*** to <u>credit spreads</u> than ***defaultable interest rate swap***, even for long maturities.
  - **<u>Short</u>** and **<u>long</u>** positions in default swaps are required to hedge ***credit spread risk***.

Explaining theta effects with and without hedging

- ***Different** **aspects*** **of "carrying" credit contracts through time.**
  - **Assume "historical" and "risk-neutral" intensities are equal.**
- **Consider a *short* position in a credit contingent contract.**
- **Present value of the deal provided by:**

$$PV(u) = E_u\left[\int_u^T \left(\exp - \int_u^t (r+\lambda)(s)ds\right) \times \left(p_T - \lambda(t)C(t)\right)dt\right]$$

- **(after computations) *Net expected capital gain*:**

$$E_u\left[PV(u+du) - PV(u)\right] = \left(r(u)+\lambda(u)\right)PV(u)du + \left(\lambda(u)C(u) - p_T\right)du$$

- ***Accrued cash-flows* (received premiums):** $p_T du$
  - **By summation, Incremental P&L (if no default between *u* and *u+du*):**

$$r(u)PV(u)du + \lambda(u)\left(C(u) + PV(u)\right)du$$

Explaining theta effects with and without hedging

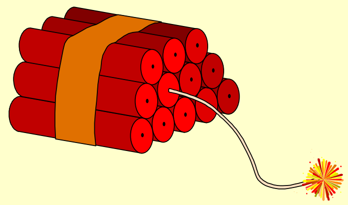
- **<u>Apparent</u> *extra return* effect :** $\lambda(u)\left(C(u)+PV(u)\right)du$
  - **But, probability of default between *u* and *u*+*du*: $\lambda(u)du$.**
  - **Losses in case of default:**
    - Commitment to pay: $C(u)$
    - Loss of PV of the credit contract: PV(u)
    - PV(u) consists in **<u>unrealised</u>** capital gains or losses in the credit derivatives book that "disappear" in case of default.
  - **Expected loss charge:** $\lambda(u)\left(C(u)+PV(u)\right)du$
- **Hedging aspects:**
  - **If we hold $C(u)+PV(u)$ short-term digital default swaps, we are protected at default-time (no jump in the P&L).**
  - **Premiums to be paid:** $\lambda(u)\left(C(u)+PV(u)\right)du$
  - **Same average rate of return, but smoother variations of the P&L.**

Hedging Default Risk in Basket Default Swaps

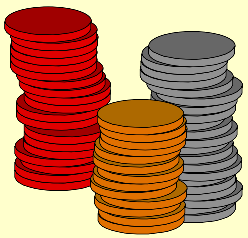
- **Example: first to default swap from a basket of two risky bonds.**
  - **If the first default time occurs before maturity,**
  - **The seller of the first to default swap pays the non recovered fraction of the defaulted bond.**
  - **Prior to that, he receives a periodic premium.**
- **Assume that the two bonds cannot default simultaneously**
  - **We moreover assume that default on one bond has *no effect* on the credit spread of the remaining bond.**
- **How can the seller be protected *at default time* ?**
  - **The only way to be protected at default time is to hold two default swaps with the *same nominal* than the *nominal* of the bonds.**
  - **The *maturity* of underlying default swaps does not matter.**

Real World hedging and risk-management issues

- **uncertainty at default time**
  - **illiquid default swaps**
  - **recovery risk**
  - **simultaneous default events**



- **Managing net premiums**
  - **Maturity of underlying default swaps**
  - **Lines of credit**
  - **Management of the carry**
  - **Finite maturity and discrete premiums**
  - **Correlation between hedging cash-flows and financial variables**



Real world hedging and risk-management issues
Case study : hedge ratios for first to default swaps

- **Consider a first to default swap associated with a basket of two defaultable loans.**
  - **Hedging portfolios based on standard underlying default swaps**
  - **Uncertain hedge ratios if:**
    - simultaneous default events
    - *Jumps* of credit spreads at default times
- **Simultaneous default events:**
  - **If counterparties default *altogether*, holding the *complete* set of default swaps is a conservative (and thus expensive) hedge.**
  - **In the *extreme* case where default *always* occur altogether, we only need a single default swap on the loan with largest nominal.**
  - **In other cases, holding a *fraction* of underlying default swaps does not hedge default risk (if *only one* counterparty defaults).**

Real world hedging and risk-management issues
Case study : hedge ratios for first to default swaps

- **What occurs if there is a *jump in the credit spread* of the second counterparty after default of the first ?**
  - **default of first counterparty means *bad news* for the second.**
- **If hedging with short-term default swaps, no capital gain at default.**
  - **Since PV of short-term default swaps is not *sensitive* to credit spreads.**
- **This is not the case if hedging with long term default swaps.**
  - **If credit spreads jump, PV of long-term default swaps jumps.**
- **Then, the amount of hedging default swaps can be reduced.**
  - **This reduction is *model-dependent*.**

On the edge of completeness ?

- **Firm-value structural default models:**
  - **Stock prices follow a diffusion processes (no jumps).**
  - **Default occurs at first time the stock value hits a barrier**
- ***In this modelling*, default credit derivatives can be completely hedged by trading the stocks:**
  - **"*Complete*" pricing and hedging model:**
- **Unrealistic features for hedging *basket default swaps*:**
  - **Because default times are predictable, *hedge ratios are close to zero* except for the counterparty with the smallest "distance to default".**

On the edge of completeness ?
<u>hazard rate</u> based models

- **In <u>hazard</u> <u>rate</u> based models :**
  - **default is a sudden, *non predictable* event,**
  - **that causes a sharp <u>jump</u> in defaultable bond prices.**
  - **Most credit contingent contracts and basket default derivatives have payoffs that are *linear* in the prices of defaultable bonds.**
  - **Thus, good news: default risk can be *hedged*.**
  - **Credit spread risk can be *substantially reduced* but not completely eliminated.**
  - **More <u>realistic</u> approach to default.**
  - ***Hedge ratios* are <u>robust</u> with respect to default risk.**

On the edge of completeness
Conclusion

- **Looking for a better understanding of credit derivatives**
  - **payments in case of default,**
  - **volatility of credit spreads.**
- **Bridge between risk-neutral valuation and the cost of the hedge approach.**
- **dynamic hedging strategy based on *standard default swaps*.**
  - **hedge ratios in order to get protection at default time.**
  - **hedging default risk is *model-independent*.**
  - **importance of quantitative models for a better management of the P&L and the residual premiums.**