Hedging Default Risks of CDOs in Markovian Contagion Models

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Presentation related to the paper
***Hedging default risks of CDOs in Markovian contagion models* (2007)**
Available on www.defaultrisk.com
to be updated soon

Overview

- **Business context**
  - **Credit and liquidity crisis**
- **Scope of the paper**
  - **Hedging CDO tranches in a complete market framework**
- **Risks at hand in CDO tranches**
  - **Default, credit spreads, dependence assumptions, recovery rates**
- **Risk Management Paradigms**
  - **In the CDO market**
- **Tree approach to hedging defaults**
  - **From theoretical ideas**
  - **To practical implementation of hedging strategies**

Business context

- **We are in the middle of a major credit and liquidity crisis**
  - **Surge in credit spreads**
  - **liquidity crunch in interbank money markets**
  - **Huge losses recorded in many major banks**
    - Raises serious doubts about the risk management processes
    - Collapses of Northenrock, IKB, Countrywide Financial, Bear Stearns
  - **Downgrading of monoline insurers (AMBAC, MBIA)**
  - **Soundness of Freddy Mac and Fannie Mae?**
  - **Private equity/ LBO nosedive:**
    - Blackstone, Carlyle
  - **Fed activism**

Business context

- **Issues about the lending process (subprime borrowers) and loan securitization**
  - **Is risk screening (FICO scores) and monitoring of credit risk efficient?**
  - **Does the securitization process enhances systemic risk and contagion effects?**
  - **Or did it avoid an even more acute crisis thanks to diversification?**
- **Regulation crisis**
  - **Liquidity management of SIV**
  - **Basel II and bank supervision in the US**
  - **Teaser rates**
  - **Capital requirements for hedge funds, SIV**
  - **Overuse of fair value accounting in illiquid markets**

Business context

- **A wide range of structured products**
  - **Wide range of loans and bonds involved**
    - Home and personal loans, corporate bonds
    - Investment grade, high yield
  - **Wide range of structures**
    - CDS, CDOs, LCDS, LCDO, CDO of ABS, CDO^2
    - Cash or synthetic CDOs, funded or unfunded
    - Bespoke CDOs or based on standard indices (CDX, iTraxx)
- **Illiquidity of structured products**
  - **Including well rated tranches**
  - **Doubts about the rating agencies process**
  - **Questions about the mark to market of complex products**
  - **Misuse of quantitative models?**

Scope of the paper

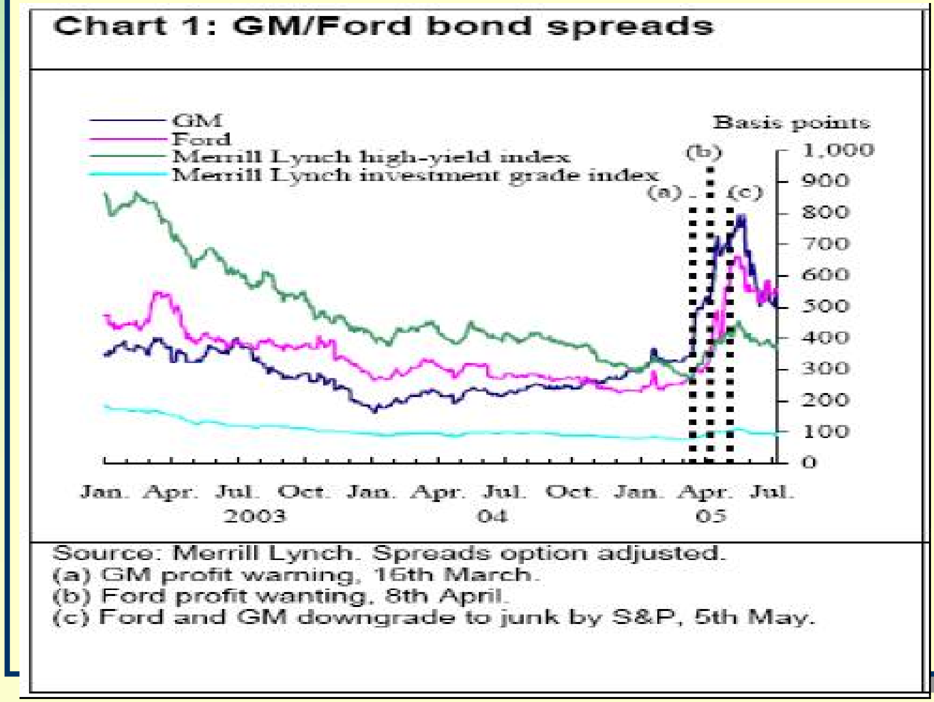
- **Risk management of standardized tranches on the iTraxx and CDX indices**
  - **The most liquid part of the CDO market**
    - A wide number of trading firms and end-users
    - CDS on underlying names are actively traded
    - Credit default swap index can also be used as a hedging tool
  - **Asymmetric information issues are not of first importance**
- **We will assume a reasonable understanding of**
  - **main market features**
  - **the one factor Gaussian copula benchmark pricing model**

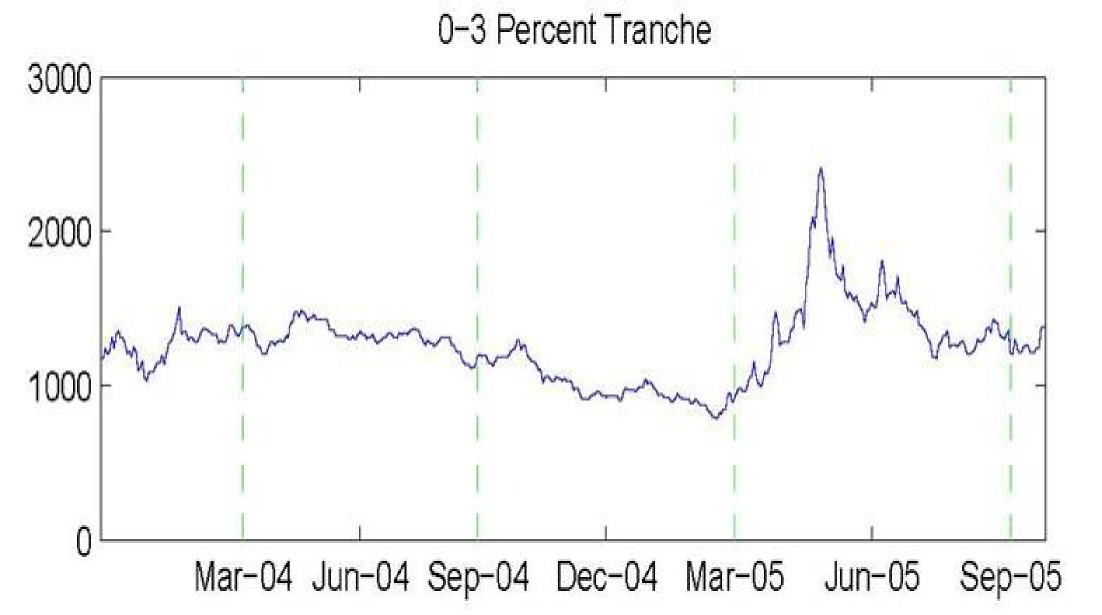
Risks at hand in CDO tranches

- Default risk
  - Default bond price jumps to recovery value at default time.
  - Drives the CDO cash-flows
- Credit spread risk
  - Changes in defaultable bond prices prior to default
    - ➢ Due to shifts in credit quality or in risk premiums
  - Changes in the marked to market of tranches
- Interactions between credit spread and default risks
  - Increase of credit spreads increases the probability of future defaults
  - Arrival of defaults may lead to jump in credit spreads
    - ➢ Contagion effects (Jarrow & Yu)
    - ➢ Enron failure was informative
    - ➢ Not consistent with the “conditional independence” assumption

Risks at hand in CDO tranches

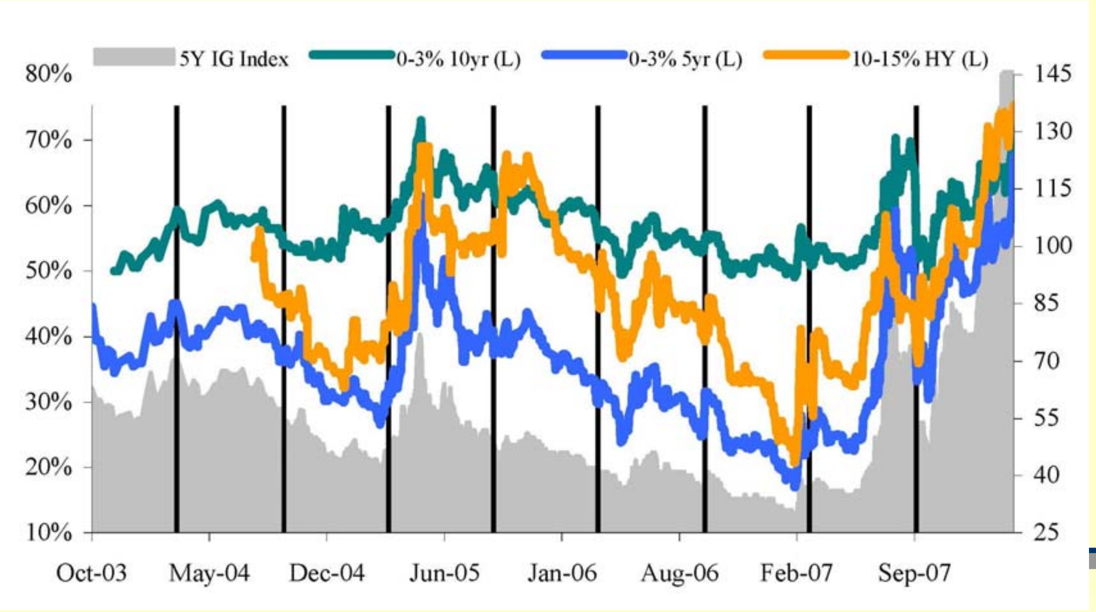
- Credit spread risk (following)
  - Idiosyncratic shift of a credit spread of a given name
    - ➢ Correlation crisis in May 2005 due to Ford and GM downgrades
    - ➢ Increase in the heterogeneity of the reference credit portfolio
    - ➢ Increase in equity tranche premiums





Risks at hand in CDO tranches

- **Parallel shifts in credit spreads**
  - ➢ As can be seen from the current crisis
  - ➢ On March 10, 2008, the 5Y CDX IG index spread quoted at 194 bp pa
  - ➢ starting from 30 bp pa on February 2007
    - – **See grey figure**
  - ➢ this is also associated with a surge in equity tranche premiums



Risks at hand in CDO tranches

- **Changes in the dependence structure between default times**
  - **In the Gaussian copula world, change in the correlation parameters in the copula**
  - **The present value of the default leg of an equity tranche decreases when correlation increases**
- **Dependence parameters and credit spreads may be highly correlated**

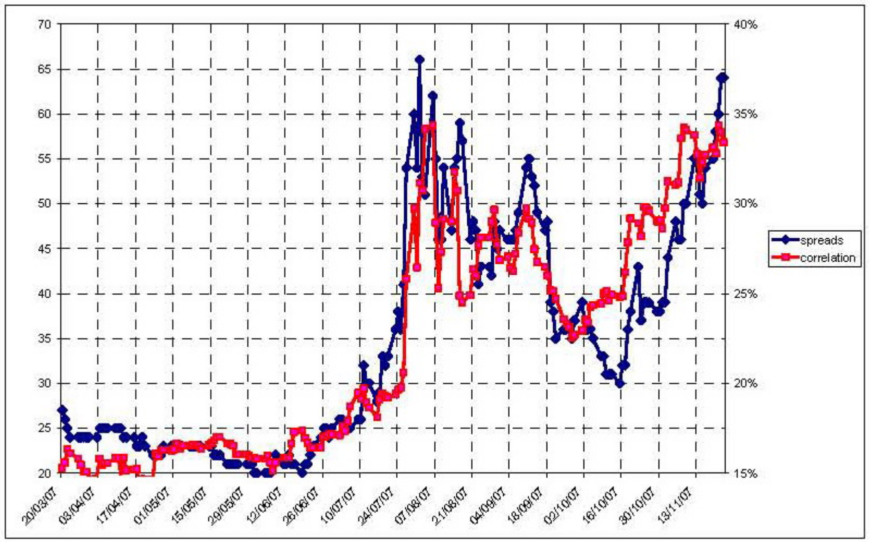
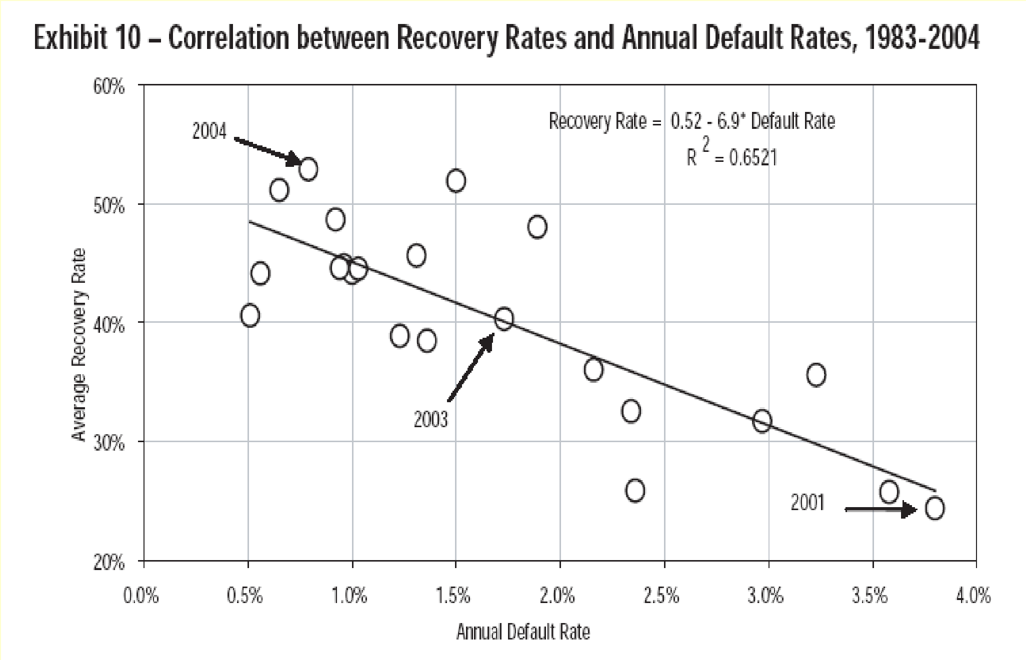


Figure 9. Credit spreads on the five years iTraxx index (Series 7) in bps on the left axis. Implied correlation on the equity tranche on the right axis

Risks at hand in CDO tranches

- **Recovery rates**
  - **Market agreement of a fixed recovery rate of 40% is inadequate**



  - **Currently a major issue in the CDX market**
    - ➢See following graph
    - ➢ Base correlations over 100% for super senior tranches...

Risks at hand in CDO tranches

Morgan Stanley

MORGAN STANLEY RESEARCH

March 10, 2008
Structured Credit Analytics

CDX and iTraxx – Correlation Analysis and Delta Neutral Return

CDX Series 9

Index	Tranche	Bid	Ask	Mid 1 Week Change	Mid 1 Month Change	Index Level	Delta	Detach Corr.[1]	Corr. Skew	1 Week Change	Corr. Skew Z Score	Corr. Skew Rel. Value	CDX vs. iTRAXX Corr Skew	Value vs. iTraxx	Delta Neutral Returns 1 Week	Delta Neutral Returns 1 Month	Delta Neutral Returns 1 Year
5yr	Index	185.0	185.0	25.0	69.0												
5yr[2]	0-3%	67.5%	68.5%	10.4%	13.0%	185 bp	2.7	38%		-7.4%	1.19	RICH	(1.10)	CHEAP	-6.6%	0.6%	50.0%
5yr	3-7%	731	738	138	219		3.0	64%	25%	2.7%	1.07	RICH	0.97	FAIR	-2.1%	3.2%	9.0%
5yr	7-10%	405	415	75	141		1.8	73%	9%	0.5%	(0.31)	FAIR	0.21	FAIR	-1.2%	1.4%	-4.7%
5yr	10-15%	207	214	12	73		1.3	87%	14%	2.1%	1.15	RICH	0.96	FAIR	0.6%	1.2%	-1.8%
5yr	15-30%	126	130	13	62		1.0	108%	21%	1.2%	(0.01)	FAIR	0.18	FAIR	0.4%	-0.3%	-1.3%
5yr	30-100%	78	80	10	45		0.7								0.3%	-0.6%	-1.6%
7yr	Index	178.0	178.0	20.0	62.0												
7yr[2]	0-3%	71.0%	72.0%	8.0%	11.3%	178 bp	2.0	39%		-4.2%	1.38	RICH			-5.0%	1.8%	42.9%
7yr	3-7%	785	795	130	193		3.2	62%	23%	1.6%	1.00	FAIR			-2.6%	5.7%	18.2%
7yr	7-10%	452	460	79	106		2.5	71%	8%	0.1%	(0.66)	FAIR			-1.8%	5.4%	-2.8%
7yr	10-15%	256	265	21	73		1.6	83%	12%	1.6%	(0.09)	FAIR			0.5%	3.0%	-2.0%
7yr	15-30%	139	144	14	65		1.1	104%	21%	0.3%	(0.73)	FAIR			0.3%	-0.1%	-1.7%
7yr	30-100%	81	83	12	48		0.7								0.1%	-0.9%	-2.1%
10yr	Index	174.0	174.0	18.0	57.0												
10yr[2]	0-3%	74.0%	74.8%	7.9%	9.9%	174 bp	1.5	39%		-5.5%	1.33	RICH	1.31	RICH	-5.2%	1.5%	31.6%
10yr	3-7%	910	920	110	204		3.4	57%	18%	3.3%	3.13	RICH	3.16	RICH	-1.6%	7.5%	37.4%
10yr	7-10%	523	533	67	98		2.5	64%	7%	0.4%	(0.64)	FAIR	(0.64)	FAIR	-0.9%	8.5%	1.8%
10yr	10-15%	300	308	22	95		1.8	76%	12%	1.1%	(0.60)	FAIR	(0.60)	FAIR	1.0%	2.6%	3.0%
10yr	15-30%	150	155	14	63		1.2	99%	23%	0.3%	(1.99)	CHEAP	(2.01)	CHEAP	0.5%	0.0%	-1.6%
10yr	30-100%	79	82	10	45		0.7								0.2%	-1.2%	-2.9%
HY	Index	718.8	718.8	23.6	83.1												
HY[3]	0-10%	92.1%	92.6%	1.6%	2.5%	719 bp	0.7	44%		-2.4%	1.05	RICH			-1.1%	-0.4%	10.2%
HY	10-15%	75.3%	75.8%	2.5%	6.1%		1.8	49%	5%	1.2%	(0.12)	FAIR			-1.4%	-1.4%	8.0%
HY	15-25%	1,290	1,305	81	222		2.3	72%	23%	1.4%	2.73	RICH			-1.0%	-1.5%	1.6%
HY	25-35%	680	695	48	145		1.5	92%	19%	0.2%	(0.21)	FAIR			-0.6%	-1.6%	-6.3%
HY	35-100%	202	208	(2)	32		0.6								0.5%	0.0%	-2.1%
LCDX	Index	440.0	440.0	7.8	29.6												
LCDX[3]	0-5%	86.6%	87.5%	3.9%	-0.2%	440 bp	1.6	78%		-6.2%					-3.5%	3.0%	
LCDX[3]	5-8%	66.9%	67.6%	4.3%	1.0%		2.7								-3.6%	3.6%	
LCDX	8-12%	1140.0	1152.0	30.0	35.0		3.5								0.0%	3.0%	
LCDX	12-15%	740.0	748.0	-7.0	107.0		2.4								0.9%	-1.4%	
LCDX	15-100%	206.0	210.0	-13.5	20.0		0.6								0.7%	-0.8%	

[1] Correlation of tranche with 0% attachment and the same detachment point as the benchmark tranche, implied from market prices of benchmark tranches
[2] Points upfront plus 500 bp running
[3] Points upfront plus 0 bp running
Source: Morgan Stanley

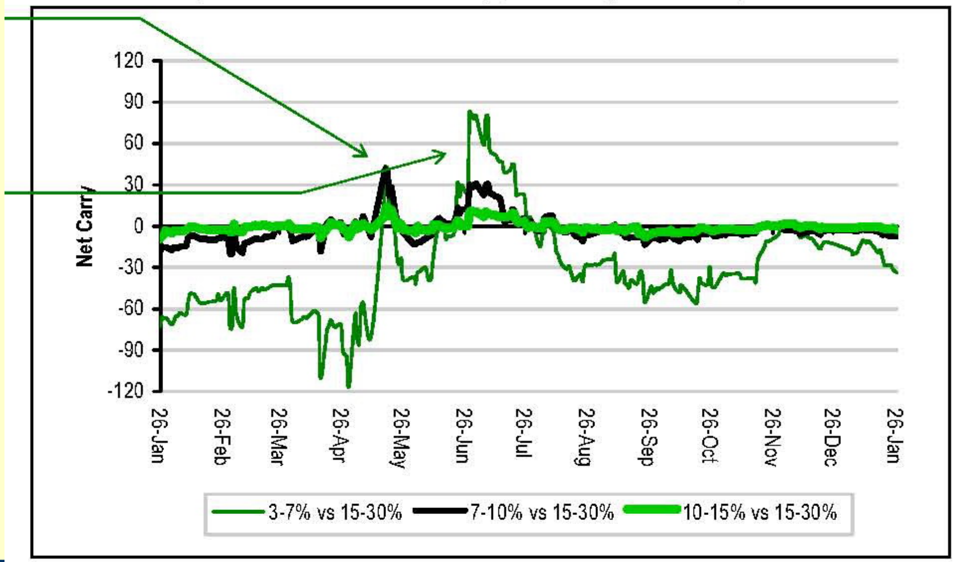
Risk Management Paradigms

- Static hedging
- Buy a portfolio of credits, split it into tranches and sell the tranches to investors
  - No correlation or model risk for market makers
  - No need to dynamically hedge with CDS
- Only « budget constraint »:
  - Sum of the tranche prices greater than portfolio of credits price
  - Similar to stripping ideas for Treasury bonds
- <u>No clear idea of relative value of tranches</u>
  - Depends of demand from investors
  - Markets for tranches might be segmented
  - Especially in turmoil times

Risk Management Paradigms

- Relative value deals may lead to an integrated tranche market
  - Trading across the capital structure
  - Example : “positive carry straddle trade”
    - ➢ Sell protection on the CDX.NA.IG [0-3%] and buy protection on the [7-10%] tranche
    - ➢ Delta neutral with respect to shifts in credit spreads
- Depends on the presence of « arbitrageurs »
  - Investors with small risk aversion
    - ➢ Trading floors, hedge funds
  - Unwinding such trades…
  - May lead to market breakdowns

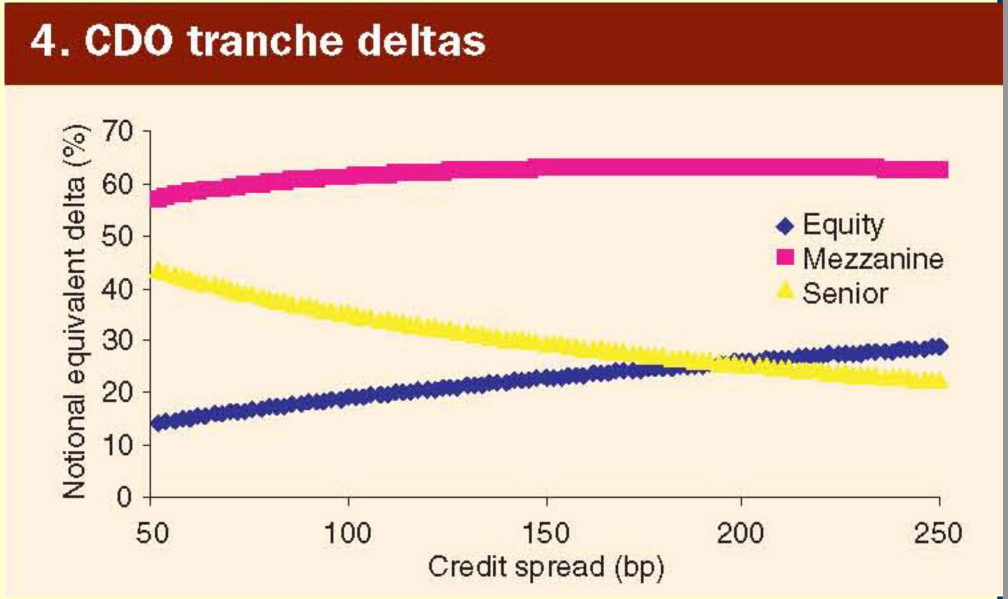
Chart 62: CDX 5y Short Correlation Net Carry (Delta-hedged tranches)



Source: Merrill Lynch

Risk Management Paradigms

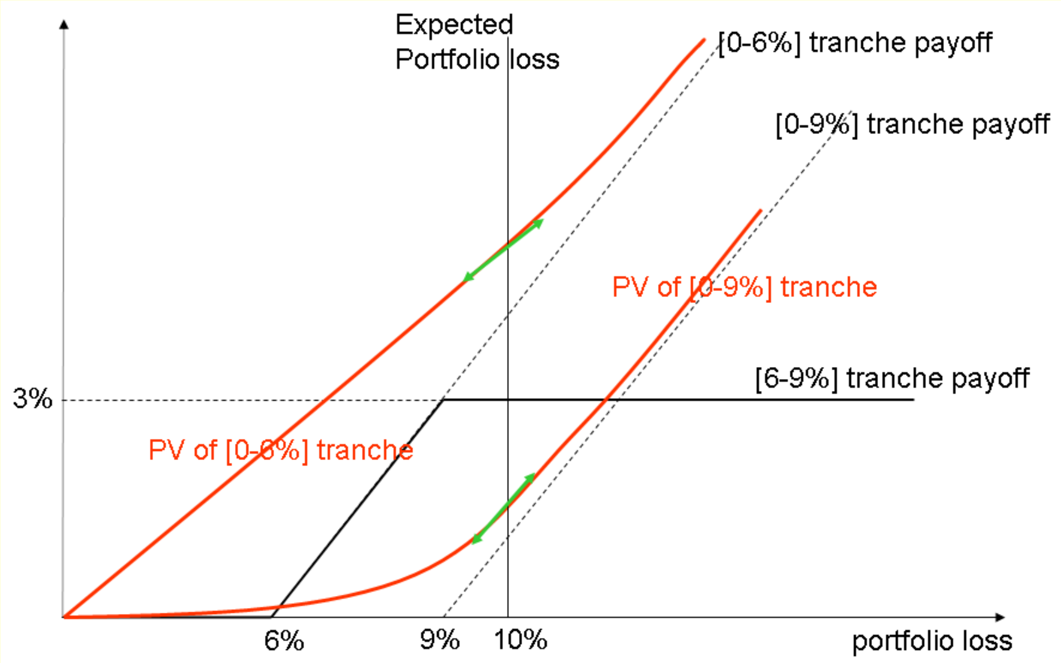
- The decline of the one factor Gaussian copula model + base correlation
- CDS hedge ratios are computed by bumping the marginal credit curves
  - Focus on credit spread risk



- Poor theoretical properties
  - Does not lead to a replication of CDO tranche payoffs
  - Not a hedge against defaults…
  - Unclear issues with respect to the management of correlation risks

Risk Management Paradigms

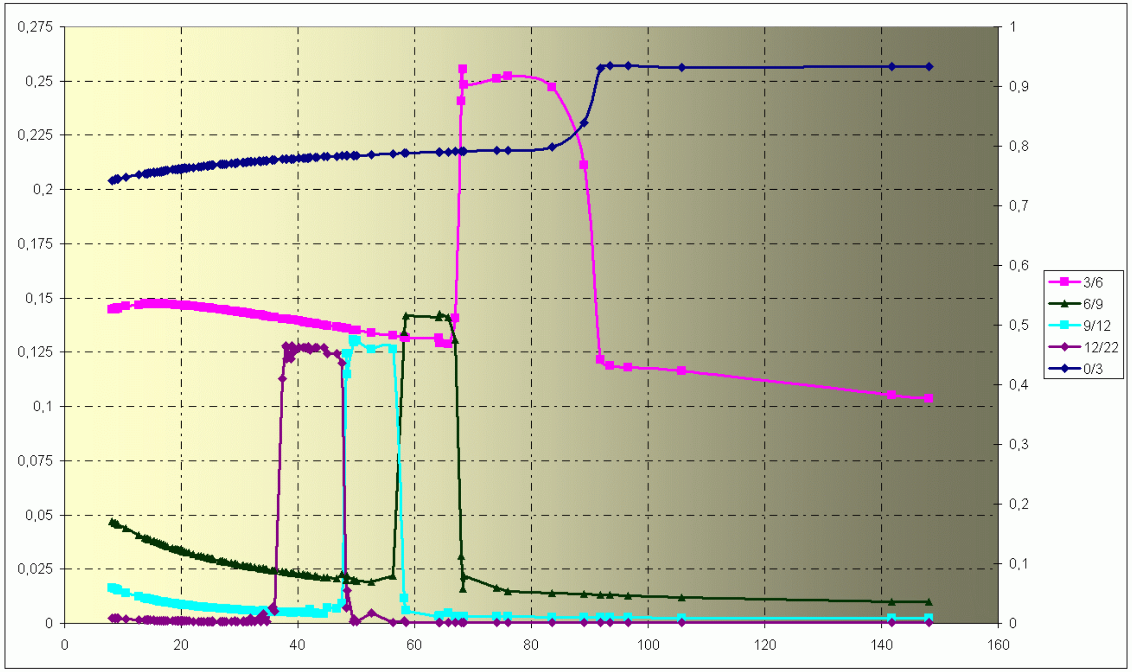
- The decline of the one factor Gaussian copula model + base correlation (following)
  - This is rather a practical than a theoretical issue
- Negative tranche deltas frequently occur
  - Especially with steep base correlations curves



  - Which is rather unlikely for out of the money call spreads

Risk Management Paradigms

- Decline of the one factor Gaussian copula model
- Credit deltas in “high correlation states”
  - Close to comonotonic default dates (current market situation)
  - Deltas are equal to zero or one depending on the level of spreads
    - ➢Individual effects are too pronounced
    - ➢Unrealistic gammas



Risk Management Paradigms

- The ultimate step : complete markets
  - As many risks as hedging instruments
  - News products are only designed to save transactions costs and are used for risk management purposes
  - Assumes a high liquidity of the market
- Perfect replication of payoffs by dynamically trading a small number of « underlying assets »
  - Black-Scholes type framework
  - Possibly some model risk
- This is further investigated in the presentation
  - Dynamic trading of CDS to replicate CDO tranche payoffs

Tree approach to hedging defaults

- What are we trying to achieve?
- Show that under some (stringent) assumptions the market for CDO tranches is complete
  - CDO tranches can be perfectly replicated by dynamically trading CDS
  - Exhibit the building of the unique risk-neutral measure
- Display the analogue of the local volatility model of Dupire or Derman & Kani for credit portfolio derivatives
  - One to one correspondence between CDO tranche quotes and model dynamics
- Show the practical implementation of the model with market data
  - Deltas correspond to “sticky implied tree”

Tree approach to hedging defaults

- Main theoretical features of the complete market model
  - No simultaneous defaults
    - **Unlike multivariate Poisson models**
  - Credit spreads are driven by defaults
    - Contagion model
      - **Jumps in credit spreads at default times**
    - Credit spreads are deterministic between two defaults
  - Bottom-up approach
    - Aggregate loss intensity is derived from individual loss intensities
  - Correlation dynamics is also driven by defaults
    - Defaults lead to an increase in dependence

Tree approach to hedging defaults

- Without additional assumptions the model is intractable
  - Homogeneous portfolio
    - Only need of the CDS index
    - No individual name effect
    - Top-down approach
      - **Only need of the aggregate loss dynamics**
  - Markovian dynamics
    - Pricing and hedging CDO tranches within a binomial tree
    - Easy computation of dynamic hedging strategies
  - Perfect calibration the loss dynamics from CDO tranche quotes
    - Thanks to forward Kolmogorov equations
  - Practical building of dynamic credit deltas
  - Meaningful comparisons with practitioner's approaches

Tree approach to hedging defaults

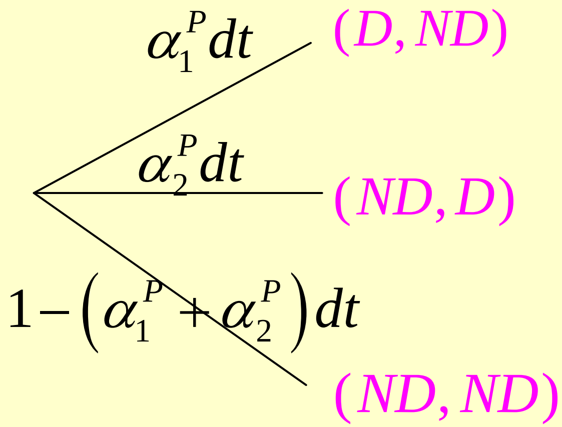
- We will start with two names only
- Firstly in a static framework
  - Look for a First to Default Swap
  - Discuss historical and risk-neutral probabilities
- Further extending the model to a dynamic framework
  - Computation of prices and hedging strategies along the tree
  - Pricing and hedging of tranchelets
- Multiname case: homogeneous Markovian model
  - Computation of risk-neutral tree for the loss
  - Computation of dynamic deltas
- Technical details can be found in the paper:
  - “hedging default risks of CDOs in Markovian contagion models”

Tree approach to hedging defaults

- Some notations :
  - τ_1, τ_2 default times of counterparties 1 and 2,
  - $\mathcal{H}_t$ available information at time t,
  - P historical probability,
  - α_1^P, α_2^P : (historical) default intensities:
    - $P\left[\tau_i \in [t, t+dt[\,\middle|\, H_t\right] = \alpha_i^P dt,\ i = 1,2$
- Assumption of « local » independence between default events
  - Probability of 1 and 2 defaulting altogether:
    - $P\left[\tau_1 \in [t, t+dt[, \tau_2 \in [t, t+dt[\,\middle|\, H_t\right] = \alpha_1^P dt \times \alpha_2^P dt \text{ in } (dt)^2$
  - Local independence: simultaneous joint defaults can be neglected

Tree approach to hedging defaults

- Building up a tree:
  - Four possible states: (*D*,*D*), (*D*,*ND*), (*ND*,*D*), (*ND*,*ND*)
  - Under no simultaneous defaults assumption $p_{(D,D)}=0$
  - Only three possible states: (*D*,*ND*), (*ND*,*D*), (*ND*,*ND*)
  - Identifying (historical) tree probabilities:



$$\begin{cases} p_{(D,D)}=0 \Rightarrow p_{(D,ND)}=p_{(D,D)}+p_{(D,ND)}=p_{(D,.)}=\alpha_1^P dt \\ p_{(D,D)}=0 \Rightarrow p_{(ND,D)}=p_{(D,D)}+p_{(ND,D)}=p_{(.,D)}=\alpha_2^P dt \\ p_{(ND,ND)}=1-p_{(D,.)}-p_{(.,D)} \end{cases}$$

Tree approach to hedging defaults

- Stylized cash flows of short term digital CDS on counterparty 1:
  - $\alpha_1^Q dt$ CDS 1 premium

0

$\alpha_1^P dt$ $\quad 1-\alpha_1^Q dt \quad (D, ND)$

$\alpha_2^P dt$ $\quad -\alpha_1^Q dt \quad (ND, D)$

$1-\left(\alpha_1^P+\alpha_2^P\right)dt$ $\quad -\alpha_1^Q dt \quad (ND, ND)$

- Stylized cash flows of short term digital CDS on counterparty 2:

0

$\alpha_1^P dt$ $\quad -\alpha_2^Q dt \quad (D, ND)$

$\alpha_2^P dt$ $\quad 1-\alpha_2^Q dt \quad (ND, D)$

$1-\left(\alpha_1^P+\alpha_2^P\right)dt$ $\quad -\alpha_2^Q dt \quad (ND, ND)$

Tree approach to hedging defaults

- Cash flows of short term digital first to default swap with premium $\alpha_F^Q dt$:

$$
0 \quad
\begin{cases}
\alpha_1^P dt & 1-\alpha_F^Q dt \quad (D, ND) \\
\alpha_2^P dt & 1-\alpha_F^Q dt \quad (ND, D) \\
1-\left(\alpha_1^P+\alpha_2^P\right)dt & -\alpha_F^Q dt \quad (ND, ND)
\end{cases}
$$

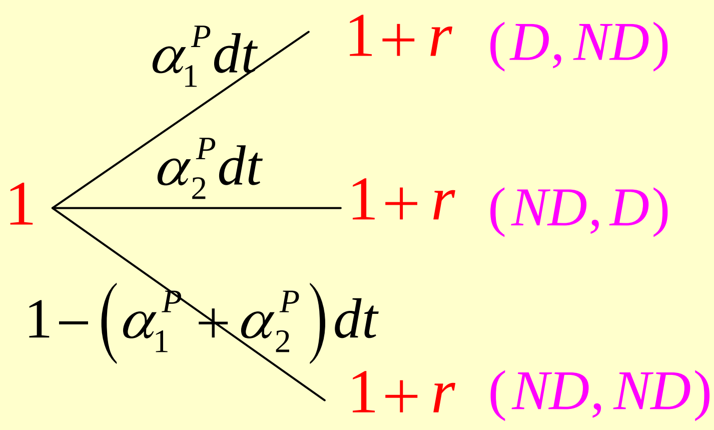
- Cash flows of holding CDS 1 + CDS 2:

$$
0 \quad
\begin{cases}
\alpha_1^P dt & 1-\left(\alpha_1^Q+\alpha_2^Q\right)dt \quad (D, ND) \\
\alpha_2^P dt & 1-\left(\alpha_1^Q+\alpha_2^Q\right)dt \quad (ND, D) \\
1-\left(\alpha_1^P+\alpha_2^P\right)dt & -\left(\alpha_1^Q+\alpha_2^Q\right)dt \quad (ND, ND)
\end{cases}
$$

- Perfect hedge of first to default swap by holding 1 CDS 1 + 1 CDS 2
  - Delta with respect to CDS 1 = 1, delta with respect to CDS 2 = 1

Tree approach to hedging defaults

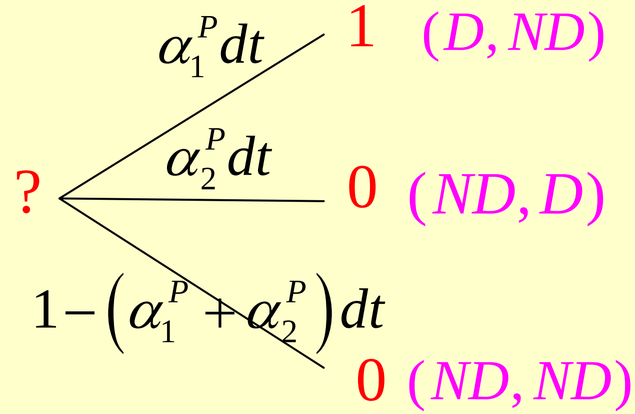
- Absence of arbitrage opportunities imply:
  - $\alpha_F^Q = \alpha_1^Q + \alpha_2^Q$
- Arbitrage free first to default swap premium
  - Does not depend on historical probabilities α_1^P, α_2^P
- Three possible states: (*D*,*ND*), (*ND*,*D*), (*ND*,*ND*)
- Three tradable assets: CDS1, CDS2, risk-free asset

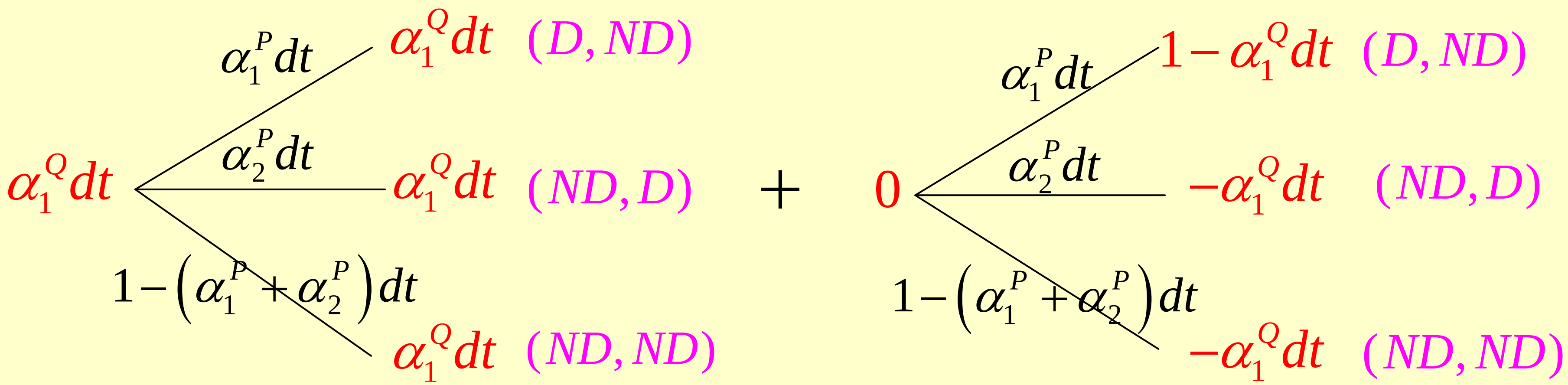


- For simplicity, let us assume $r = 0$

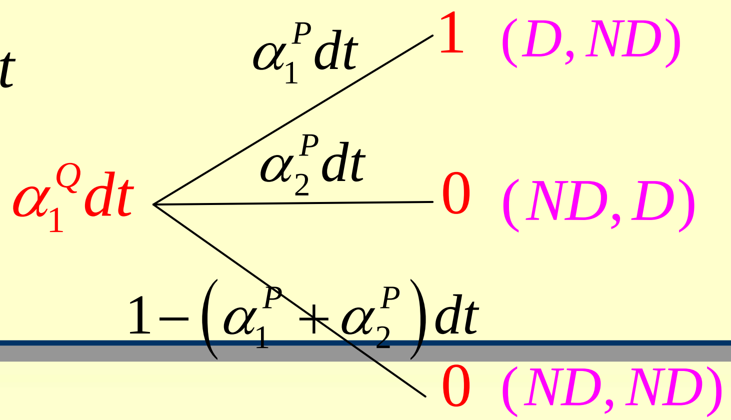
Tree approach to hedging defaults

- Three state contingent claims
  - Example: claim contingent on state (D, ND)
  - Can be replicated by holding
  - 1 CDS 1 + $\alpha_1^Q dt$ risk-free asset



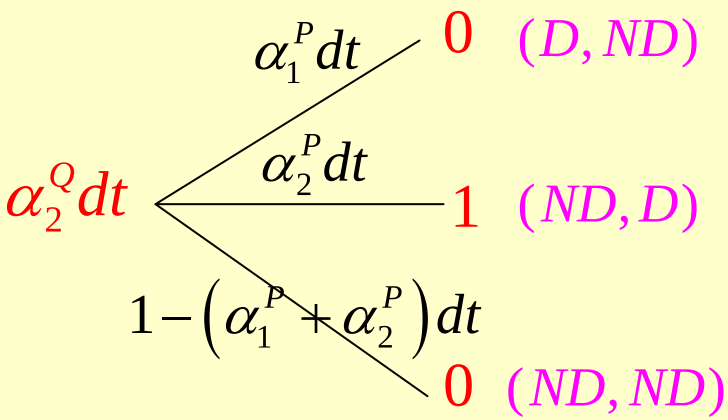


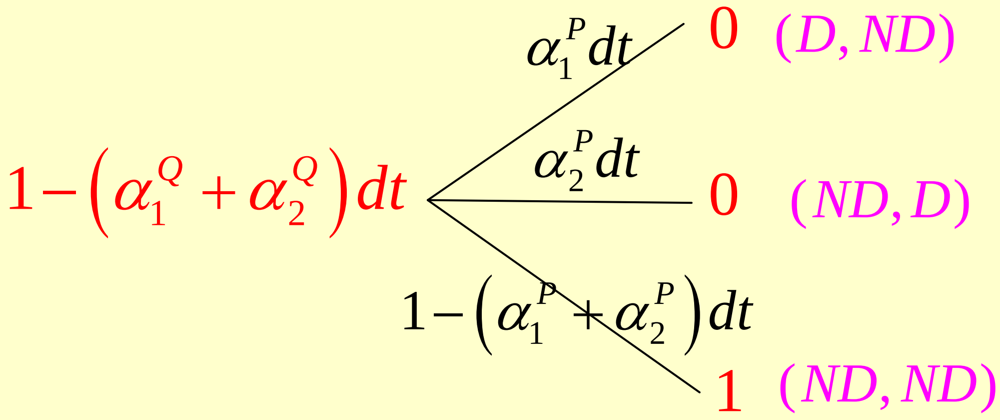
  - Replication price = $\alpha_1^Q dt$



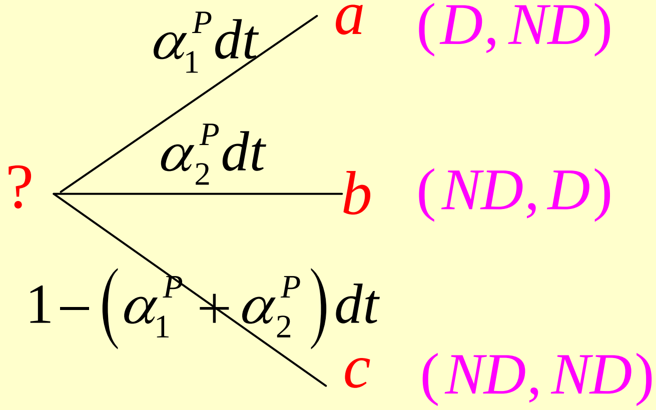
Tree approach to hedging defaults

- Similarly, the replication prices of the (ND, D) and (ND, ND) claims





- Replication price of:

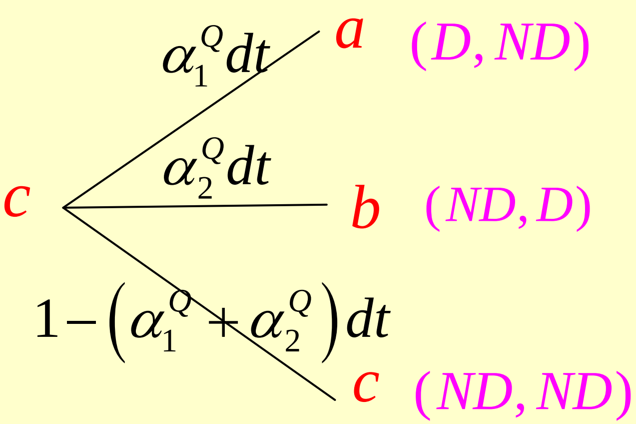


- Replication price = $\alpha_1^Q dt \times a + \alpha_2^Q dt \times b + \left(1-(\alpha_1^Q+\alpha_2^Q)dt\right)c$

Tree approach to hedging defaults

- Replication price obtained by computing the expected payoff
  - Along a risk-neutral tree

$$\alpha_1^Q dt \times a + \alpha_2^Q dt \times b + \left(1-(\alpha_1^Q + \alpha_2^Q)dt\right)c$$



- Risk-neutral probabilities
  - Used for computing replication prices
  - Uniquely determined from short term CDS premiums
  - No need of historical default probabilities

Tree approach to hedging defaults

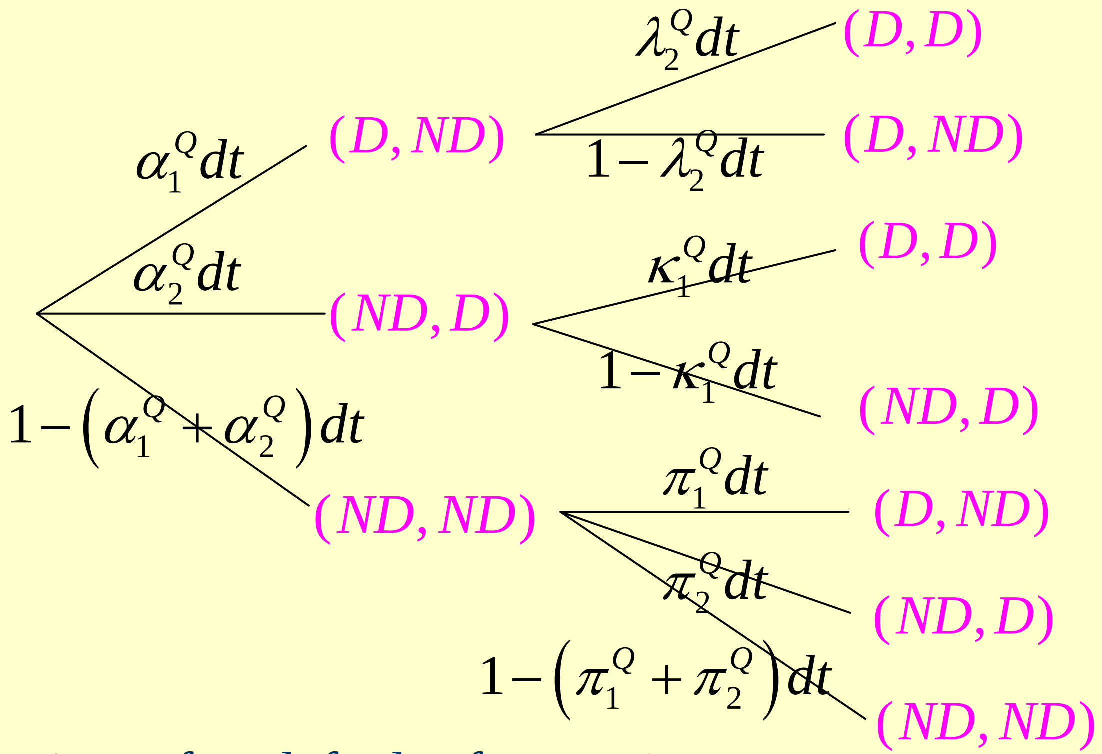
- Computation of deltas
  - Delta with respect to CDS 1: δ_1
  - Delta with respect to CDS 2: δ_2
  - Delta with respect to risk-free asset: p
    - p also equal to up-front premium

$$\begin{cases} a = p + \delta_1 \times \overbrace{\left(1 - \alpha_1^Q dt\right)}^{\text{payoff CDS 1}} + \delta_2 \times \overbrace{\left(-\alpha_2^Q dt\right)}^{\text{payoff CDS 2}} \\ b = p + \delta_1 \times \left(-\alpha_1^Q dt\right) + \delta_2 \times \left(1 - \alpha_2^Q dt\right) \\ c = p + \delta_1 \times \underbrace{\left(-\alpha_1^Q dt\right)}_{\text{payoff CDS 1}} + \delta_2 \times \underbrace{\left(-\alpha_2^Q dt\right)}_{\text{payoff CDS 2}} \end{cases}$$

  - As for the replication price, deltas only depend upon CDS premiums

Tree approach to hedging defaults

- Dynamic case:



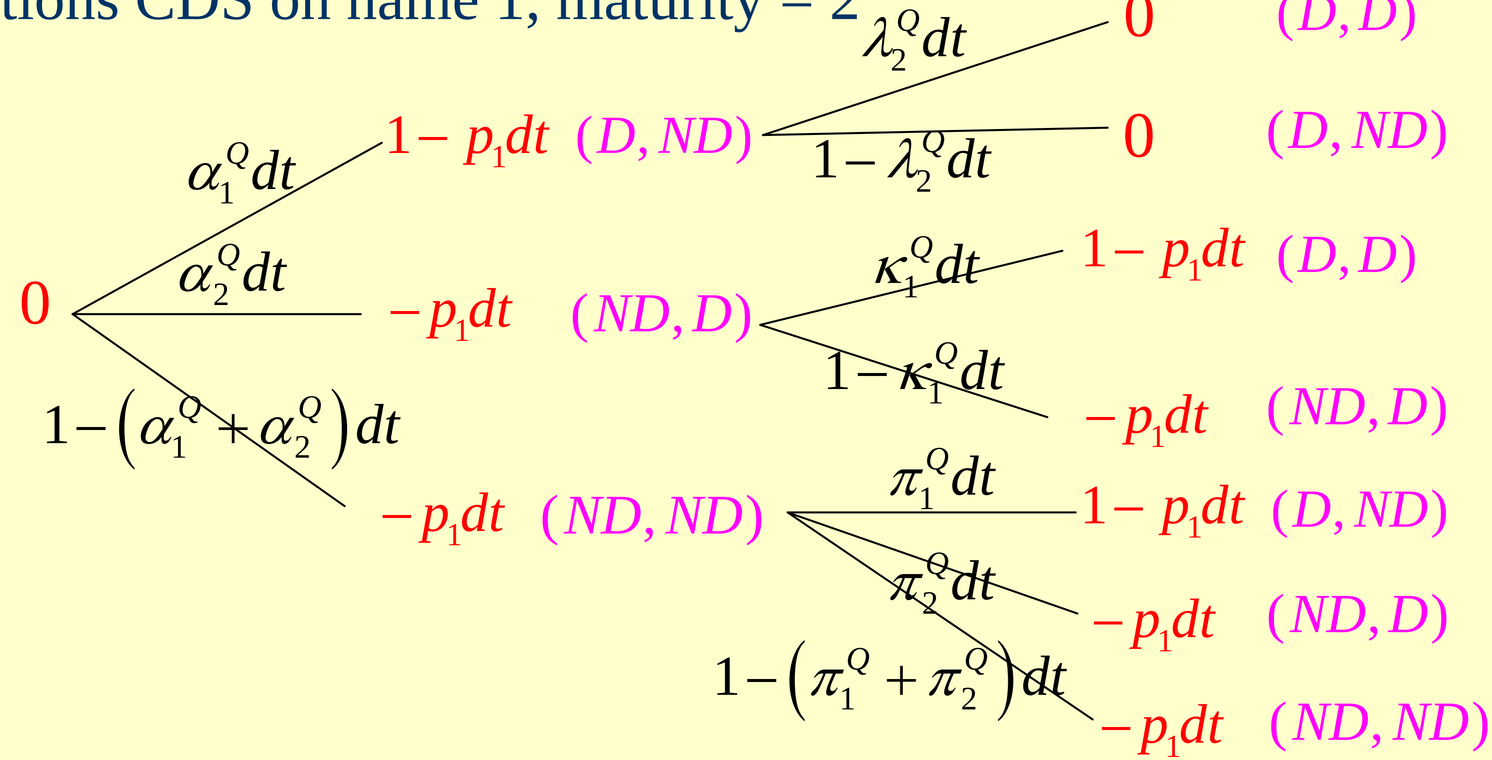
  - $\lambda_2^Q dt$ CDS 2 premium after default of name 1
  - $\kappa_1^Q dt$ CDS 1 premium after default of name 2
  - $\pi_1^Q dt$ CDS 1 premium if no name defaults at period 1
  - $\pi_2^Q dt$ CDS 2 premium if no name defaults at period 1
- Change in CDS premiums due to contagion effects
  - Usually, $\pi_1^Q < \alpha_1^Q < \kappa_1^Q$ and $\pi_2^Q < \alpha_2^Q < \lambda_2^Q$

Tree approach to hedging defaults

- Computation of prices and hedging strategies by backward induction
  - use of the dynamic risk-neutral tree
  - Start from period 2, compute price at period 1 for the three possible nodes
  - + hedge ratios in short term CDS 1,2 at period 1
  - Compute price and hedge ratio in short term CDS 1,2 at time 0
- Example: term structure of credit spreads
  - computation of CDS 1 premium, maturity = 2
  - $p_1 dt$ will denote the periodic premium
  - Cash-flow along the nodes of the tree

Tree approach to hedging defaults

- Computations CDS on name 1, maturity = 2



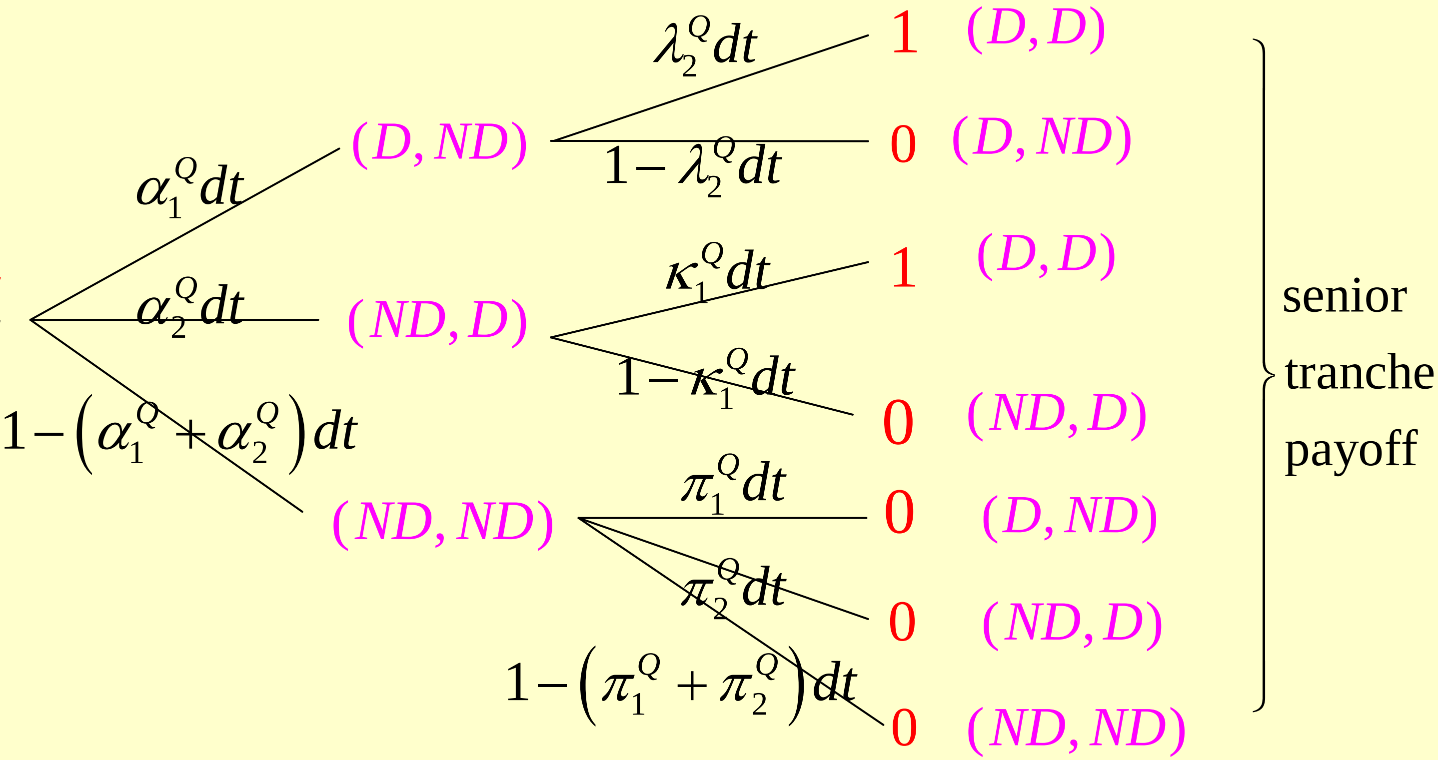
- Premium of CDS on name 1, maturity = 2, time = 0, $p_1 dt$ solves for:

$$0=\left(1-p_1\right)\alpha_1^Q+\left(-p_1+\left(1-p_1\right)\kappa_1^Q-p_1\left(1-\kappa_1^Q\right)\right)\alpha_2^Q$$
$$+\left(-p_1+\left(1-p_1\right)\pi_1^Q-p_1\pi_2^Q-p_1\left(1-\pi_1^Q-\pi_2^Q\right)\right)\left(1-\alpha_1^Q-\alpha_2^Q\right)$$

Tree approach to hedging defaults

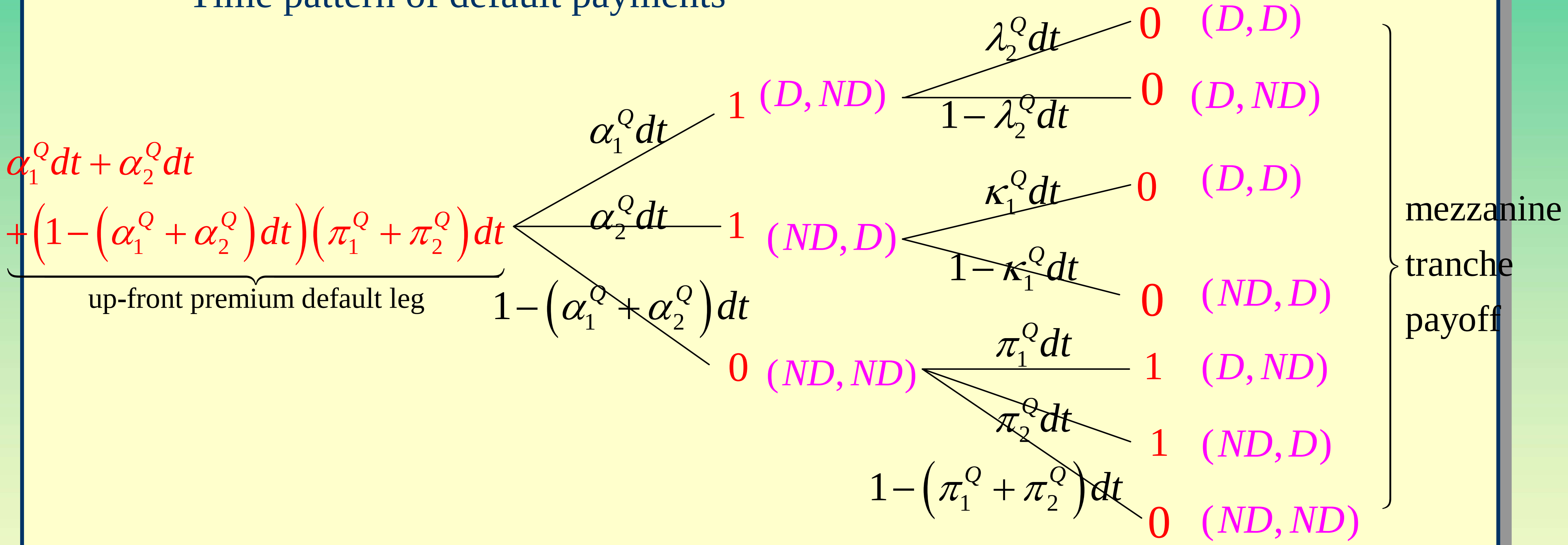
- **Stylized example: default leg of a senior tranche**
  - Zero-recovery, maturity 2
  - Aggregate loss at time 2 can be equal to 0,1,2
    - ➢ Equity type tranche contingent on no defaults
    - ➢ Mezzanine type tranche : one default
    - ➢ Senior type tranche : two defaults

$$\underbrace{\alpha_1^Q dt \times \kappa_2^Q dt + \alpha_2^Q dt \times \kappa_1^Q dt}_{\text{up-front premium default leg}}$$



Tree approach to hedging defaults

- **Stylized example: default leg of a mezzanine tranche**
  - Time pattern of default payments



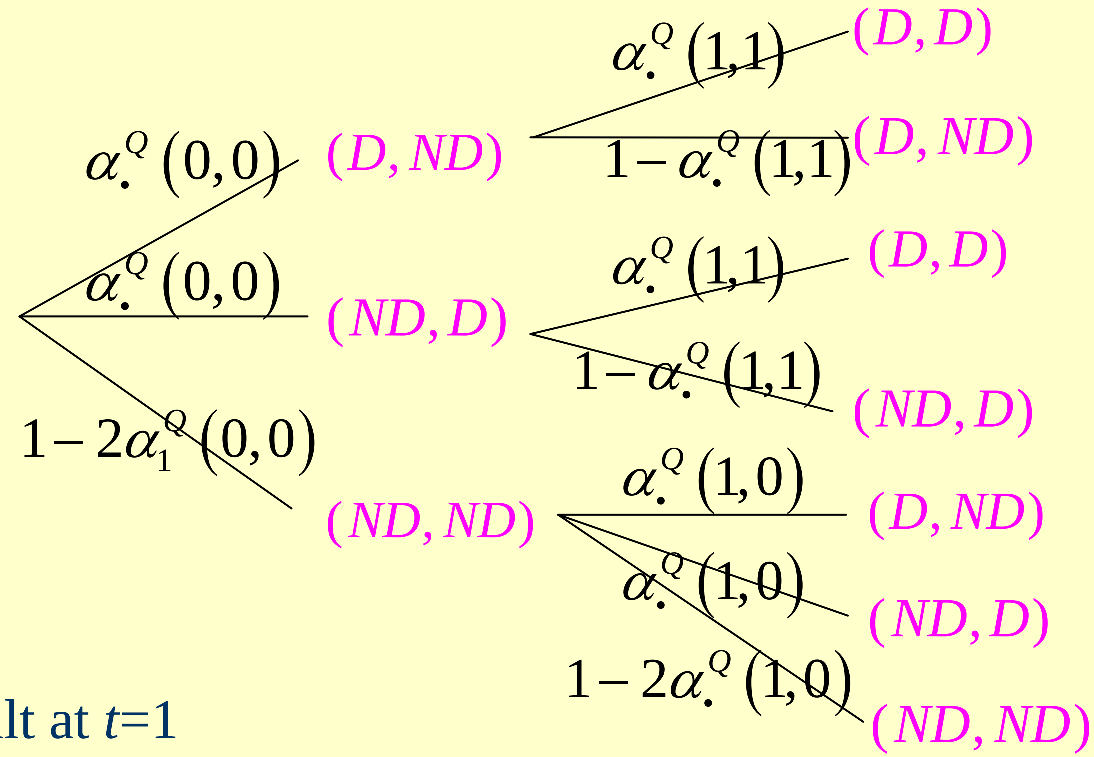
  - Possibility of taking into account discounting effects
  - The timing of premium payments
  - Computation of dynamic deltas with respect to short or actual CDS on names 1,2

Tree approach to hedging defaults

- In theory, one could also derive dynamic hedging strategies for standardized CDO tranches
  - Numerical issues: large dimensional, non recombining trees
  - Homogeneous Markovian assumption is very convenient
    - CDS premiums at a given time *t* only depend upon the current number of defaults $N(t)$
  - CDS premium at time 0 (no defaults) $\alpha_1^Q dt = \alpha_2^Q dt = \alpha_\bullet^Q \left(t = 0, N(0) = 0\right)$
  - CDS premium at time 1 (one default) $\lambda_2^Q dt = \kappa_1^Q dt = \alpha_\bullet^Q \left(t = 1, N(t) = 1\right)$
  - CDS premium at time 1 (no defaults) $\pi_1^Q dt = \pi_2^Q dt = \alpha_\bullet^Q \left(t = 1, N(t) = 0\right)$

Tree approach to hedging defaults

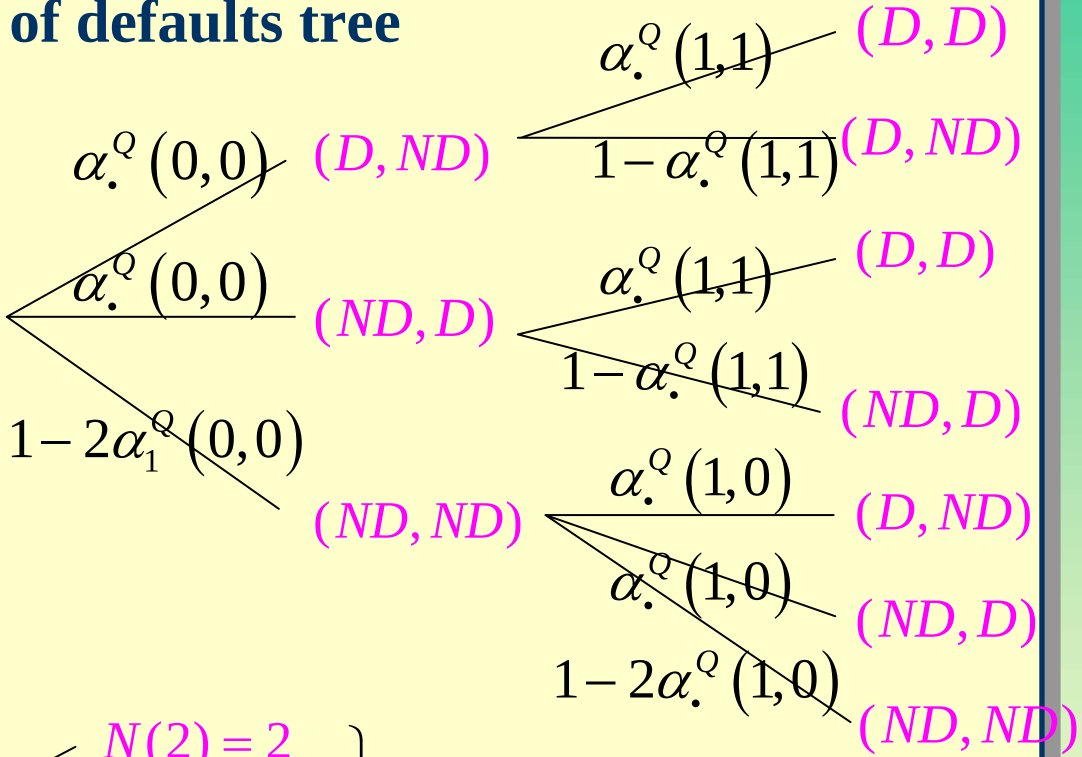
- Tree in the homogeneous case

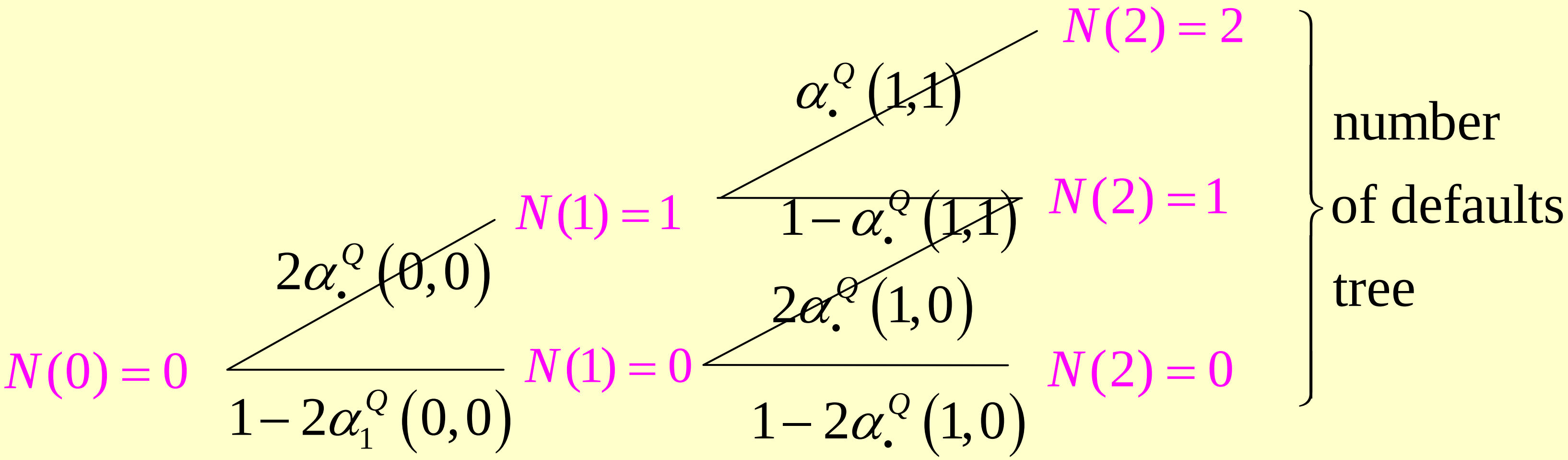


  - If we have $N(1)=1$, one default at t=1
  - The probability to have $N(2)=1$, one default at t=2…
  - Is $1-\alpha_{\bullet}^{Q}(1,1)$ and does not depend on the defaulted name at t=1
  - $N(t)$ is a **Markov process**
  - **Dynamics of the number of defaults can be expressed through a binomial tree**

Tree approach to hedging defaults

- **From name per name to number of defaults tree**





Tree approach to hedging defaults

- Easy extension to n names
  - Predefault name intensity at time t for $N(t)$ defaults: $\alpha_{\bullet}^{Q}\left(t, N(t)\right)$
  - Number of defaults intensity : sum of surviving name intensities:

$$\lambda\left(t, N(t)\right) = \left(n - N(t)\right)\alpha_{\bullet}^{Q}\left(t, N(t)\right)$$

$N(0)=0$ $\xrightarrow{n\alpha_{\bullet}^{Q}(0,0)}$ $N(1)=1$; $N(0)=0$ $\xrightarrow{1-n\alpha_{1}^{Q}(0,0)}$ $N(1)=0$

$N(1)=1$ $\xrightarrow{(n-1)\alpha_{\bullet}^{Q}(1,1)}$ $N(2)=2$; $N(1)=1$ $\xrightarrow{1-(n-1)\alpha_{\bullet}^{Q}(1,1)}$ $N(2)=1$

$N(1)=0$ $\xrightarrow{n\alpha_{\bullet}^{Q}(1,0)}$ $N(2)=1$; $N(1)=0$ $\xrightarrow{1-n\alpha_{\bullet}^{Q}(1,0)}$ $N(2)=0$

$N(2)=2$ $\xrightarrow{(n-2)\alpha_{\bullet}^{Q}(2,2)}$ $N(3)=3$; $N(2)=2$ $\xrightarrow{1-(n-1)\alpha_{\bullet}^{Q}(2,2)}$ $N(3)=2$

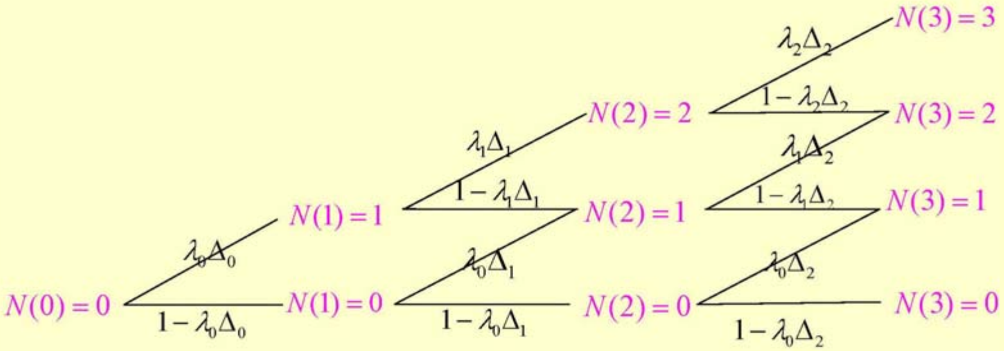
$N(2)=1$ $\xrightarrow{(n-1)\alpha_{\bullet}^{Q}(2,1)}$ $N(3)=2$; $N(2)=1$ $\xrightarrow{1-(n-1)\alpha_{\bullet}^{Q}(2,1)}$ $N(3)=1$

$N(2)=0$ $\xrightarrow{n\alpha_{\bullet}^{Q}(2,0)}$ $N(3)=1$; $N(2)=0$ $\xrightarrow{1-n\alpha_{\bullet}^{Q}(2,0)}$ $N(3)=0$

  - $\alpha_{\bullet}^{Q}\left(0,0\right), \alpha_{\bullet}^{Q}\left(1,0\right), \alpha_{\bullet}^{Q}\left(1,1\right), \alpha_{\bullet}^{Q}\left(2,0\right), \alpha_{\bullet}^{Q}\left(2,1\right), \ldots$ can be easily calibrated
  - on marginal distributions of $N(t)$by forward induction.

Tree approach to hedging defaults

- Calibration of the tree example
  - Number of names: 125
  - Default-free rate: 4%
  - 5Y credit spreads: 20 bps
  - Recovery rate: 40%



3%	6%	9%	12%	22%
18%	28%	36%	42%	58%

Table 8. Base correlations with respect to attachment points.

- Loss intensities with respect to the number of defaults
  - For simplicity, assumption of time homogeneous intensities
  - Increase in intensities: contagion effects
  - Compare flat and steep base correlation structures

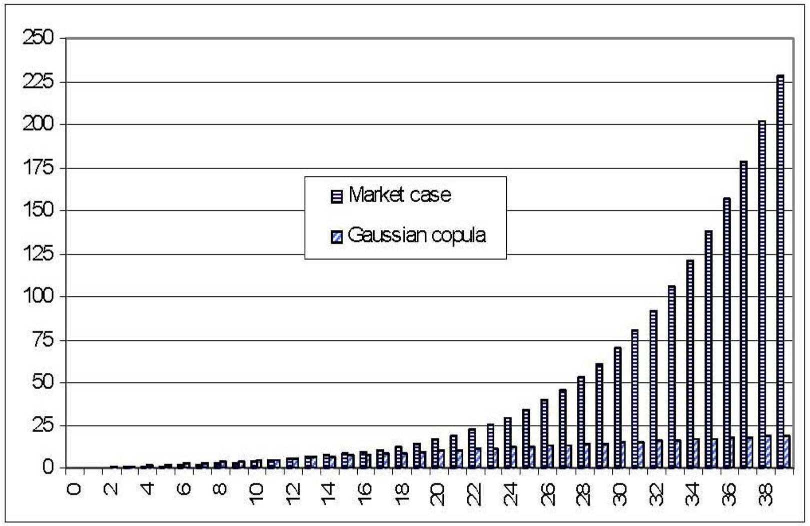


Figure 6. Loss intensities for the Gaussian copula and market case examples. Number of defaults on the $x-$axis.

Tree approach to hedging defaults

- **Dynamics of the credit default swap index in the tree**

Nb Defaults	Weeks			
	0	**14**	**56**	**84**
0	20	19	17	16
1	0	31	23	20
2	0	95	57	43
3	0	269	150	98
4	0	592	361	228
5	0	1022	723	490
6	0	1466	1193	905
7	0	1870	1680	1420
8	0	2243	2126	1945
9	0	2623	2534	2423
10	0	3035	2939	2859

Table 9. Dynamics of credit default swap index spread $s_{IS}(i,k)$ in basis points per annum.

  - The first default leads to a jump from 19 bps to 31 bps
  - The second default is associated with a jump from 31 bps to 95 bps
  - Explosive behavior associated with upward base correlation curve

Tree approach to hedging defaults

- What about the credit deltas?
  - In a homogeneous framework, deltas with respect to CDS are all the same
  - Perfect dynamic replication of a CDO tranche with a credit default swap index and the default-free asset
  - Credit delta with respect to the credit default swap index
  - = change in PV of the tranche / change in PV of the CDS index

Nb Defaults	OutStanding Nominal	Weeks			
		0	14	56	84
0	**3.00%**	0.541	0.617	0.823	0.910
1	**2.52%**	0	0.279	0.510	0.690
2	**2.04%**	0	0.072	0.166	0.304
3	**1.56%**	0	0.016	0.034	0.072
4	**1.08%**	0	0.004	0.006	0.012
5	**0.60%**	0	0.002	0.002	0.002
6	**0.12%**	0	0.001	0.000	0.000
7	**0.00%**	0	0	0	0

Table 11. Delta of the default leg of the $[0,3\%]$ equity tranche with respect to the credit default swap index ($\delta_d(i,k)$).

Tree approach to hedging defaults

- Dynamics of credit deltas:

Nb Defaults	OutStanding Nominal	Weeks			
		0	14	56	84
0	3.00%	0.541	0.617	0.823	0.910
1	2.52%	0	0.279	0.510	0.690
2	2.04%	0	0.072	0.166	0.304
3	1.56%	0	0.016	0.034	0.072
4	1.08%	0	0.004	0.006	0.012
5	0.60%	0	0.002	0.002	0.002
6	0.12%	0	0.001	0.000	0.000
7	0.00%	0	0	0	0

Table 11. Delta of the default leg of the $[0,3\%]$ equity tranche with respect to the credit default swap index ($\delta_d(i,k)$).

  - Deltas are between 0 and 1
  - Gradually decrease with the number of defaults
    - ➢Concave payoff, negative gammas
  - When the number of defaults is > 6, the tranche is exhausted
  - Credit deltas increase with time
    - ➢Consistent with a decrease in time value

Tree approach to hedging defaults

- **Market and tree deltas at inception**
- **Market deltas computed under the Gaussian copula model**
  - Base correlation is unchanged when shifting spreads
  - “Sticky correlation” rule
  - Standard way of computing CDS index hedges in trading desks

	[0-3%]	[3-6%]	[6-9%]	[9-12%]	[12-22%]
market deltas	27	4.5	1.25	0.6	0.25
model deltas	21.5	4.63	1.63	0.9	NA

- **Smaller equity tranche deltas for in the tree model**
  - How can we explain this?

Tree approach to hedging defaults

- **Smaller equity tranche deltas in the tree model (cont.)**
  - **Default is associated with an increase in dependence**
    - ➢Contagion effects

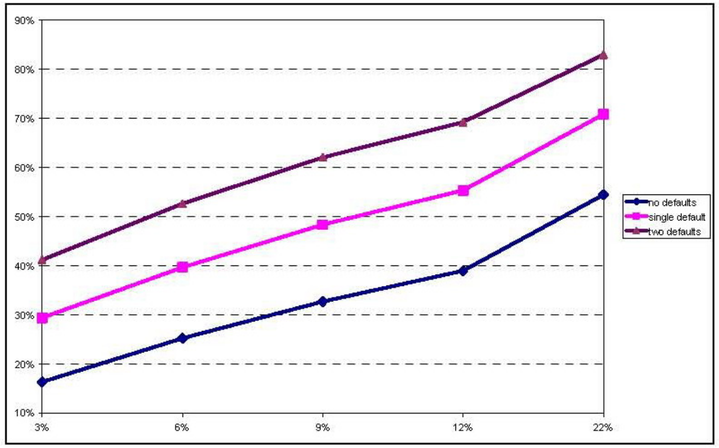


Figure 8. Dynamics of the base correlation curve with respect to the number of defaults. Detachment points on the x–axis. Base correlations on the y–axis.

  - **Increasing correlation leads to a decrease in the PV of the equity tranche**
    - ➢Sticky implied tree deltas
  - **Recent market shifts go in favour of the contagion model**

Tree approach to hedging defaults

- **The current crisis is associated with joint upward shifts in credit spreads**
  - **Systemic risk**
- **And an increase in base correlations**

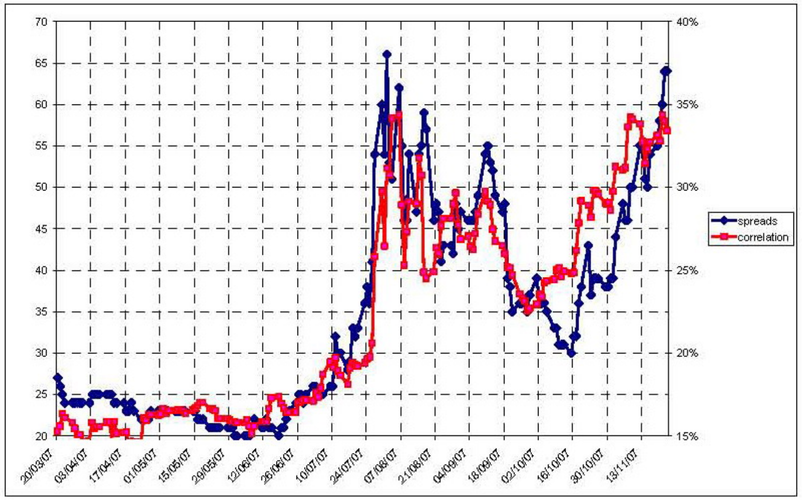


Figure 9. Credit spreads on the five years iTraxx index (Series 7) in bps on the left axis. Implied correlation on the equity tranche on the right axis

- **Sticky implied tree deltas are well suited in regimes of fear (Derman)**

Tree approach to hedging defaults

- What do we learn from this hedging approach?
  - Thanks to stringent assumptions:
    - **credit spreads driven by defaults**
    - **homogeneity**
    - **Markov property**
  - It is possible to compute a dynamic hedging strategy
    - **Based on the CDS index**
  - That fully replicates the CDO tranche payoffs
    - **Model matches market quotes of liquid tranches**
    - **Very simple implementation**
    - **Credit deltas are easy to understand**
  - Improve the computation of default hedges
    - **Since it takes into account credit contagion**
  - Provide some meaningful results in the current credit crisis