KIER-TMU International Workshop on Financial Engineering 2010

repossession

Pricing CDOs with state dependent stochastic recovery rates

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Pricing CDOs with state dependent stochastic recovery rates

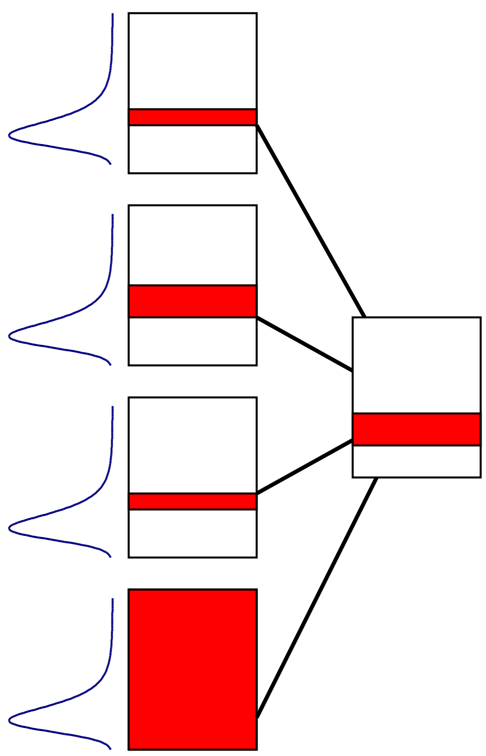
- Main practical issue
  - *Better understanding of large credit portfolio losses*
  - *After the credit and liquidity crisis*
  - *By introducing stochastic recovery rates*
  - *« correlated » together*
  - *And « correlated » with default dates*
  - *Through dependence upon common factor(s)*
  - *Study the properties of such (bottom-up) models*
- Results of interest for market risk assessment
  - *And not only portfolio credit risk*

Pricing CDOs with state dependent stochastic recovery rates

- Need to distinguish CDOs of subprimes
  - *Overestimated ratings for AAA senior tranches*
    - Comonotonic losses
      - Related to real estate market in the US
      - Overestimation of diversification effects amongst assets
    - Underestimation of marginal default probabilities
      - Huge adverse selection problems with originate and distribute system especially in the low-quality
  - *Huge losses borne by so-called "sophisticated investors"*
    - … such as regional banks in Europe
    - "*Because of the dispersion of financial risks to those more willing and able to bear them, the economy and the financial system are more resilient,*"
      - Ben Bernanke keynote address, Federal Reserve Bank of Chicago's annual conference on bank structure and competition on May 18, 2006

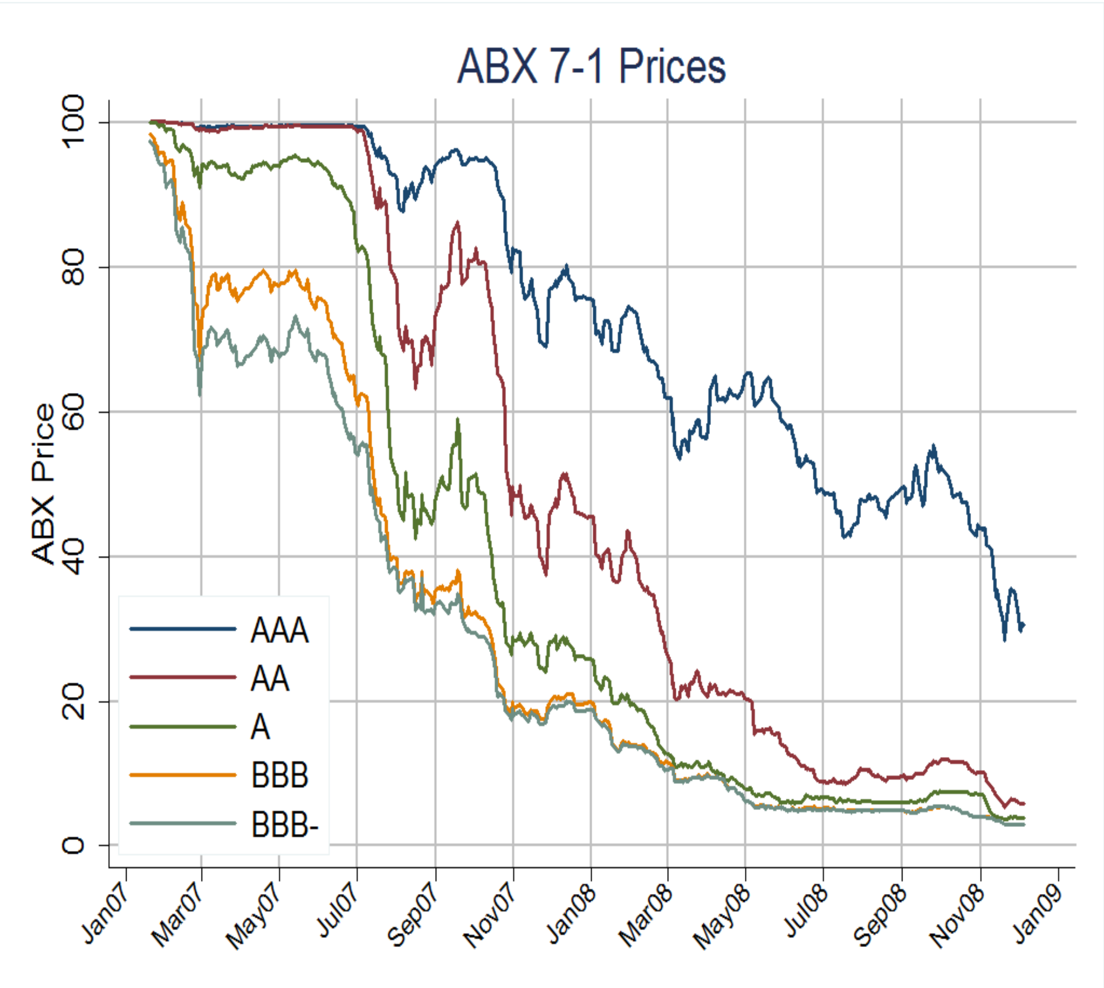
Pricing CDOs with state dependent stochastic recovery rates

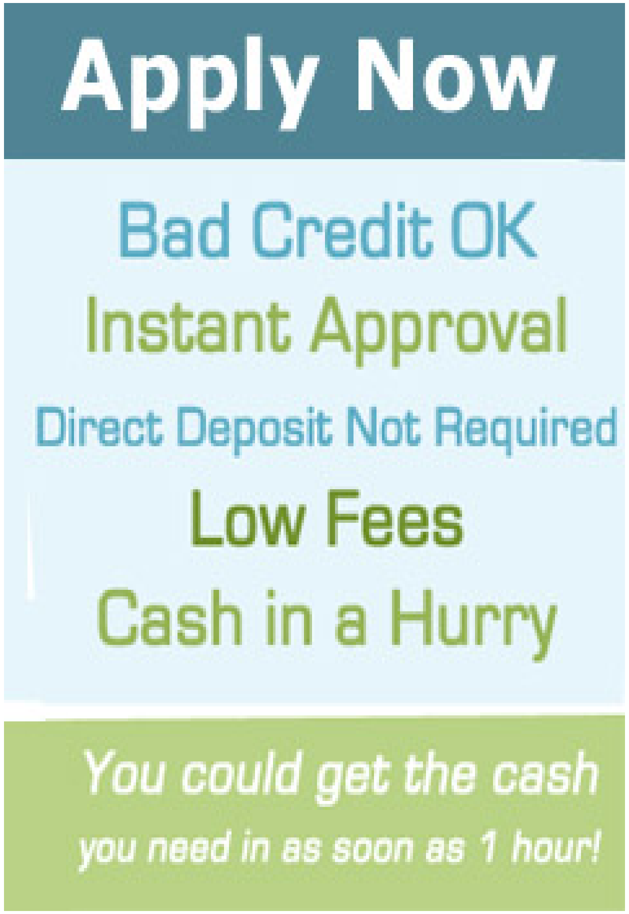
- Need to distinguish between CDOs of subprimes and corporate CDOs
  - *CDO of subprimes are CDO squared*



Pricing CDOs with state dependent stochastic recovery rates

- **Need to distinguish between CDOs of subprimes and corporate CDOs**





Pricing CDOs with state dependent stochastic recovery rates

- Huge losses to “sponsors” of SIV
  - *Mainly US banks actively operating in private securitization of subprime mortgages*
  - *A SIV being a shadow bank, with highly illiquid low rated MBS on the asset side and on the liability side, no core equity, funding itself issuing short-term CP*
    - Obvious **solvency and liquidity** issues for such SIV
  - *How did it infect the sponsor banks?*
  - *through “accounting engineering” such as 365 days lines of credit*

Pricing CDOs with state dependent stochastic recovery rates

- Huge losses to "sponsors" of SIV
  - *Credit and liquidity exposures unconsolidated?*
  - *poor regulation (Basel I) and banking supervision*
  - *"Citigroup has agreed to pay **$75m** to settle **civil** charges that it misled investors over potential losses from high-risk mortgages"*
  - *Citigroup had said in 2007 that its exposure was **$13bn** or less. The SEC said it exceeded **$50bn**.*
  - *SEC Enforcement Director Robert Khuzami said Citigroup had misled analysts and the market of its ability to reduce its subprime exposure.*

"It's up to you now, Miller. The only thing that can save us is an accounting breakthrough."

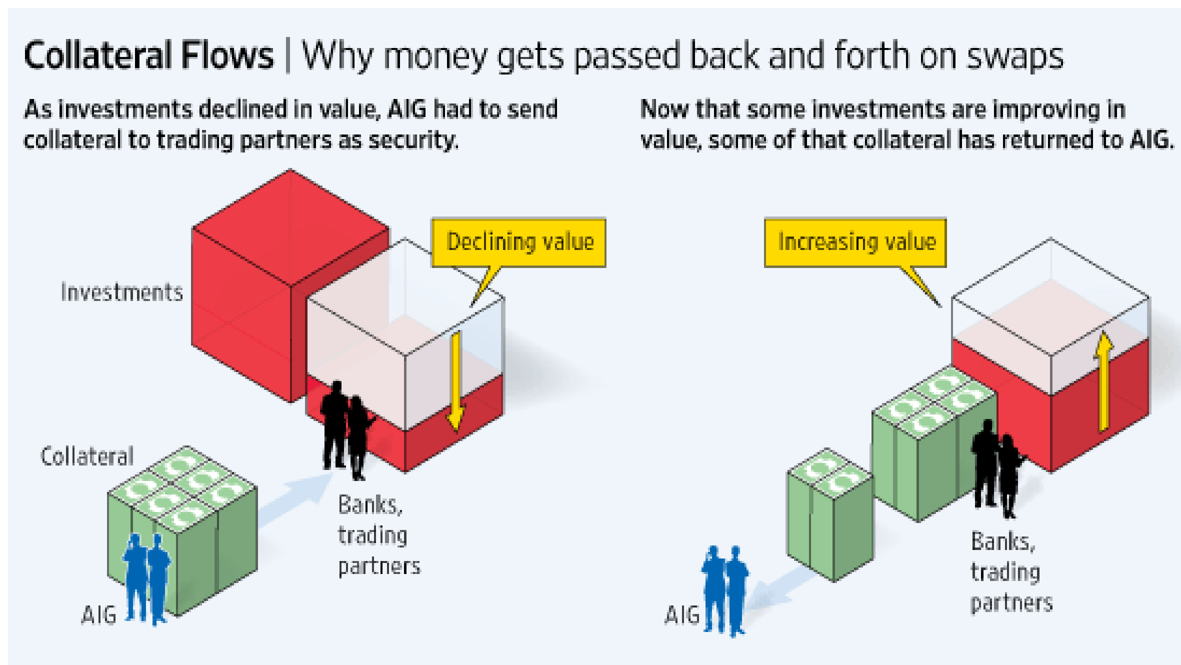
State dependent recovery rates

- Practical context
  - *Calibration of super senior tranches during the liquidity and credit crisis*
    - Insurance against very large credit losses
    - [30-100] tranche on CDX starts to pay when (approximately) 50% of the 125 major companies in North America are in default
      - Contributed to the collapse of AIG
    - AIG reinsurer of major banks
      - Sold protection through AIG Financial Products (London) and Banque AIG (Paris)
      - Between 440 and 500 billion “CDS” outstanding
      - Issues with accounting, counterparty risk, collateral management and liquidity.
        - Large MTM losses
        - Though no insurance payments were to be made

State dependent recovery rates

THE SECRET FORMULA
That Destroyed Wall Street
$P=\Phi(A,B,\gamma)$

- Asymmetric CSA and downgrading of AIG triggered huge collateral posting
  - *30 billion USD of collateral to be posted for super senior tranches*
  - *Not corresponding to actual credit losses on tranches but to « mark to market » of highly illiquid insurance policies*
  - *What occurred when US Treasury took over AIG?*



Pricing CDOs with state dependent stochastic recovery rates

- Practical context: high spreads on senior tranches
- Increase of risk for individual losses leads to increase of risk in aggregate losses
  - *For proper positive dependence*
  - *General results likely to be useful for market risk analysis*
- Comparing risks when claim frequency increase and claim amount decrease (with equal mean)
  - *Analysis of changing recovery rate assumptions on convex measures of risk*
- Comparing risks for granular portfolios sharing the same large portfolio limit
  - *Stochastic recovery rate versus recovery markdown*
- Numerical issues
  - *Expansion techniques vs recursion techniques*

State dependent recovery rates

- High spreads on super senior tranches
  - *Could not be calibrated with a standard 40% recovery rate*

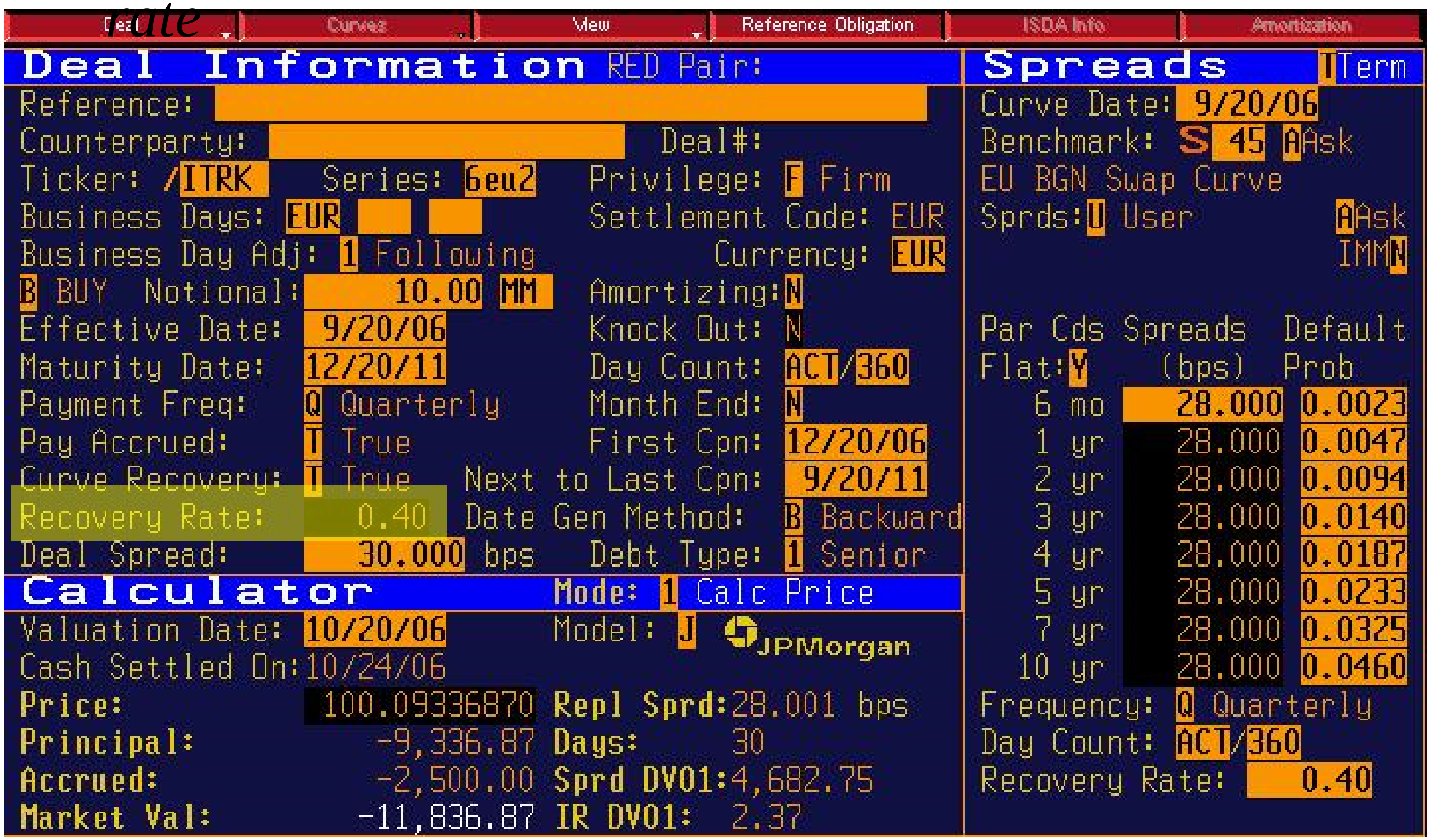
Average Sr. Unsecured Bond Recovery Rates by Year Prior to Default, 1982-2008[1]

	Year 1	Year 2	Year 3	Year 4	Year 5
Aaa	n.a.	3.33%[2]	n.a.	97.00%	85.55%
Aa	43.60%	40.15%	43.45%	57.61%	43.40%
A	42.48%	45.45%	44.50%	38.28%	40.95%
Baa	41.85%	44.56%	44.09%	45.44%	42.68%
Ba	48.00%	42.68%	41.58%	41.15%	41.12%
B	36.98%	35.41%	35.88%	36.91%	40.68%
Caa-C	33.96%	33.25%	33.11%	39.59%	41.94%
Investment-Grade	42.05%	44.23%	44.24%	44.57%	43.37%
Speculative-Grade	36.26%	35.71%	36.30%	38.26%	40.90%
All Rated	36.56%	36.65%	37.50%	39.52%	41.51%

1. *Issuer-weighted, based on 30-day post default market prices.*
2. *Based on three Icelandic bank defaults.*

State dependent recovery rates

- High spreads on super senior tranches
  - *Could not be calibrated with a standard 40% recovery rate*



State dependent recovery rates

- High spreads on super senior tranches
  - *Could not be calibrated with a standard 40% recovery rate*

Table 1: Tranche Quotes and Base Correlations for CDX.NA.IG Series 9 5Y

Tranche	Spread (bps)	Base Correlation (%)
0-3%	500, Upfront 68.51 points	39.45
3-7%	773.99	67.12
7-10%	435.52	72.58
10-15%	240.05	85.18
15-30%	126.50	-
30-100%	69.57	-

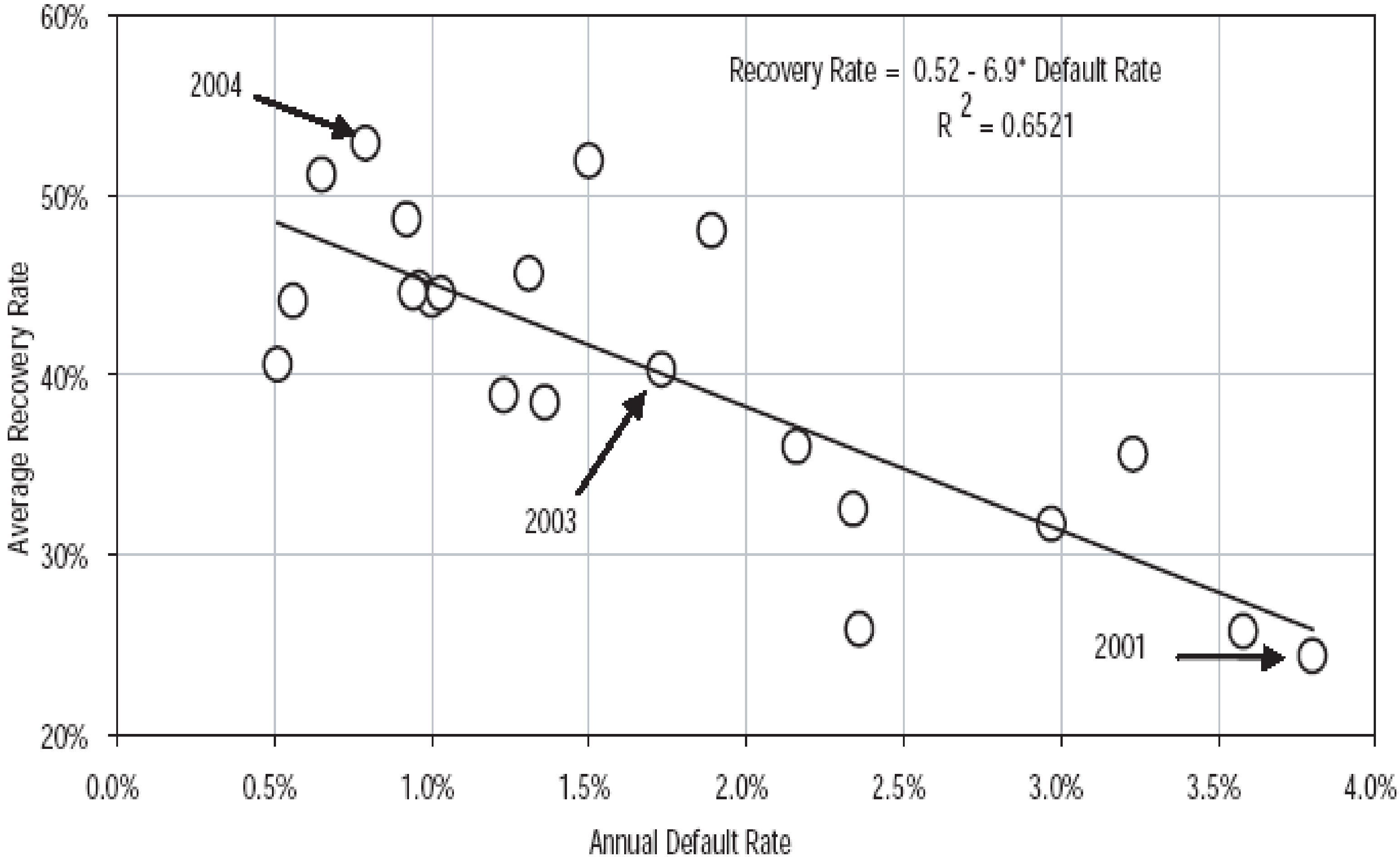
Source: Data from March 12, supplied by ***Markit***

State dependent recovery rates

- Practical context
  - *Steep "base correlations"*
  - *Implied dependence as measured by implied Gaussian copula correlation*
  - *Increases strongly with respect to attachment point*
    - Reflecting "fat tails" in aggregate loss distributions
    - A bunch of issues of trading desks
      - Negative tranchelet prices
      - Delta discriminance
      - Weird Idiosyncratic gamma
  - *These issues are (partly) solved in a stochastic recovery rate approach*
  - *Main issue during 2008 and 2009 for investment banks*

State dependent recovery rates

Exhibit 10 – Correlation between Recovery Rates and Annual Default Rates, 1983-2004



State dependent recovery rates and credit modelling

- Credit models often focus on the dependence between default dates
  - *Bottom-up models*
    - Well-suited to analyze changes of portfolio allocation
  - *Top-down models*
    - Markov models for aggregate losses
    - Dependence through contagion effects : jumps in aggregate loss intensity at default times
    - It is not obvious to relate risks to portfolio structure
    - Unit losses are capped by credit nominal, aggregate loss is also capped
- Our approach is (currently) related to bottom-up approach
  - *When clustering comes (only) through simultaneous defaults*
  - *It can actually create huge dependence effects (common shocks)*
  - *For example, possibility of an Armageddon risk*
    - Is this building really safe regarding earthquakes?

State dependent recovery rates and credit modelling

- Competing approaches for modelling default date dependencies
  - *Joint defaults : common shock models*
    - Starts from Duffie (1999), then Lindskog & McNeil (2003)
  - *Multivariate structural models*
    - CreditMetrics, Basel II, Moody's KMV
  - *Correlated intensities*
    - Multivariate Cox processes
  - *Frailty models (Archimedean copulas)*
    - Hierarchical Archimedean copulas (partially nested)
  - *Gaussian copula*
    - Li (2000)
    - Intra & inter sector correlations: Gregory & Laurent (2004)
  - *Factor copulas*
    - Associated with a wide range of dependence structures

State dependent recovery rates and credit modelling

  - *Markov Copulae*
    - Bielecki and co-authors
    - In between top-down and bottom-up
    - Small homogeneous portfolios may be considered as Markov
    - Dependence comes from simultaneous defaults (related with paper?)
  - *GPL: Brigo et al.*
- No embedding framework
- Large credit losses can also come from stochastic recovery rates
    - "collateral damage"
    - Consider a model with factor dependence
    - Large homogeneous approximation with factor dependent recovery rate
    - Change of mixing distribution for defaults or change recovery rates ?

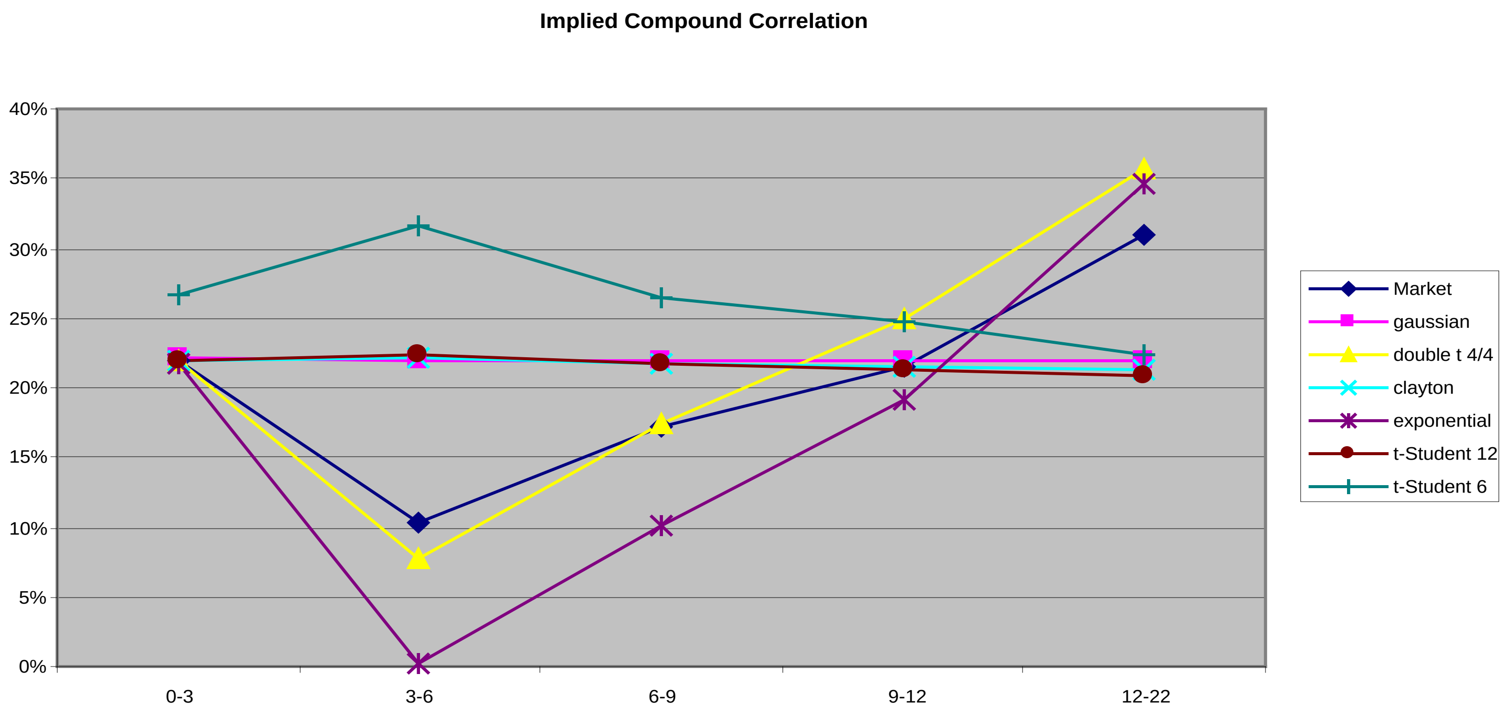
Identification issue $\longleftrightarrow$ $\left(1-\delta\left(V\right)\right)p^{|V}$

State dependent recovery rates and credit modelling

- Dependence in large dimension
  - *The puzzling issue of parametrization*
    - Take the Gaussian copula case as the simplest example
- Homogeneous portfolios (static case)
  - *De Finetti theorem*
  - *One factor*
    - Partially exchangeable portfolios
    - A number of ways to introduce sector-based effects
      - Homogeneous sub-portfolios
- Common shock model is rather well-known
  - *Multivariate exponential distributions*
  - *Marshall Olkin copulas*
  - *Within the factor copula framework*
  - *This eases CDO computations and model analysis*

State dependent recovery rates and credit modelling

- *Common shock models developed for CDOs by Elouerkhaoui*
- *The model can be associated with very large dependence*
  - Much higher than Cox process models and even that frailty models
  - Allows to control for loss distributions (here small mezzanine tranches)



State dependent recovery rates and credit modelling

- Properties of the common shock model
- Specifying the dependence structure
  - *Huge overfitting*
  - *n names can lead to 2^n intensities!*
  - *Checking model restrictions?*
- Dynamics of credit spreads
  - *No contagion effects*
- Dependence only due to simultaneous defaults
- Due to the large number of states, incomplete markets
  - *Requires more involved techniques to construct risk-mitigating dynamic strategies*

State dependent recovery rates and credit modelling

- What are we looking at?
  - *Risk measurement*
    - At which time horizon ?
    - Need to account for rating migration, changes in credit spreads
    - (not only defaults)
    - Possible changes in the (local) correlation structure.
    - Static versus dynamic
  - *CDO pricing*
  - *Investment grade names (100 names), medium size corporate portfolios, mortgages*
- Not the same inputs
  - *historical default data, recovery rates, definition of a default, credit spreads, ratings, bond prices, etc.*

State dependent recovery rates and credit modelling

- Coping with Basel 2 “++”
  - *Capital requirements for CDS and CDO trading books*
  - *CRM : Comprehensive Risk Measure*
  - *Incremental Risk Capital Charge (IRC)*
  - *Stressed VaR : 99.9%, 1 Year time horizon*
  - *Must take into account dynamic hedging with CDO tranches, credit migration, credit spread volatility, stochastic correlation, stochastic recovery rates,…*
  - *Urgent action required (completion by end of year 2010)*
- Moody’s KMV, CreditMetrics and related packages are frontrunners

State dependent recovery rates and credit modelling

- Timing of defaults and default date definition
  - *Not that clear in the corporate world*
  - *Costly non-defaults, costless defaults*
  - *For example, is a bail-out a default?*
    - What has occurred to Merrill Lynch counterparties after BofA stepped-in?
      - Then, it is associated with a joint default event, together with Lehman
  - *Credit migration?*
    - Prior to Bear Stearns bail out by JP Morgan, many counterparties transferred their OTC exposures to thirds parties
    - Novation: transfer rights and obligations to a third party
    - *"In the three weeks preceding Bear Stearns's collapse, GS, Citadel and Paulson exited about 400 trades where Bear Stearns was the trading partner, more than any other firms did."*
    - *GS unloaded a number of swap contracts. Positions were transferred to a variety of players, including Lehman Brothers and Morgan Stanley.*

State dependent recovery rates and credit modelling

- *(Almost) costless defaults : Fannie Mae Subordinated,*
  - Final price, 6^{th} October CDS auction : 99.9
- Jarrow et al. (2008)
  - *Distressed Debt Prices and Recovery Rate Estimation*
- Large discrepancies between economic and recorded default dates
  - *Likely to be a major issue when dealing with the estimation of a model with simultaneous defaults*
  - *more problematic then in the case of no simultaneous defaults*
- Recovery rates also contribute to dependence between individual default dates

State dependent recovery rates

- Theoretical context
  - *Aggregate loss = sum of individual losses*
  - *Individual loss = default indicator times loss given default*
  - *Recovery rate = 1 – loss given default / credit notional*
  - *Recovery rates are stochastic*
- Cross dependencies
  - *Amongst default events (copula models, etc.)*
  - *Between default events and recovery rates*
  - *Amongst recovery rates*
- Dependence through common latent factors
  - *For convenience*

State dependent recovery rates

- When does an increase in individual risk leads to an increase in the risk on the aggregate portfolio (sum of individual risks) ?
  - *(Multivariate) Gaussian risks*
    - Individual risks with same expectation
    - Increase in risk = increase in variance
    - Increase in aggregate portfolio risk occurs if and only if pairwise correlations are non negative
  - *What about the general case ?*
    - Stochastic orders
      - Univariate case : convex order (close to second order stochastic dominance)
    - Positive dependence between individual risks

State dependent recovery rates

- Positive dependence
  - *MTP2: Multivariate Total Positivity of Order 2 (Karlin & Rinott (1980))*
    - Log-density is supermodular
  - *Conditionally Increasing*
    - $X = (X_1, \ldots, X_n)$ is CI if and only if $E\left[\phi(X_i) \middle| (X_j)_{j \in J}\right]$ is increasing in $(X_j)_{j \in J}$ for increasing ϕ
  - *Positive association (Esary, Proschan &Walkup (1967))*
  - *PSMD: positive supermodular dependent*
- Gaussian copula
  - *Positive association = PSMD = positive pairwise correlations*
  - *MTP2 = CI (Müller & Scarsini (2001))*

State dependent recovery rates

- Theoretical context
  - *Non Gaussian framework*
    - Individual risks have a probability mass at 0
  - *Increase of risk of individual risks: convex order*
  - *Theorem (Müller & Scarsini (2001))*
    - X and Y random vectors with common conditionally increasing copula
    - X_i smaller than Y_i for all i
    - Then X smaller than Y with respect to dcx (directionally convex) order
      - Then X smaller than Y with respect to stop-loss order
  - *Gaussian copula dependence*
    - Conditionally increasing if and only if the inverse of covariance matrix is a M-matrix
    - Σ non singular, entrywise non negative, Σ^{-1} has positive non diagonal entries

State dependent recovery rates

JOURNAL OF ECONOMIC THEORY 13, 341–360 (1976)

The Arbitrage Theory of Capital Asset Pricing

STEPHEN A. ROSS*

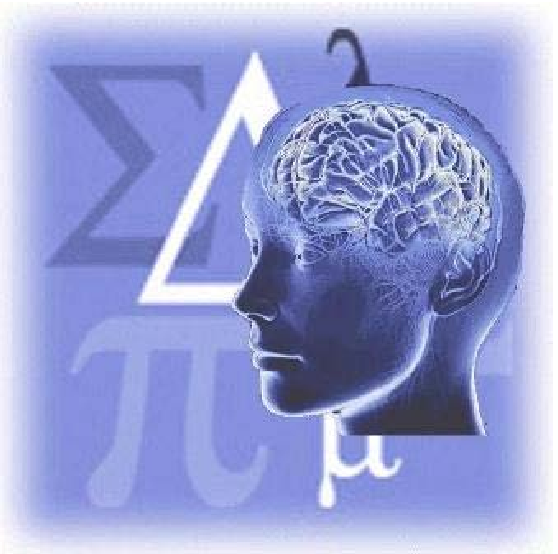
Departments of Economics and Finance, University of Pennsylvania, The Wharton School, Philadelphia, Pennsylvania 19174

Received March 19, 1973; revised May 19, 1976

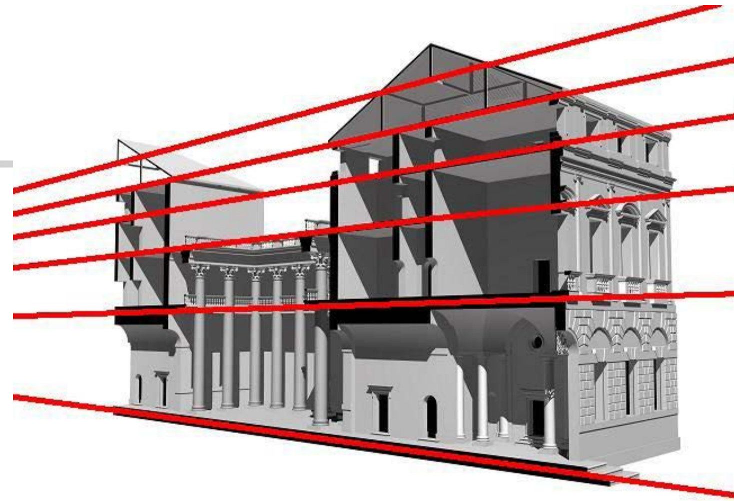
- *Dependence in large dimension*
- *Well known to finance people*
- *Factor models*
  - Arbitrage pricing theory, asymptotic portfolios
    - Chamberlain & Rothschild (1983)
  - Large portfolio approximations (infinite granular portfolios)
    - Conditional law of large numbers
  - Qualitative data with spatial dependence
    - CreditRisk + (Binomial mixtures), Creditmetrics, Basel II (Gaussian copula)
    - Gordy (2000, 2003) Crouhy et al. (2000)
  - Factor models may not be related to a causal view upon dependence
    - De Finetti, exchangeable sequences of Bernoulli variables are Binomial mixtures
    - Mixing random variable latent factor

State dependent recovery rates

- Spatial dependence with qualitative data
  - *Factor models have been used for long in other fields*
    - IQ tests (differential psychology), Bock & Lieberman (1970), Holland (1981)
    - Item Response Models
    - Latent Monotone Univariate Models, Holland (1981), Holland & Rosenbaum (1986)
- Stochastic recovery rates
  - *Modeling of cross dependencies*



State dependent recovery rates



- Stochastic recovery rates
  - *Modeling of cross dependencies*
    - Individual loss = default indicator times loss given default
    - What is important for the computation of tranche premiums (or risk measures) is the joint distribution of individual losses
    - Direct approach: (discretized) individual loss seen as a polychotomous (or multinomial) variable
      - Multivariate Probit model (Krekel (2008))
      - Dual view of Creditmetrics (default side versus ratings)
    - Sequential models
      - Probit or logit models for default events (dichotomous model)
      - Modeling of loss given default : Amraoui & Hitier (2008)

State dependent recovery rates

- Gaussian copula
  - *When is it conditionally increasing?*
  - *One factor case (positive betas)*
    - Gaussian copula is Conditionally Increasing (proof based on Holland & Rosenbaum (1986))
  - *Multifactor case : more intricate, even if all betas are positive, Gaussian copula may not be Conditionally Increasing*
    - Counterexamples
      - Gaussian copula with positive pairwise correlation
      - Increase of marginal risk (convex order)
      - May lead to a decrease of convex risk measures on aggregate portfolio
      - Constraints on conditional covariance matrix
  - *Hierarchical Gaussian copulas*
      - Intra and intersector correlations, Gregory & Laurent (2004)
      - Conditionally Increasing copula (proof based upon Karlin & Rinott (1980))

State dependent recovery rates

- Consequences of previous analysis
  - *Other examples of Conditionally Increasing copulas*
  - *Archimedean copulas, Müller & Scarsini (2005)*
  - *Dichotomous models with monotone unidimensional representation*
    - Default indicators conditionally independent upon scalar V
    - Conditional default probabilities are non decreasing in V
    - Most known and used models
    - Includes additive factor copula models (Cousin & Laurent (2008)), such as generic one factor Levy model of Albrecher et al. (2007).

State dependent recovery rates

- Consequences of previous analysis
  - *Non stochastic recovery rates*
  - *Analysis of a "recovery markdown"*
  - *Change recovery rate assumption from 40% to 30% (say)*
  - *Change marginal default probability so that expected loss unit is unchanged*
  - *Lemma : increase of marginal risk with respect to convex order*
- Then, given a CI copula, increase of risk of the aggregate portfolio with respect to convex order
  - *Increase in senior tranche premiums*
  - *Or CDO senior tranche spreads*

State dependent recovery rates

- Consequences of previous analysis
  - *Stochastic recovery rate of Amraoui and Hitier (2008)*
  - *Depends only upon latent factor*
    - As in Altman et al (JoB 2005)
  - *Specification of recovery rate is such that conditional upon latent factor is the same as in a recovery mark-down case*
  - *Same conditional expected losses*
    - Same large portfolio approximations
    - Same “infinitely granular” portfolios
    - When number of names tends to infinity, strong convergence of aggregate losses to large portfolio limits
- Stochastic recovery rate (AH) versus recovery markdown
  - *Same infinitely granular portfolios*
  - *But finitely granular portfolios behave (slightly) differently*

State dependent recovery rates

- Stochastic recovery rate (AH) vs recovery markdown
  - *Main comparison result*
  - *Aggregate losses are ordered with respect to convex order*
  - *Smaller risks in stochastic recovery rate specification*
  - *Smaller spreads on senior tranches*
  - *Small numerical discrepancies*
- Numerical issues
  - *Computation of aggregate loss distributions in individual loss model with spatial dependence (factor models)*
  - *Actuarial methods (recursions, etc.)*
  - *FFT, inverse of Laplace transforms*
  - *Expansions (Stein's method, Gram-Charlier expansions)*

State dependent recovery rates

- Numerical issues
  - *Lots of smuggling around*
  - *Key issues for implementation*
    - Computation of prices
    - Much quicker than Monte Carlo
      - Issues for the use of Hierarchical Archimedean Copulas
    - More importantly computations of Greeks
    - Risk Management
    - Maximum Likelihood methods
  - *Needs to be reassessed in case of stochastic recovery models*