Aggregation and Credit Risk Measurement in Retail Banking

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Purpose of our study

- **<u>Improve credit risk measurement</u>**
  - ***Better assessment of correlation effects between credit portfolios.***
- **<u>Practical consequences of the choice of a risk measure</u>**
  - ***VaR, expected shortfall,***
  - ***Loss vs unexpected loss,***
  - ***Sensitivity analysis.***

Plan

- Credit loss modelling
  - Basel II
  - Extended approach.
- A case study in retail banking
  - Risk measures
  - Computation of capital requirements
  - Sensitivity analysis of risk measures

Default modelling : homogeneous portfolios

- In portfolio k, borrower i defaults with probability PD_k when :

$$\boxed{Z_{k,i} = \sqrt{\rho_k}\,\Psi_k + \sqrt{1-\rho_k}\,\varepsilon_{k,i} < \Phi^{-1}(PD_k)}$$

- ***Common portfolio factor** Ψ_k .*
- ***Specific independent factor** $\varepsilon_{k,i}$ **for a borrower i.***
- **Assumption** ***: factors follow standard Gaussian distributions.***
- ***Gaussian cdf :** Φ .*
- ***Correlation** ρ_k .*

Loss distribution : homogeneous portfolio

- ***Risk components*** **for portfolio k:**
  - Marginal Probability of Default PD_k .
  - Marginal Loss Given Default LGD_k .
  - Portfolio Exposure At Default EAD_k .

- **Infinite granularity:**
  - Total loss L_k = sum of individual losses.
  - When the number of borrowers is high, specific risk is diversified away (Gordy, 2000).

$$\boxed{L_k(\Psi_k) = EAD_k \times LGD_k \times \Phi\left(\frac{\Phi^{-1}(PD_k) - \sqrt{\rho_k}\Psi_k}{\sqrt{1-\rho_k}}\right)}$$

Aggregation of homogeneous portfolios

- **Homogeneous portfolios 1,…,K.**
- **<u>Aggregate loss:</u>**

$$L = \sum_{k=1}^{K} EAD_k \times LGD_k \times \Phi\left(\frac{\Phi^{-1}(PD_k) - \sqrt{\rho_k}\Psi_k}{\sqrt{1-\rho_k}}\right)$$

- **<u>Portfolio factors :</u>**
  - $$\Psi_k = \sqrt{\rho}\eta + \sqrt{1-\rho}\eta_k$$
  - $(\eta_k)_{1 \le k \le K}$ and η follow standard Gaussian distributions.
  - Systemic correlation between factors: ρ

- VaR of the loss distribution at the confidence level q :

$$\boxed{VaR_q(L) = \inf(l, P(L \leq l) \geq q).}$$

- Expected Shortfall (the loss has a density) :

$$\boxed{ES_q(L) = E(L \mid L \geq VaR_q(L))}$$

- « Unexpected loss » :

$$\boxed{UL_q(L) = VaR_q(L) - E(L)}$$

- **Purpose of the case study**
  - Comparison of the regulatory model and its extended version,
  - Assessment of correlation effects,
  - Assessment of risk measure choice on capital allocation.
- **Input data**
  - 14 credit lines, typical of retail banking.

Portfolio structure

credit line	EAD	PD	LGD	correlation
1	14%	0,1%	60%	16,7%
2	20%	0,2%	60%	16,1%
3	7%	0,2%	60%	15,8%
4	10%	0,4%	60%	14,9%
5	10%	0,6%	60%	14,2%
6	7%	0,8%	60%	13,2%
7	8%	1,4%	60%	11,1%
8	2%	3,2%	60%	6,9%
9	6%	3,2%	60%	6,8%
10	1%	4,6%	60%	5,0%
11	1%	7,2%	60%	3,2%
12	5%	7,3%	60%	3,2%
13	7%	16,0%	60%	2,1%
14	3%	55,0%	60%	2,0%

- ***Capital requirements:***

	VaR	ES
Basel, systemic correlation = 100%	6,1%	6,9%
Multifactor, systemic correlation = 50%	4,6%	5,0%
Absolute variation	-1,5%	-1,9%
Relative variation	-24,7%	-27,6%

- ***Basel II vs multifactor model:***
  - Overestimation of capital of an order of magnitude of 25%, either with VaR or Expected Shortfall.

- ***Expected Shortfall vs VaR:***
  - Expected Shortfall : 10% higher than VaR, in both setups.

- EAD_i **: exposure of portfolio i.**

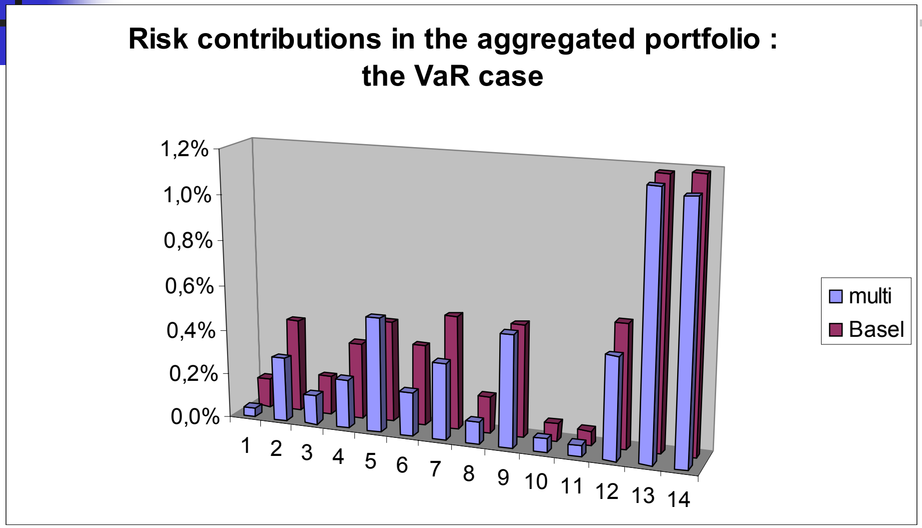
- **<u>Risk contribution for subportfolio i:</u>**
  - ***VaR based risk measure:***

$$\boxed{EAD_i \frac{\partial VaR_q(L)}{\partial EAD_i}}$$

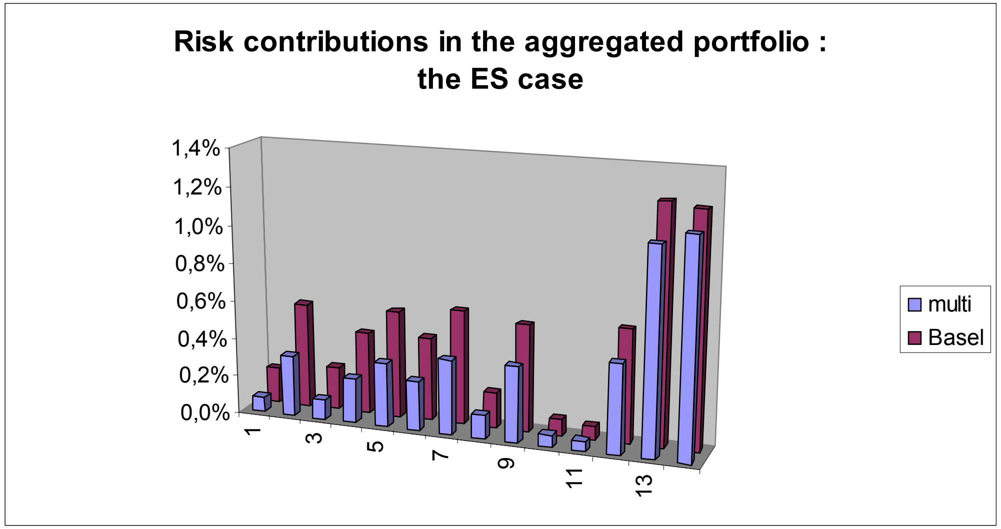
  - ***ES based risk measure:***

$$\boxed{EAD_i \frac{\partial ES_q(L)}{\partial EAD_i}}$$

Risk contributions based on total loss



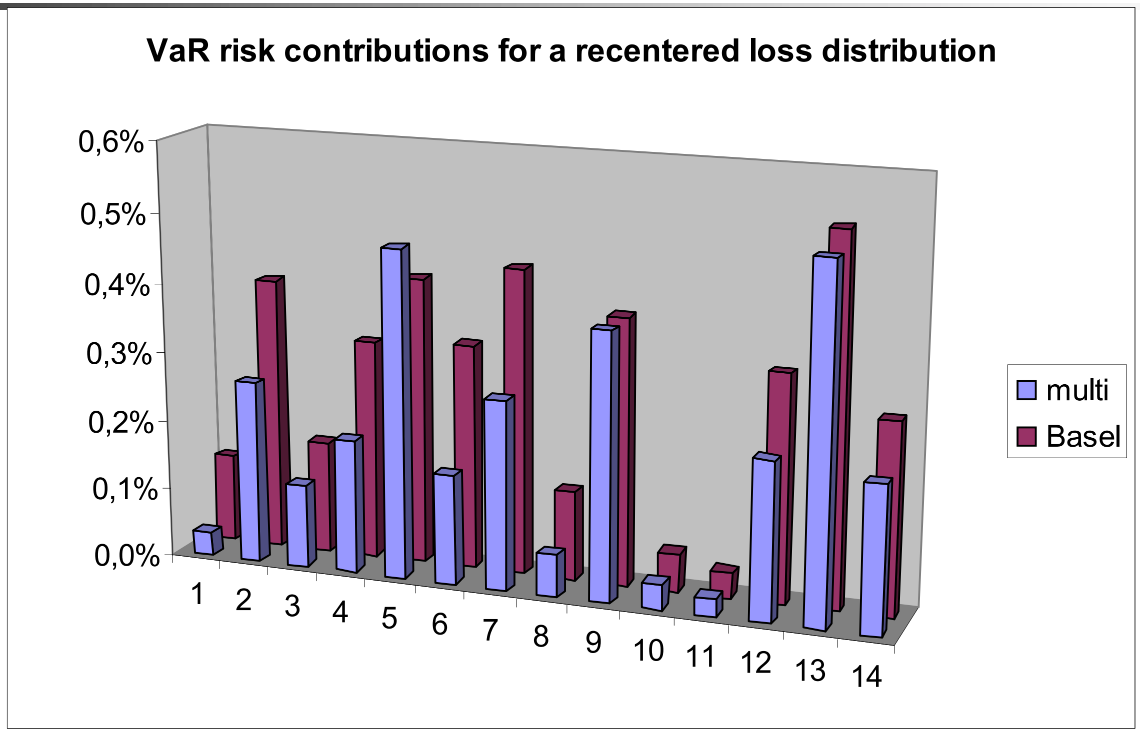
The VaR case



- *The Expected Shortfall case*

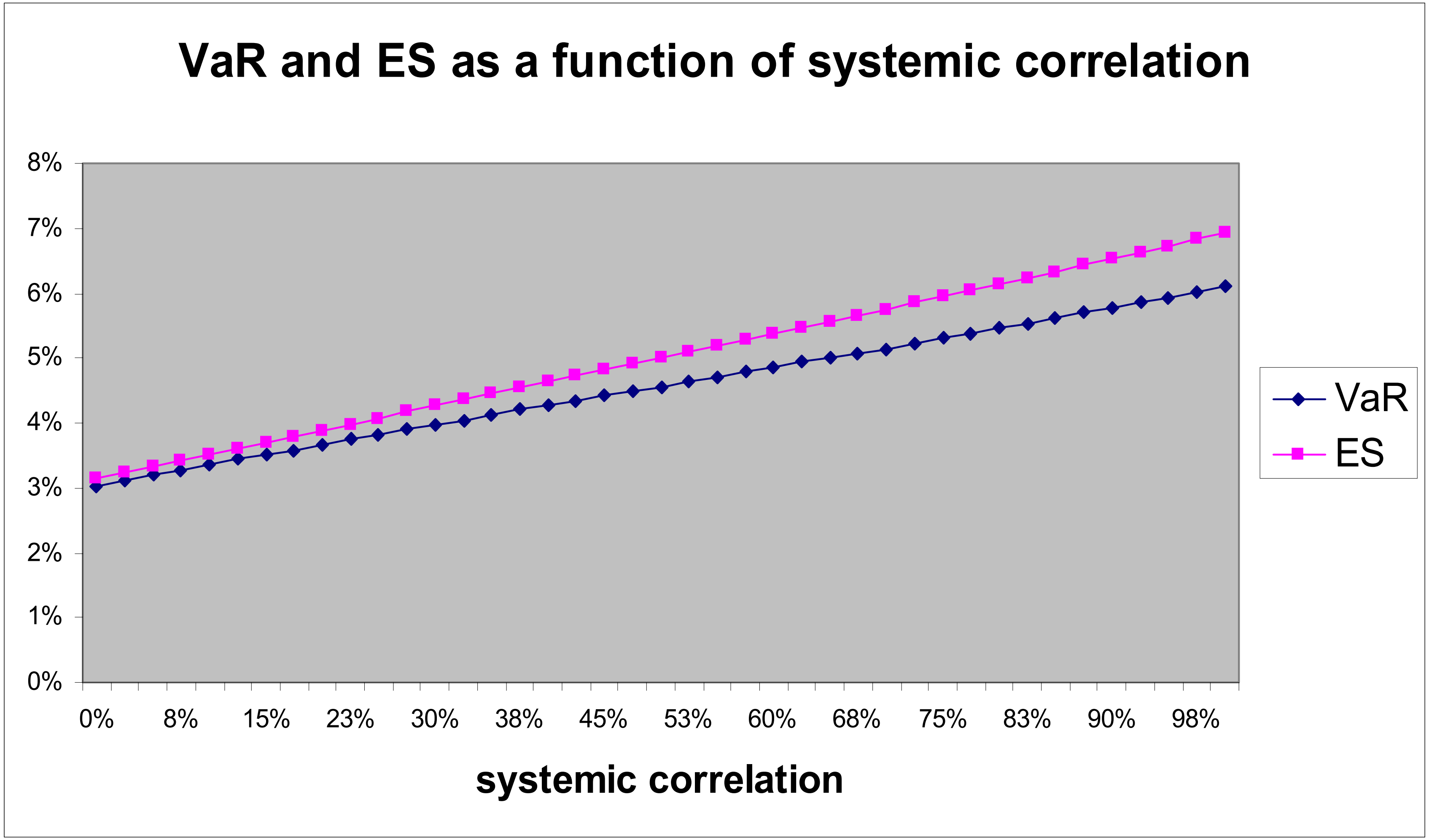
- Systemic correlation :
  - *Basel : 100%,*
  - *Multi : 50%.*

Risk contributions based on unexpected loss



- **<u>Unexpected loss</u>:** $\boxed{UL(L) = VaR_q(L) - E(L)}$
- **<u>Risk contribution of portfolio i</u>:** $\boxed{EAD_i \times \frac{\partial VaR_q(L)}{\partial EAD_i} - LGD_i \times PD_i}$
- Systemic correlation : 100% (Basel) and 50% (multi).

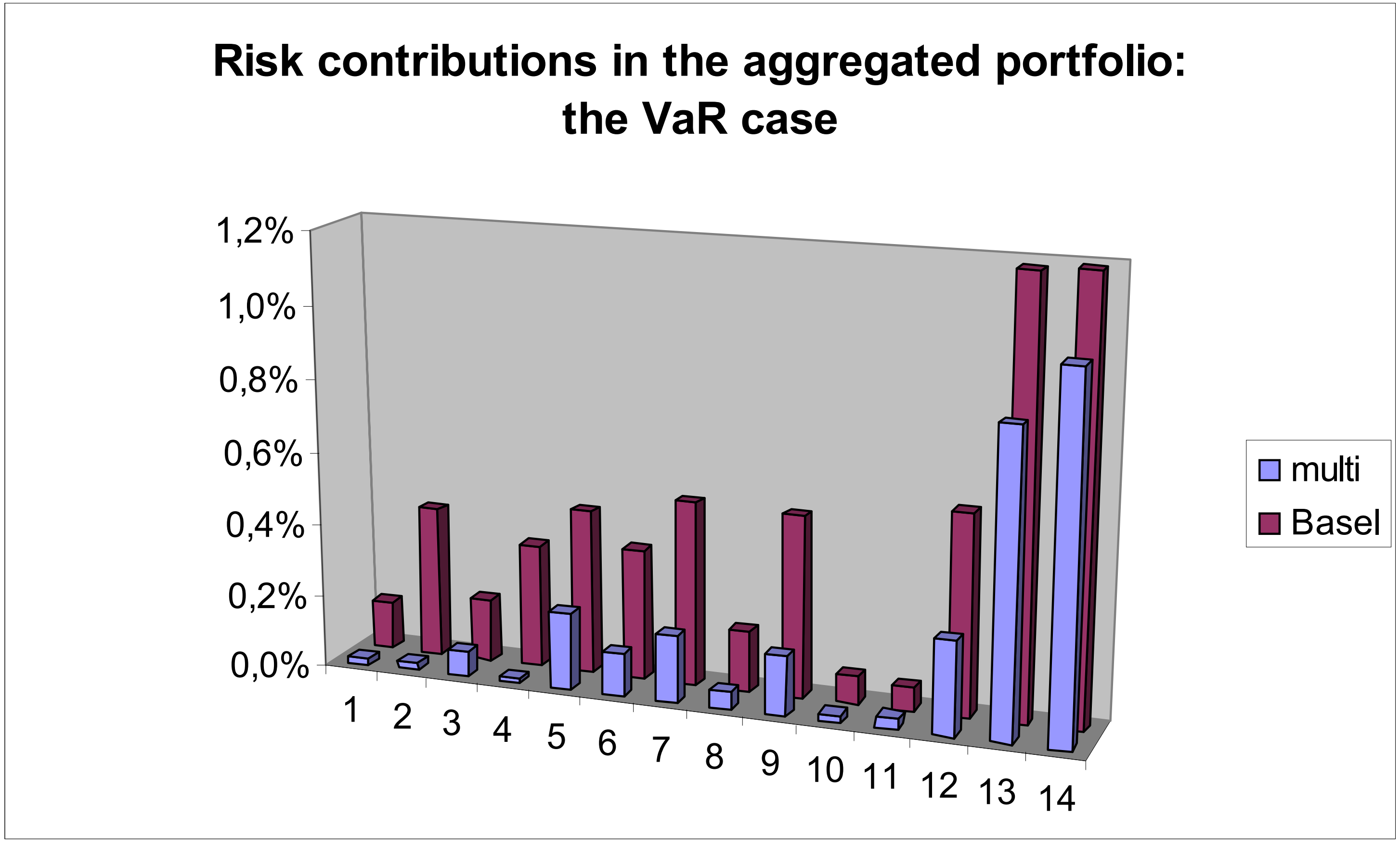
Sensitivity analysis : systemic correlation



Summary

- Extension of the regulatory model,
- Importance of risk diversification in an heterogeneous portfolio,
- Similitude between VaR and Expected Shortfall *in the studied case*,
- Taking into account expected loss…or not !

Annex : impact of low systemic correlation



- Systemic correlation ρ : 100% (Basel) and 5% (multi).
- Computation with total loss L.