Aggregation and Credit Risk Measurement in Retail Banking

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Purpose of our study

- **Improve credit risk measurement**
  - Better assessment of correlation effects between credit portfolios.

- **Practical consequences of the choice of a risk measure**
  - VaR, expected shortfall,
  - Loss vs unexpected loss,
  - Sensitivity analysis.
Plan

- Credit loss modelling
  - Basel II
  - Extended approach.

- A case study in retail banking
  - Risk measures
  - Computation of capital requirements
  - Sensitivity analysis of risk measures
Default modelling: homogeneous portfolios

In portfolio k, borrower i defaults with probability $PD_k$ when:

$$Z_{k,i} = \sqrt{\rho_k} \Psi_k + \sqrt{1 - \rho_k} \varepsilon_{k,i} < \Phi^{-1}(PD_k)$$

- **Common portfolio factor** $\Psi_k$.
- **Specific independent factor** $\varepsilon_{k,i}$ for a borrower i.
- **Assumption**: factors follow standard Gaussian distributions.
- **Gaussian cdf**: $\Phi$.
- **Correlation** $\rho_k$. 

Correlation $\rho_k$.
Loss distribution: homogeneous portfolio

**Risk components for portfolio k:**

- Marginal Probability of Default $PD_k$.
- Marginal Loss Given Default $LGD_k$.
- Portfolio Exposure At Default $EAD_k$.

**Infinite granularity:**

- Total loss $L_k$ = sum of individual losses.
- When the number of borrowers is high, specific risk is diversified away (Gordy, 2000).

\[
L_k(\Psi_k) = EAD_k \times LGD_k \times \Phi\left(\frac{\Phi^{-1}(PD_k) - \sqrt{\rho_k \Psi_k}}{\sqrt{1 - \rho_k}}\right)
\]
Aggregation of homogeneous portfolios

- Homogeneous portfolios 1,...,K.
- Aggregate loss:

\[ L = \sum_{k=1}^{K} EAD_k \times LGD_k \times \Phi \left( \frac{\Phi^{-1}(PD_k) - \sqrt{\rho_k} \Psi_k}{\sqrt{1-\rho_k}} \right) \]

- Portfolio factors:

\[ \Psi_k = \sqrt{\rho \eta} + \sqrt{1-\rho} \eta_k \]

- \( (\eta_k)_{1\leq k \leq K} \) and \( \eta \) follow standard Gaussian distributions.
- Systemic correlation between factors: \( \rho \)
Risk measures

- VaR of the loss distribution at the confidence level \( q \) :
  \[
  \text{VaR}_q(L) = \inf(l, P(L \leq l) \geq q).
  \]

- Expected Shortfall (the loss has a density) :
  \[
  \text{ES}_q(L) = E(L \mid L \geq \text{VaR}_q(L))
  \]

- « Unexpected loss » :
  \[
  \text{UL}_q(L) = \text{VaR}_q(L) - E(L)
  \]
Our case study

Purpose of the case study

- Comparison of the regulatory model and its extended version,
- Assessment of correlation effects,
- Assessment of risk measure choice on capital allocation.

Input data

- 14 credit lines, typical of retail banking.
## Portfolio structure

<table>
<thead>
<tr>
<th>credit line</th>
<th>EAD</th>
<th>PD</th>
<th>LGD</th>
<th>correlation</th>
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<tbody>
<tr>
<td>1</td>
<td>14%</td>
<td>0,1%</td>
<td>60%</td>
<td>16,7%</td>
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<tr>
<td>2</td>
<td>20%</td>
<td>0,2%</td>
<td>60%</td>
<td>16,1%</td>
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<tr>
<td>3</td>
<td>7%</td>
<td>0,2%</td>
<td>60%</td>
<td>15,8%</td>
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<tr>
<td>4</td>
<td>10%</td>
<td>0,4%</td>
<td>60%</td>
<td>14,9%</td>
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<td>5</td>
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<td>0,6%</td>
<td>60%</td>
<td>14,2%</td>
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<tr>
<td>6</td>
<td>7%</td>
<td>0,8%</td>
<td>60%</td>
<td>13,2%</td>
</tr>
<tr>
<td>7</td>
<td>8%</td>
<td>1,4%</td>
<td>60%</td>
<td>11,1%</td>
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<tr>
<td>8</td>
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<td>3,2%</td>
<td>60%</td>
<td>6,9%</td>
</tr>
<tr>
<td>9</td>
<td>6%</td>
<td>3,2%</td>
<td>60%</td>
<td>6,8%</td>
</tr>
<tr>
<td>10</td>
<td>1%</td>
<td>4,6%</td>
<td>60%</td>
<td>5,0%</td>
</tr>
<tr>
<td>11</td>
<td>1%</td>
<td>7,2%</td>
<td>60%</td>
<td>3,2%</td>
</tr>
<tr>
<td>12</td>
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<tr>
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<td>16,0%</td>
<td>60%</td>
<td>2,1%</td>
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<tr>
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<td>3%</td>
<td>55,0%</td>
<td>60%</td>
<td>2,0%</td>
</tr>
</tbody>
</table>
Capital requirements

- **Capital requirements:**

- **Basel II vs multifactor model:**
  - Overestimation of capital of an order of magnitude of 25%, either with VaR or Expected Shortfall.

- **Expected Shortfall vs VaR:**
  - Expected Shortfall: 10% higher than VaR, in both setups.
Risk contributions based on total loss

- $EAD_i$: exposure of portfolio $i$.

### Risk contribution for subportfolio $i$:

- **VaR based risk measure:**
  $$EAD_i \frac{\partial VaR_q(L)}{\partial EAD_i}$$

- **ES based risk measure:**
  $$EAD_i \frac{\partial ES_q(L)}{\partial EAD_i}$$
Risk contributions based on total loss

The VaR case

- The Expected Shortfall case

- Systemic correlation:
  - Basel: 100%,
  - Multi: 50%.
**Risk contributions based on unexpected loss**

- **Unexpected loss**: 
  \[ UL(L) = \text{VaR}_q (L) - E(L) \]

- **Risk contribution of portfolio i**: 
  \[ EAD_i \times \frac{\partial \text{VaR}_q (L)}{\partial EAD_i} - LGD_i \times PD_i \]

- Systemic correlation: 100% (Basel) and 50% (multi).
Sensitivity analysis: systemic correlation

VaR and ES as a function of systemic correlation

- VaR
- ES
Summary

- Extension of the regulatory model,

- Importance of risk diversification in an heterogeneous portfolio,

- Similitude between VaR and Expected Shortfall in the studied case,

- Taking into account expected loss…or not!
Annex: impact of low systemic correlation

- Systemic correlation $\rho$: 100% (Basel) and 5% (multi).
- Computation with total loss L.