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Modelling Credit Spread Behaviour

FIRST Credit, Insurance and Risk

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Overview

- Part I
  – Need for Credit Models
- Part II
  – Simple Binomial Model
- Part III
  – Jump-Diffusion Model
- Part IV
  – Credit Migration Model
- Part V
  – Estimating Credit Spread Volatilities

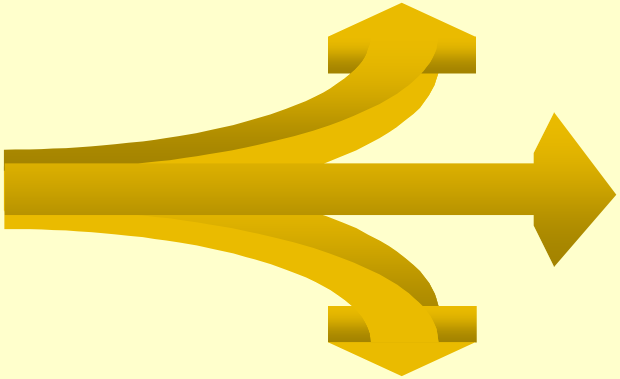
Part I

Need for credit spread models

Need For Credit Models (I)

- Credit derivatives market
- Active management of loan portfolios

Why?



Growth of emerging markets

Active management of counterparty risk in standard derivatives portfolios

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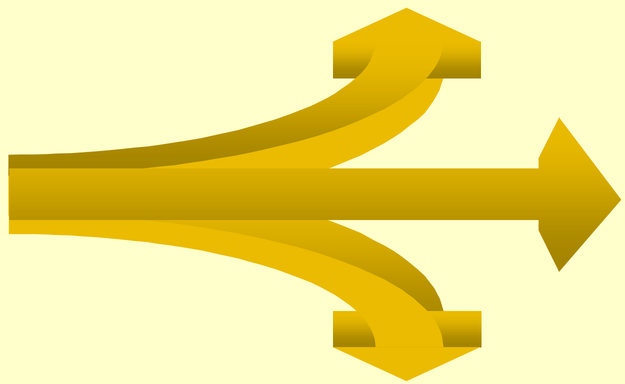
Need For Credit Models (II)

Valuing credit derivatives, options on risky bonds, vulnerable derivatives

What for?



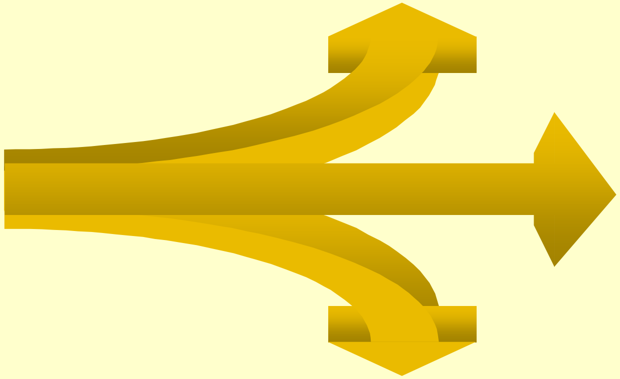
Assessing the credit risk of portfolios - spread and event risk

- Optimising portfolio risk / return profile
- Relative value analysis

Need For Credit Models (III)

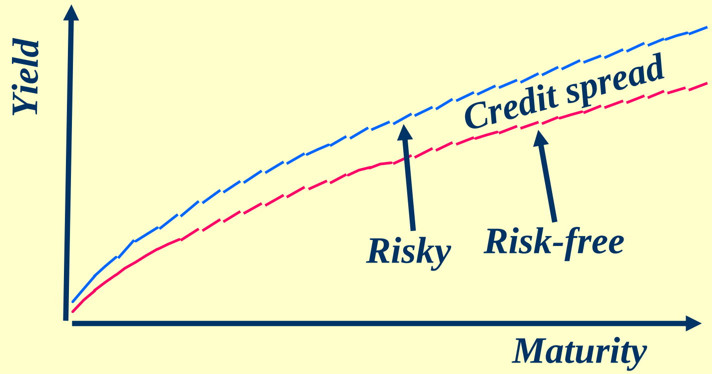
Estimate the current risk free and risky term structures

How?



Model the evolution of the risk free rate and the credit spread

Calibrate to observed bond and option prices



page 6

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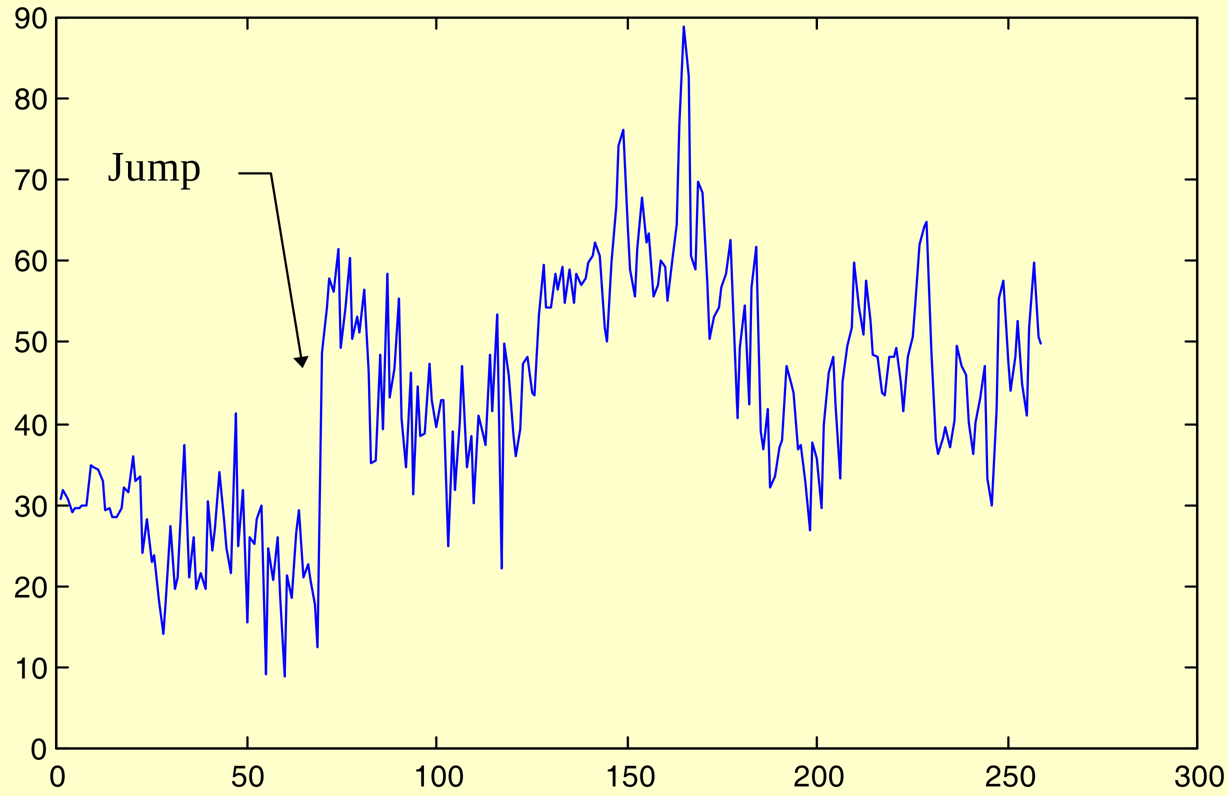
Credit Data

- Limited / crude data available on credit
- Moody's historical data (annual)
  - Default probability $0 \le p_i \le 25\%$
  - Pairwise default correlation $0 \le \rho_{ij} \le 5\%$
  - Credit migration $0 \le q_{kl} \le 20\%$
  - Loss given default $0 \le l_i \le 100\%$
- Default correlation and recovery rate difficult to estimate
- Credit crashes - high default correlation

Credit spread for an AA bond



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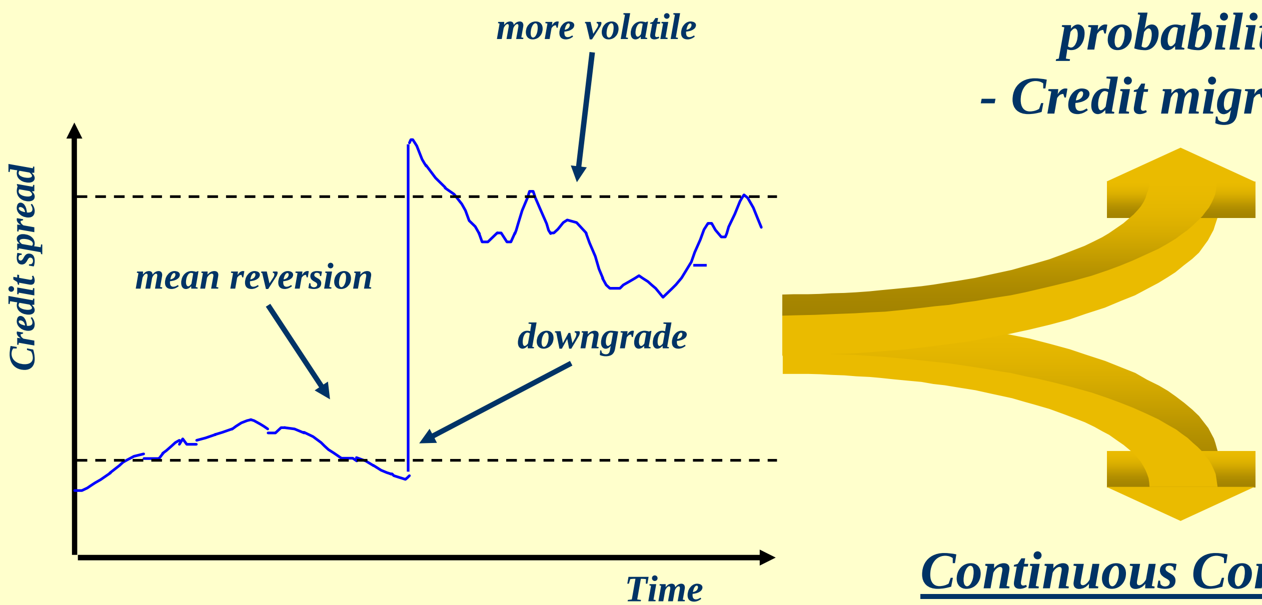
Properties of Credit Spreads



Jump Component
- Discrete change in default probability
- Credit migration

Continuous Component
- Mean reverting
- Change in market price of risk - risk premia

Modelling Credit Spread

$$r_{risky} = r_{risk\ free} + \tilde{\lambda}$$

Credit Spread

Constant
Simple binomial model
(Part II)

Continuous and jump components
Jump-diffusion model
(Part III)

Model underlying
credit migration process
(Part IV)

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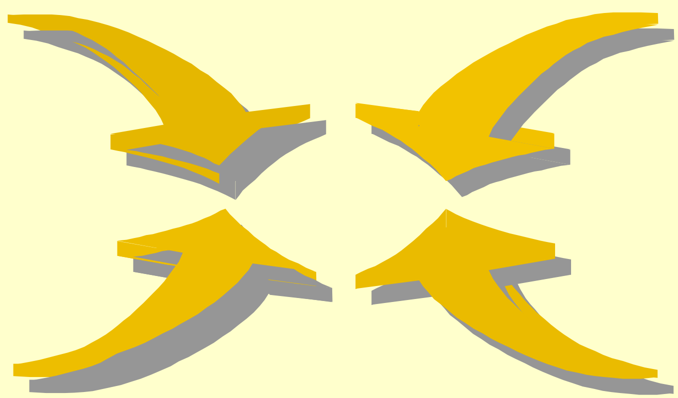
Part II

Simple Binomial Model

Simple Binomial Model (I)

- Constant risk free term structure
- Constant recovery rate

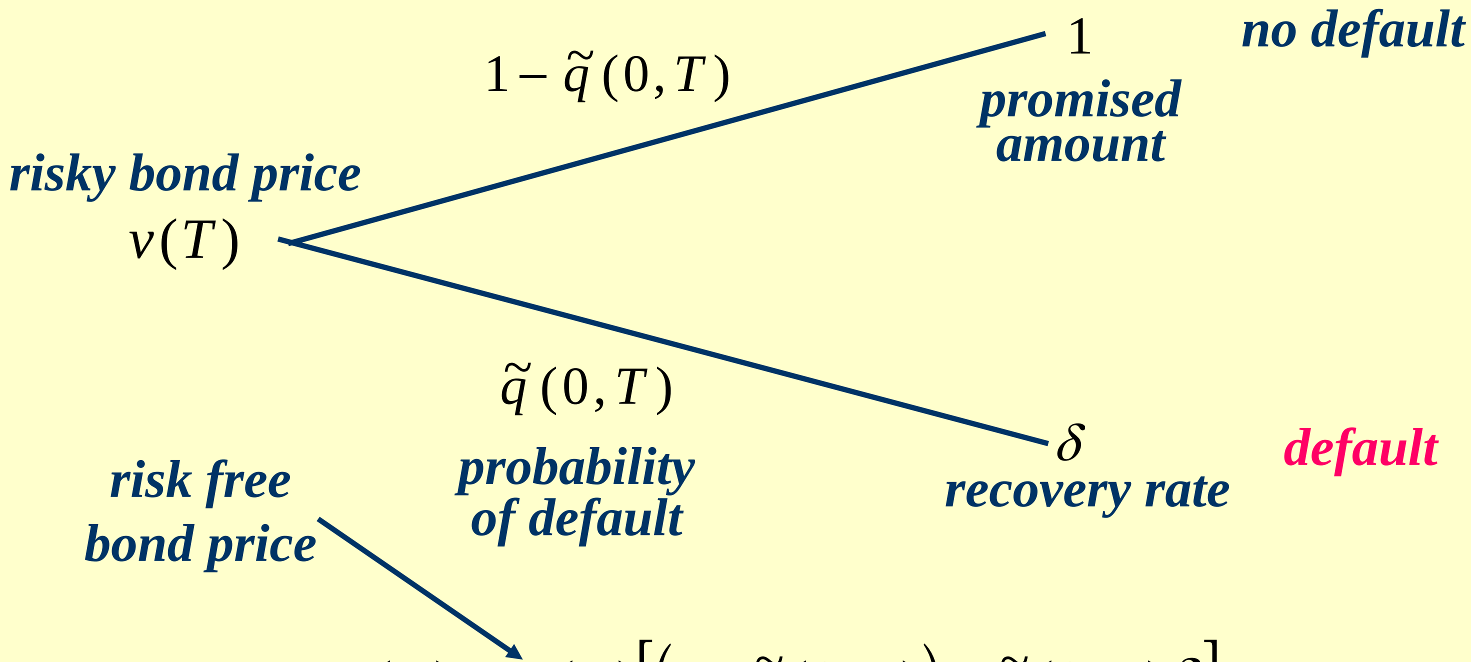
- Constant credit spread if no default
- Jump in credit spread if default occurs



- Derive risk neutral default probabilities from risky and risk- free bond prices

<u>Risk neutral default probabilities</u>
- Actual default probabilities
- Risk premia
- Liquidity
- Uncertainty over recovery rate

Simple Binomial Model (II)



$$v(T) = p(T)\left[\left(1-\tilde{q}(0,T)\right)+\tilde{q}(0,T)\delta\right]$$

$$\tilde{q}(0,T) = \frac{1-v(T)/p(T)}{(1-\delta)}$$

- price any product with payoff contingent on default event

Part III
Jump-Diffusion Model

Jump-Diffusion Model

Continuous component
- Positive and mean reverting
- Correlated with interest rates

Jump Component
- Jumps of random size occur at random times
- Jumps in only one direction

- Standard implementation and calibration
- Standard numerical pricing algorithms can be used

Risk-free interest rate
-Continuous and mean reverting

Risk Free Term Structure (I)

- **Assumptions on the future evolution of the instantaneous risk free rate**
  - **–Volatility $\sigma_r \sqrt{r(t)}$ (normal, lognormal, square root, …)**
  - **–Drift / mean reversion**
    - Long term mean $\bar{r}(t)$
    - Rate of mean reversion $k_r(t)$

$$r(t) = r(0) + \int_0^t k_r(t)\left(\bar{r}(t) - r(t)\right)dt + \int_0^t \sigma_r \sqrt{r(t)}\,dW_r(t)$$

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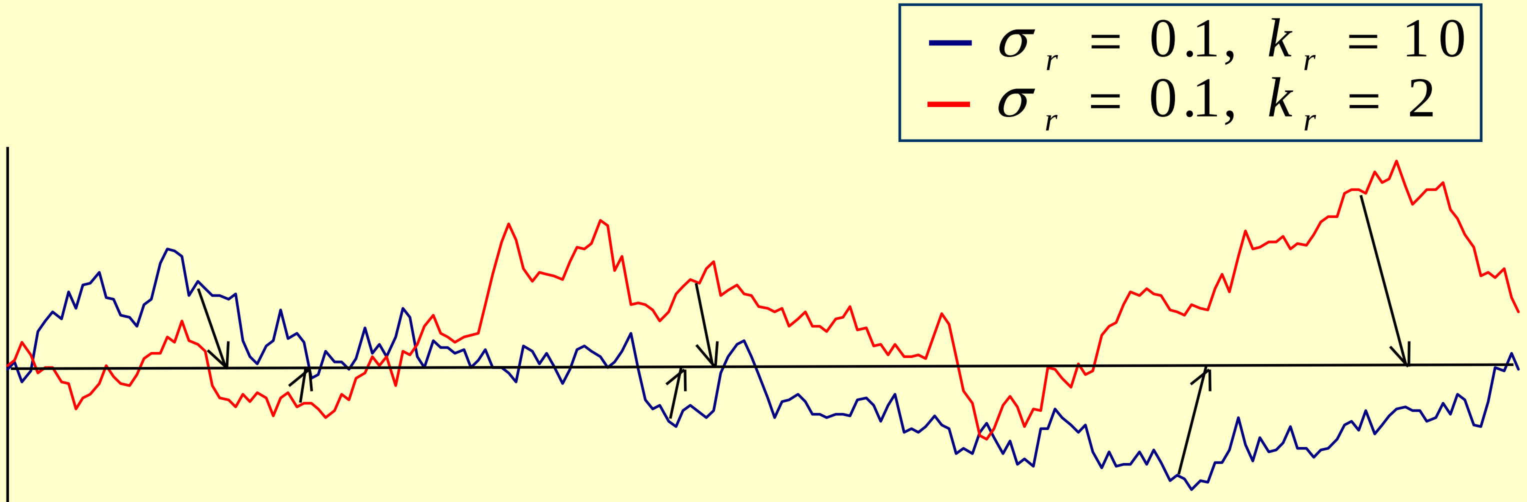
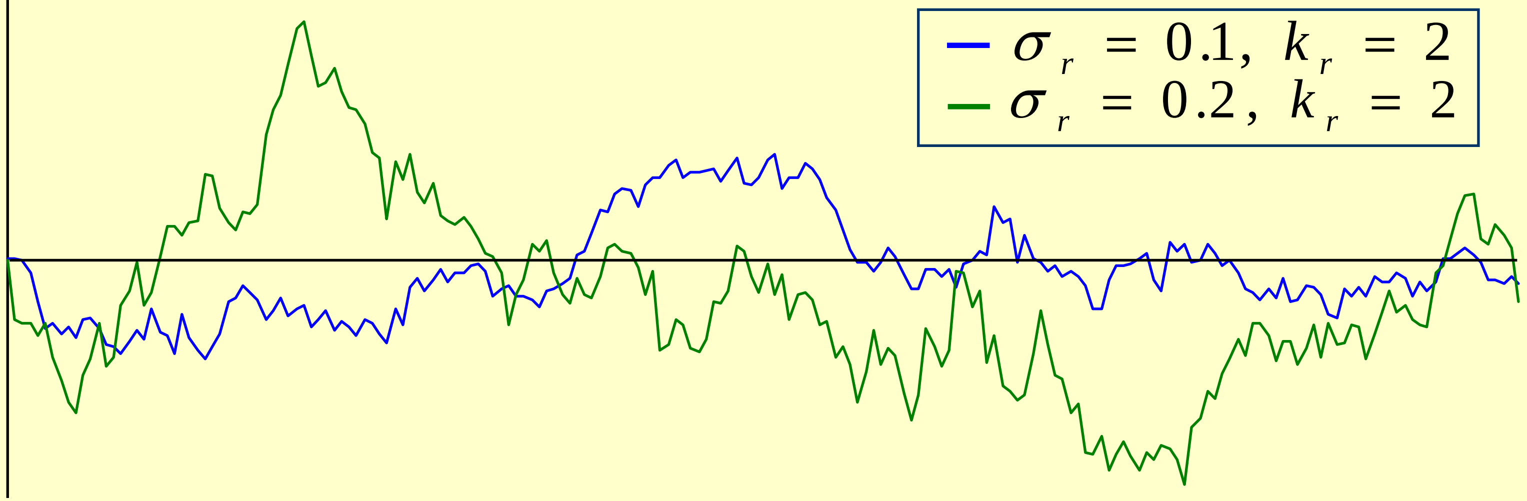
Risk Free Term Structure (II)





Credit Spread Term Structure (I)

$$\tilde{\lambda}(t) = \rho r(t) + x(t) \qquad corr\big(r(t), x(t)\big) = 0$$

determines correlation between $\tilde{\lambda}(t)$ and $r(t)$

- Uncorrelated with interest rates
- Continuous and jump component

- **Random jump size *z*, exponentially distributed**

$$\theta e^{-\theta z}, z > 0$$

- **Random number of jumps - follows Poisson process**

$$e^{-\lambda\tau}(\lambda\tau)^n / n! \quad n = 0,1,2,\ldots \quad \tau = \text{time interval}$$

Credit Spread Term Structure (II)

- **Credit spread component uncorrelated with the risk free interest rate**

$$x(t) = x(0) + \int_0^t \bar{k}_x(s)\left(\bar{x}(s) - x(s)\right)ds$$
$$+ \int_0^t \sigma_x(s)\sqrt{x(s)}\,dW_x(s) + \sum_{i;\tau(i)\le t} Z(i)$$

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Credit Spread Term Structure (III)

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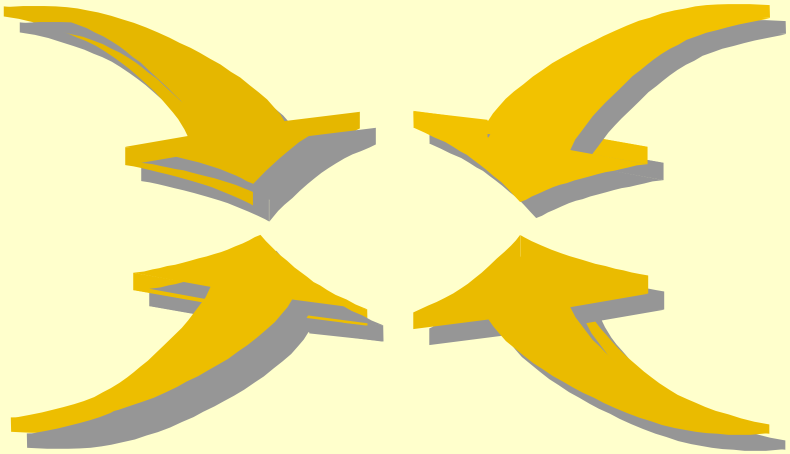
Part IV

Credit Migration Model

Credit Migration Model

- *Jumps* modelled as changes in credit ratings and defaults
- *Continuous part* modelled as continually changing risk premia

- Model jointly assets in various credit classes
- Portfolio management and risk analysis



- Calibration - incorporate economic and historical information

- Flexible in terms of data requirements and number of states

Markov Chains - Generator Matrix (I)

- **Continuous time Markov chain**
- **Discrete state space**

$$
\tilde{\Lambda} =
\begin{array}{c}
 \\ 1 \\ 2 \\ \\ K-1 \\ K
\end{array}
\begin{array}{c}
\begin{array}{ccccc} 1 & 2 & & K-1 & K \end{array} \\
\begin{pmatrix}
\tilde{\lambda}_{1} & \tilde{\lambda}_{12} & . & \tilde{\lambda}_{1,K-1} & \tilde{\lambda}_{1K} \\
\tilde{\lambda}_{21} & \tilde{\lambda}_{2} & . & \tilde{\lambda}_{2,K-1} & \tilde{\lambda}_{2K} \\
. & . & . & . & . \\
\tilde{\lambda}_{K-1,1} & \tilde{\lambda}_{K-1,2} & . & \tilde{\lambda}_{K-1} & \tilde{\lambda}_{K-1,K} \\
0 & 0 & . & 0 & 0
\end{pmatrix}
\end{array}
$$

constant over time

absorbing state (default)

Markov Chains - Generator Matrix (II)

- $I + \tilde{\Lambda} dt$ **, transition matrix over short period dt**
- $\lambda_{ij} \geq 0$ **, non-negative transition probabilities**
- $\tilde{\lambda}_i = -\sum_{\substack{i=1 \\ j \neq i}}^{K} \tilde{\lambda}_{ij}$ **, sum of all probabilities equals 1**

$$\sum_{j \geq i}^{K} \tilde{\lambda}_{ij} \leq \sum_{j \geq k} \tilde{\lambda}_{i+1,j}, \quad \forall\ i,k \quad k \neq i+1$$

A state *i+1* is always more risky than state *i*

Markov Chains - Transition Matrix

- **Transition matrix for the period t to *T***
- **Explicit computation**

$$\tilde{\Lambda} = \Sigma^{-1} D \Sigma$$

$$\tilde{Q}(t,T) = \Sigma^{-1} \exp\left[D(T-t)\right]\Sigma$$

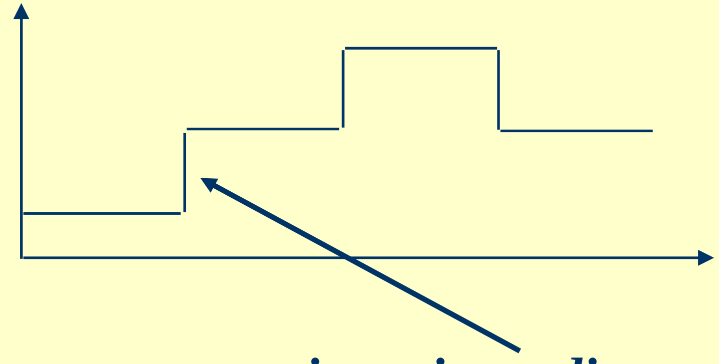
$$\tilde{Q}(t,T) = \begin{pmatrix} \tilde{q}_1(t,T) & . & \tilde{q}_{1,K-1}(t,T) & \tilde{q}_{1K}(t,T) \\ \tilde{q}_{21}(t,T) & . & \tilde{q}_{2,K-1}(t,T) & \tilde{q}_{2K}(t,T) \\ . & . & . & . \\ \tilde{q}_{K-1,1}(t,T) & . & \tilde{q}_{K-1}(t,T) & \tilde{q}_{K-1,K}(t,T) \\ 0 & . & 0 & 1 \end{pmatrix}$$

Model Structure (I)

- States : uniquely determine default probability
- Credit ratings - can incorporate past credit rating transitions - non-Markovian model

$\tilde{\Lambda}$ - Risk neutral generator matrix

$\tilde{\Lambda}$ - constant



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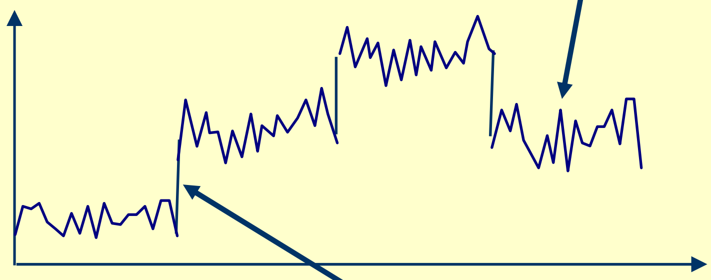
Model Structure (II)

- **Incorporate stochastic risk premia**

$$\tilde{\Lambda}_{stochastic} = \tilde{\Lambda} \times U(t)$$

$\tilde{\Lambda}_{stochastic}$



Stochastic Generator Matrix

- Stochastic generator matrix arises from randomly changing risk premia

$$\tilde{\Lambda}_{stochastic} = \tilde{\Lambda} \times U(t)$$

Stochastic component

$$U(t) = U(0) + \int_0^t \left(a - kU(t)\right)dt + \int_0^t \sigma\sqrt{U(t)}\,dW_t$$

Mean reverting process

- Closed form formulae for bond prices

page 28

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Stochastic Risk Premia

- **If eigenvectors are constant, can pose**

$$\Lambda(t,T) = \Sigma^{-1}\mathbf{D}(t)\Sigma$$

- **Possible evolution of eigenvalues**

$$dX_j = (a_j - b_j X_j)dt + \sigma_j dw, \quad \mathbf{D}(t) = \mathrm{diag}(X_j(t))_{j=1}^{K}$$

- **Pricing equation is now modified to**

$$q_{iK}(t,T) = \sum_{j=1}^{K} (\Sigma^{-1})_{ij} \mathrm{E}\left[\exp\left(\int_t^T X_j(s)ds \right) \middle| \mathbf{D}(t) \right] \Sigma_{jK}$$

- **Expectation has closed (algebraic) form**
  - depends on parameters a b σ and on $\mathbf{D}(t)$

Calibration (I)

Prices of risky bonds for various credit classes and maturities

$B^i(0,T)$

- Least squares estimation
- Adjust historical generator matrix to fit market prices
- Achieve fit closest to historical data

$\tilde{\Lambda}_{stochastic}$

(Λ, a, k, σ)

Simulate Credit Spread

Λ

- Historical generator matrix (estimated from one year transition matrix)
- Credit spread historical time series

Price exotic structures

Calibration (II)

- Least squares fit to match directly observed coupon bond prices (any number)

$$\min_{\tilde{\Lambda}}\left\{\sum_{i=1}^{K}\sum_{j=1}^{J^i}\left(P_j^i-\sum_{h=1}^{T}F_j^i(h)v^i(h;\tilde{\Lambda})\right)^2+\sum_{i,j=1}^{K}\left(\frac{\left(\tilde{\lambda}_{ij}-\lambda_{ij}\right)^2}{\beta_{ij}}\right)\right\}$$

market price of coupon bond for class i → P_j^i

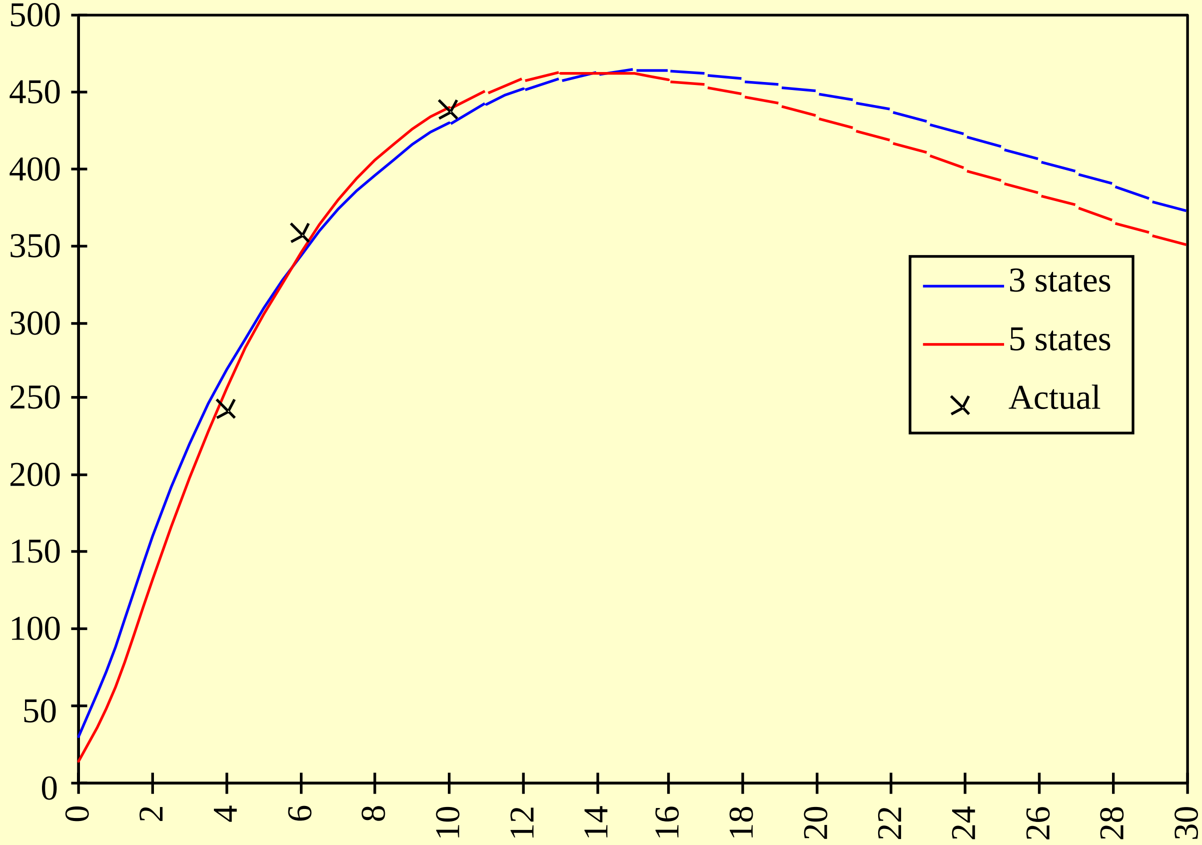
coupon at date h → $F_j^i(h)$

prior generator matrix → λ_{ij}

confidence level → β_{ij}

- Obtain solution closest to the historical generator matrix Λ - stable calibration

Calibration - Emerging Markets (II)



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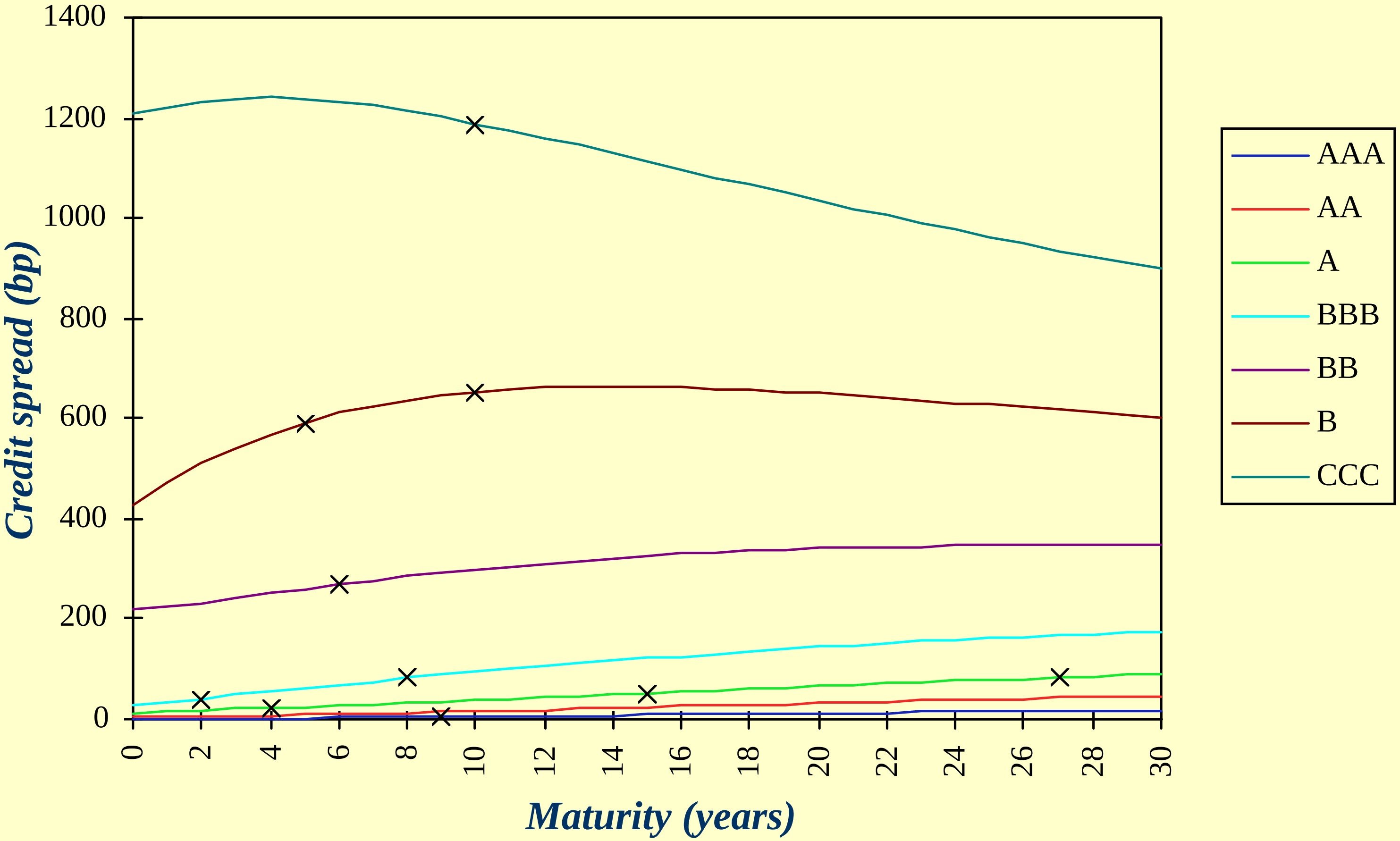
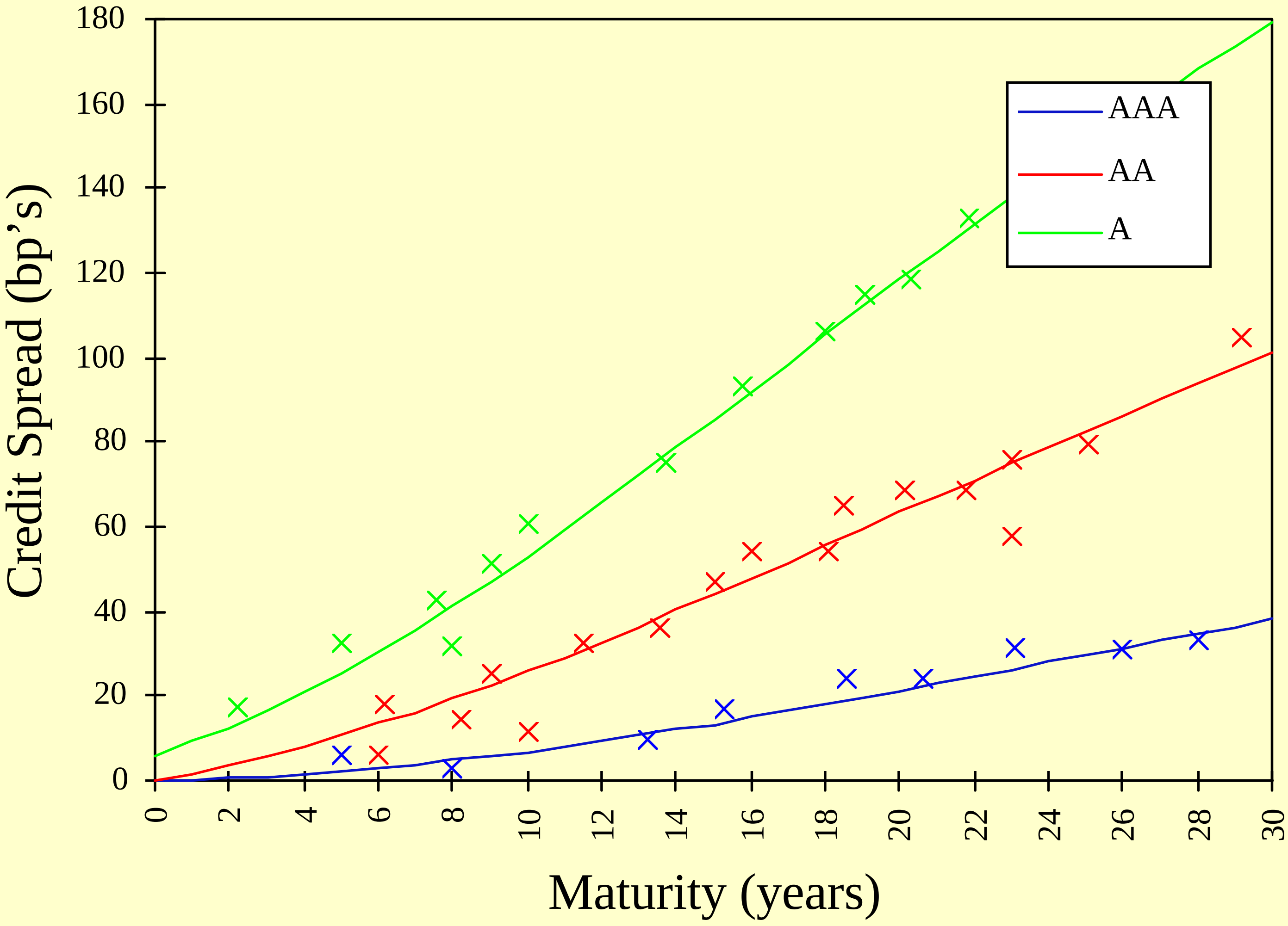
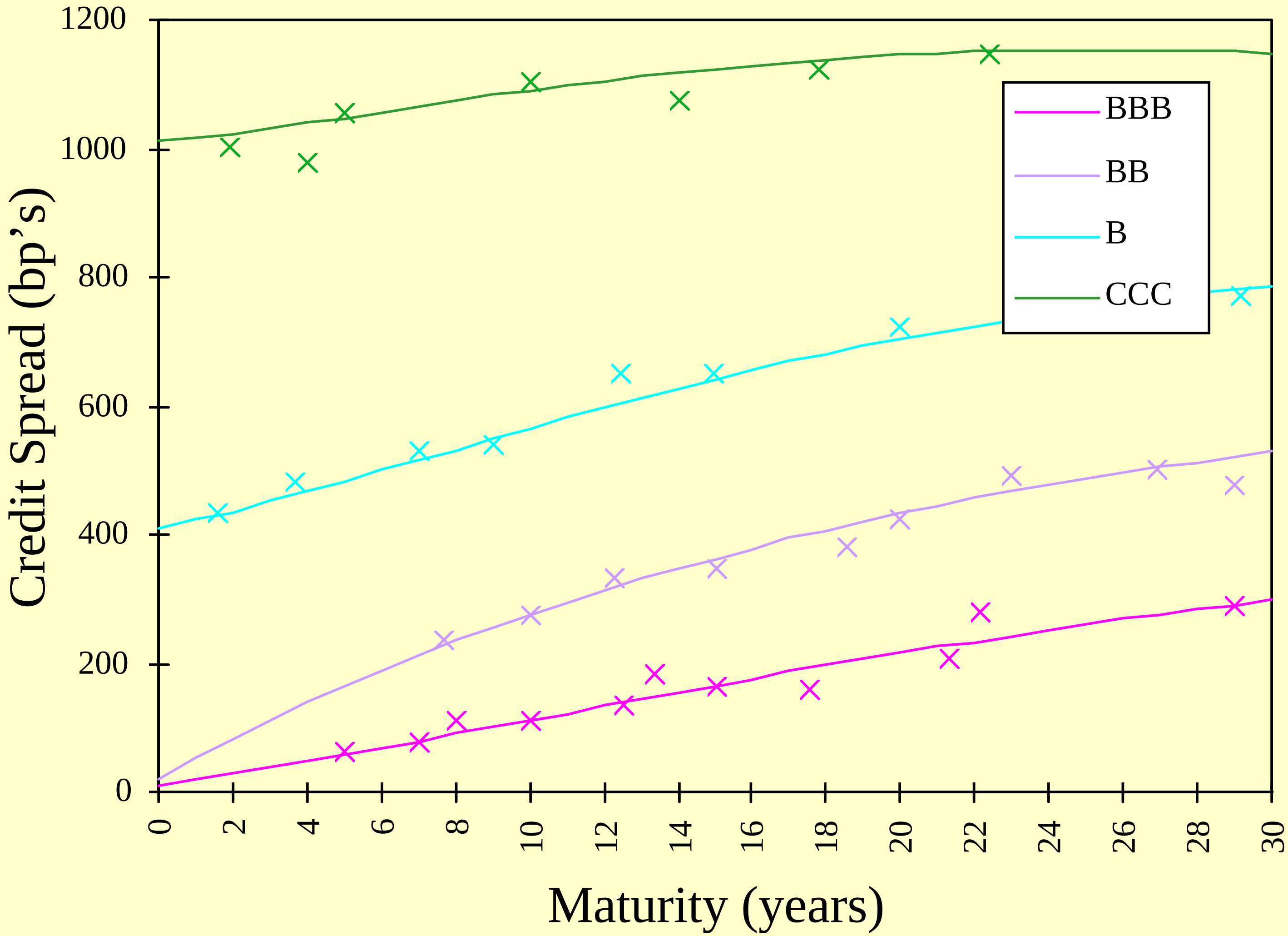
Calibration - Corporate Market (II)



Calibration - US Industrials (I)



Calibration - US Industrials (II)



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Part V
Estimating Credit Spread Volatility

Credit spread volatilities estimates

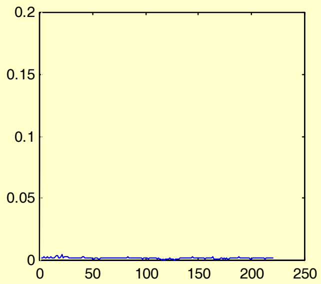
- 74 Bonds
  - 67 Investment grade (Baa and above) US Industrial bonds
  - 7 Speculative grade (Ba and below) Emerging Market bonds

- 8 Model states
  - 1 Moody's Aaa
  - 2 Moody's Aa1 - Aa3
  - 3 Moody's A1 - A3
  - 4 Moody's Baa1 - Baa3

  5 Moody's Ba1 - Ba3
  6 Moody's B1 - B3
  7 Moody's CCC
  8 Default

- Historical generator matrix from Moody's average 1y transition matrix 1920 - 1996

Estimated short term spreads



Credit ratings

AA A

BBB BB

B CCC

Lower ratings have

- higher spread
- higher volatility

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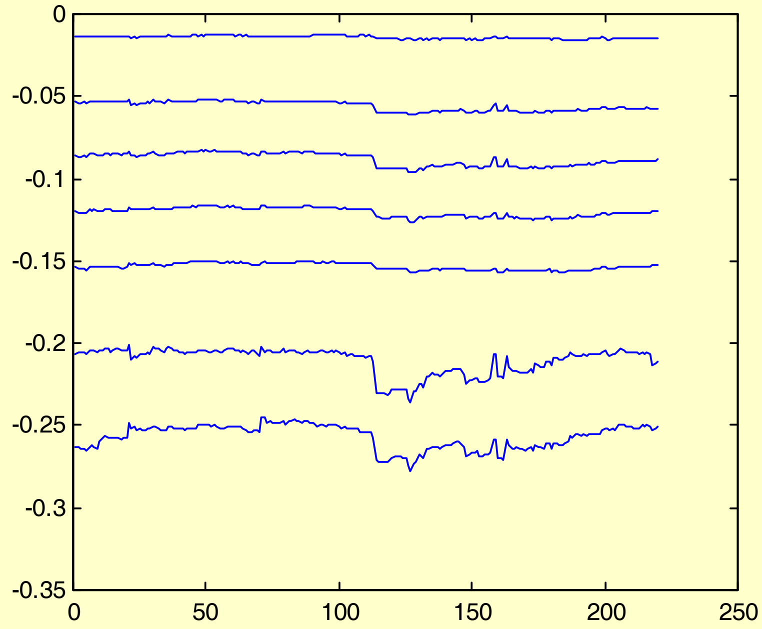
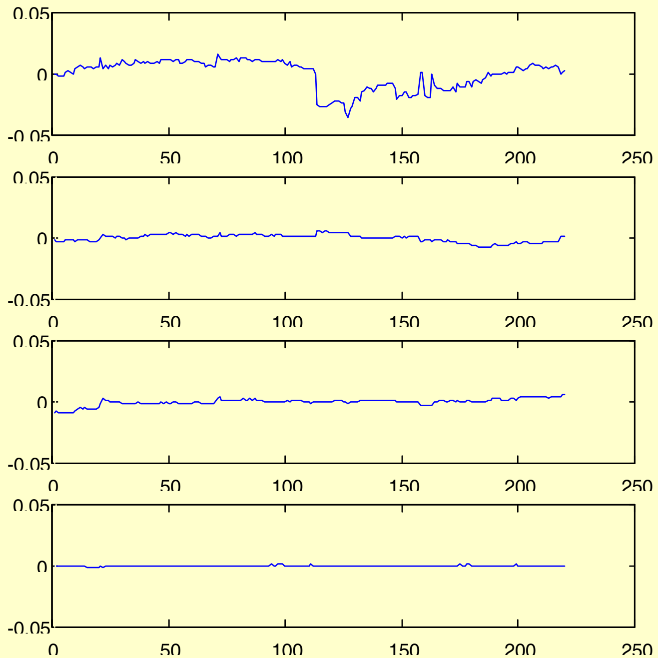
Eigenvalues of generator matrix

Eigenvalues



Principal components



- first 3 principal components account for 99% of the variance

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Advanced Modelling Issues

Stochastic Recovery Rates

- Recovery rates are random with high variance
- Exogenous
- Endogenous - depend on the severity of default

Credit Events Correlated with Interest Rates

- Credit migration and defaults depend on interest rates
- Joint state variables for interest rates and credit spreads
- Incorporate business cycles

Non - Markovian Bankruptcy Process

- Autocorrelated migration process
- Markovian in state space augmented with lagged values

Second Generation Products

- Basket options - credit spread, default correlations
- Multiple Currencies
- Quantos