Modelling Credit Spread Behaviour

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Overview

- Part I
  - Need for Credit Models

- Part II
  - Simple Binomial Model

- Part III
  - Jump-Diffusion Model

- Part IV
  - Credit Migration Model

- Part V
  - Estimating Credit Spread Volatilities
Part I

Need for credit spread models
Need For Credit Models (I)

- Credit derivatives market
- Active management of loan portfolios

Why?

Growth of emerging markets

Active management of counterparty risk in standard derivatives portfolios
Need For Credit Models (II)

Valuing credit derivatives, options on risky bonds, vulnerable derivatives

What for?

- Optimising portfolio risk / return profile
- Relative value analysis

Assessing the credit risk of portfolios - spread and event risk
Need For Credit Models (III)

Estimate the current risk free and risky term structures

Model the evolution of the risk free rate and the credit spread

Calibrate to observed bond and option prices

How?
Credit Data

- Limited / crude data available on credit
- Moody’s historical data (annual)
  - Default probability \(0 \leq p_i \leq 25\%\)
  - Pairwise default correlation \(0 \leq \rho_{ij} \leq 5\%\)
  - Credit migration \(0 \leq q_{kl} \leq 20\%\)
  - Loss given default \(0 \leq l_i \leq 100\%\)
- Default correlation and recovery rate difficult to estimate
- Credit crashes - high default correlation

Credit Modelling
Credit spread for an AA bond

Credit Modelling
Properties of Credit Spreads

**Jump Component**
- Discrete change in default probability
- Credit migration

**Continuous Component**
- Mean reverting
- Change in market price of risk - risk premia

Credit Modelling
Modelling Credit Spread

\[ r_{\text{risky}} = r_{\text{risk free}} + \tilde{\lambda} \]

- Credit Spread
- Constant
- Simple binomial model (Part II)
- Continuous and jump components
- Jump-diffusion model (Part III)
- Model underlying credit migration process (Part IV)

Credit Modelling
Part II

Simple Binomial Model
Simple Binomial Model (I)

- Constant risk free term structure
- Constant recovery rate
- Constant credit spread if no default
- Jump in credit spread if default occurs

- Derive risk neutral default probabilities from risky and risk-free bond prices

Risk neutral default probabilities
- Actual default probabilities
- Risk premia
- Liquidity
- Uncertainty over recovery rate
Simple Binomial Model (II)

\[ v(T) = p(T) \left[ (1 - \tilde{q}(0, T)) + \tilde{q}(0, T)\delta \right] \]

\[ \tilde{q}(0, T) = \frac{1 - v(T) / p(T)}{(1 - \delta)} \]

- price any product with payoff contingent on default event
Part III
Jump-Diffusion Model
Jump-Diffusion Model

Continuous component
- Positive and mean reverting
- Correlated with interest rates

Jump Component
- Jumps of random size occur at random times
- Jumps in only one direction

- Standard implementation and calibration
- Standard numerical pricing algorithms can be used

Risk-free interest rate
- Continuous and mean reverting

Credit Modelling
Risk Free Term Structure (I)

- Assumptions on the future evolution of the instantaneous risk free rate
  - Volatility $\sigma_r \sqrt{r(t)}$ (normal, lognormal, square root, …)
  - Drift / mean reversion
    - Long term mean $\bar{r}(t)$
    - Rate of mean reversion $k_r(t)$

\[
r(t) = r(0) + \int_0^t k_r(t)(\bar{r}(t) - r(t))dt + \int_0^t \sigma_r \sqrt{r(t)}dW_r(t)
\]
Risk Free Term Structure (II)

\[ \sigma_{rr} = 0.1, \ k_r = 10 \]

\[ \sigma_{rr} = 0.1, \ k_r = 2 \]

\[ \sigma_{rr} = 0.2, \ k_r = 2 \]
\[ \tilde{\lambda}(t) = \rho r(t) + x(t) \]
\[ \text{corr}(r(t), x(t)) = 0 \]

- Uncorrelated with interest rates
- Continuous and jump component

- Random jump size \( z \), exponentially distributed
  \[ \theta e^{-\theta z}, z > 0 \]

- Random number of jumps - follows Poisson process
  \[ e^{-\lambda \tau} (\lambda \tau)^n / n! \quad n = 0,1,2,\ldots \quad \tau = \text{time interval} \]
Credit spread component uncorrelated with the risk free interest rate

\[ x(t) = x(0) + \int_0^t k_x(s)(\bar{x}(s) - x(s))\,ds \]
\[ + \int_0^t \sigma_x(s)\sqrt{x(s)}\,dW_x(s) + \sum_{i; \tau(i) \leq t} Z(i) \]
Credit Spread Term Structure (III)

more frequent and larger jumps

Credit Modelling
Part IV
Credit Migration Model
Credit Migration Model

- Jumps modelled as changes in credit ratings and defaults
- Continuous part modelled as continually changing risk premia
- Model jointly assets in various credit classes
- Portfolio management and risk analysis

- Calibration - incorporate economic and historical information
- Flexible in terms of data requirements and number of states
**Markov Chains - Generator Matrix (I)**

- Continuous time Markov chain
- Discrete state space

\[ \tilde{\Lambda} = \begin{pmatrix}
1 & 2 & \cdots & K-1 & K \\
\tilde{\lambda}_1 & \tilde{\lambda}_{12} & \cdot & \tilde{\lambda}_{1,K-1} & \tilde{\lambda}_{1K} \\
\tilde{\lambda}_{21} & \tilde{\lambda}_2 & \cdot & \tilde{\lambda}_{2,K-1} & \tilde{\lambda}_{2K} \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
\tilde{\lambda}_{K-1,1} & \tilde{\lambda}_{K-1,2} & \cdot & \tilde{\lambda}_{K-1} & \tilde{\lambda}_{K-1,K} \\
0 & 0 & \cdot & 0 & 0 
\end{pmatrix} \]

Constant over time

Absorbing state (default)

**Credit Modelling**
Markov Chains - Generator Matrix (II)

- \( I + \tilde{\Lambda} dt \), transition matrix over short period \( dt \)

- \( \lambda_{ij} \geq 0 \), non-negative transition probabilities

- \( \tilde{\lambda}_i = -\sum_{i=1}^{K} \tilde{\lambda}_{ij} \), sum of all probabilities equals 1

\[
\sum_{j \geq i}^{K} \tilde{\lambda}_{ij} \leq \sum_{j \geq k}^{\tilde{\lambda}_{i+1,j}}, \quad \forall \ i, k \quad k \neq i + 1
\]

A state \( i+1 \) is always more risky than state \( i \)

Credit Modelling
**Markov Chains - Transition Matrix**

- **Transition matrix for the period** \( t \) **to** \( T \)
- **Explicit computation**

\[
\tilde{\Lambda} = \Sigma^{-1} D \Sigma \\
\tilde{Q}(t,T) = \Sigma^{-1} \exp[D(T-t)] \Sigma
\]

\[
\tilde{Q}(t,T) = \begin{pmatrix}
\tilde{q}_1(t,T) & \tilde{q}_{1,K-1}(t,T) & \tilde{q}_{1K}(t,T) \\
\tilde{q}_{21}(t,T) & \tilde{q}_{2,K-1}(t,T) & \tilde{q}_{2K}(t,T) \\
\vdots & \vdots & \vdots \\
\tilde{q}_{K-1,1}(t,T) & \tilde{q}_{K-1,K}(t,T) & \tilde{q}_{K-1,K}(t,T) \\
0 & 0 & 1
\end{pmatrix}
\]
• States: uniquely determine default probability
• Credit ratings - can incorporate past credit rating transitions - non-Markovian model

\[ \tilde{\Lambda} \] - Risk neutral generator matrix

\[ \bar{\Lambda} \] - constant

jump in credit spread due to downgrade
• Incorporate stochastic risk premia

\[ \tilde{\Lambda}_{stochastic} = \tilde{\Lambda} \times U(t) \]

continuous process for risk premia

jump in credit spread due to downgrade
**Stochastic Generator Matrix**

- Stochastic generator matrix arises from randomly changing risk premia

\[
\tilde{\Lambda}_{stochastic} = \tilde{\Lambda} \times U(t)
\]

\[
U(t) = U(0) + \int_0^t \left(a - kU(t)\right) dt + \int_0^t \sigma \sqrt{U(t)} dW_t
\]

- Closed form formulae for bond prices
Stochastic Risk Premia

• If eigenvectors are constant, can pose

\[ \Lambda(t, T) = \Sigma^{-1} D(t) \Sigma \]

• Possible evolution of eigenvalues

\[ dX_j = (a_j - b_j X_j) dt + \sigma_j dw, \quad D(t) = \text{diag}(X_j(t))_{j=1}^K \]

• Pricing equation is now modified to

\[ q_{ik}(t, T) = \sum_{j=1}^{K} (\Sigma^{-1})_{ij} E \left[ \exp \left( \int_t^T X_j(s) ds \right) \bigg| D(t) \right] \Sigma_{jK} \]

• Expectation has closed (algebraic) form
  – depends on parameters \( a \, b \, \sigma \) and on \( D(t) \)
Calibration (I)

Prices of risky bonds for various credit classes and maturities $B'(0,T)$

- Least squares estimation
- Adjust historical generator matrix to fit market prices
- Achieve fit closest to historical data

$\tilde{\Lambda}_{stochastic}$

$\Lambda$

- Historical generator matrix (estimated from one year transition matrix)
- Credit spread historical time series

Simulate Credit Spread

Price exotic structures
Calibration (II)

- Least squares fit to match directly observed coupon bond prices (any number)

\[
\min_\Lambda \left\{ \sum_{i=1}^{K} \sum_{j=1}^{J_i} \left( P_{ij}^i - \sum_{h=1}^{T} F_{ij}^i (h) v^i (h; \Lambda) \right)^2 + \sum_{i,j=1}^{K} \frac{(\tilde{\lambda}_{ij} - \lambda_{ij})^2}{\beta_{ij}} \right\}
\]

- Market price of coupon bond for class i
- Coupon at date h
- Prior generator matrix
- Confidence level

- Obtain solution closest to the historical generator matrix \( \Lambda \) - stable calibration

Credit Modelling
Credit Modelling
Credit Modelling
**Calibration - US Industrials (II)**

Credit Spread (bp's) vs. Maturity (years)

- BBB
- BB
- B
- CCC

Credit Modelling
Part V
Estimating Credit Spread Volatility
Credit spread volatilities estimates

• 74 Bonds
  • 67 Investment grade (Baa and above) US Industrial bonds
  • 7 Speculative grade (Ba and below) Emerging Market bonds

• 8 Model states
  • 1 Moody’s Aaa
  • 2 Moody’s Aa1 - Aa3
  • 3 Moody’s A1 - A3
  • 4 Moody’s Baa1 - Baa3
  • 5 Moody’s Ba1 - Ba3
  • 6 Moody’s B1 - B3
  • 7 Moody’s CCC
  • 8 Default

• Historical generator matrix from Moody’s average 1y transition matrix 1920 - 1996

Credit Modelling
Estimated short term spreads

Credit ratings

- AA
- A
- BBB
- BB
- B
- CCC

Lower ratings have
- higher spread
- higher volatility
Eigenvalues of generator matrix

Eigenvalues

Principal components

- first 3 principal components account for 99% of the variance

Credit Modelling
Advanced Modelling Issues

Stochastic Recovery Rates
- Recovery rates are random with high variance
- Exogenous
- Endogenous - depend on the severity of default

Credit Events Correlated with Interest Rates
- Credit migration and defaults depend on interest rates
- Joint state variables for interest rates and credit spreads
- Incorporate business cycles

Non-Markovian Bankruptcy Process
- Autocorrelated migration process
- Markovian in state space augmented with lagged values

Second Generation Products
- Basket options - credit spread, default correlations
- Multiple Currencies
- Quantos

Credit Modelling