Applying Dynamic Hedging Techniques to Credit Derivatives

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Jean-Paul LAURENT
Professor, ISFA Actuarial School, University of Lyon,
Scientific Advisor, BNP PARIBAS, Fixed Income Research and Strategies

Correspondence
laurent.jeanpaul@online.fr or jean-paul.laurent@univ-lyon1.fr
Web page: http://laurent.jeanpaul.free.fr/
On the Edge of Completeness: Purpose and main ideas

- **Purpose:**
  - risk-analysis of exotic credit derivatives:
    - dynamic default swaps, credit spread options, basket default swaps.
  - pricing and hedging exotic credit derivatives.

- **Main ideas:**
  - distinguish between credit spread volatility and default risk.
  - dynamic hedge of exotic default swaps with standard default swaps.

On the Edge of completeness: Overview

- Modelling credit derivatives: the state of the art
- Trading credit risk: closing the gap between supply and demand
- A new approach to credit derivatives modelling:
  - closing the gap between pricing and hedging
  - disentangling default risk and credit spread risk
Modelling credit derivatives: the state of the art

- Modelling credit derivatives: Where do we stand?
- Financial industry approaches
  - Plain default swaps and risky bonds
  - Credit risk management approaches

- The Noah’s arch of credit risk models
  - “firm-value” models
  - Risk-intensity based models
  - Looking desperately for a hedging based approach to pricing.
Modelling credit derivatives: Where do we stand?

Plain default swaps

- Static arbitrage of plain default swaps with short selling underlying bond
  - plain default swaps hedged using underlying risky bond
  - “bond strippers”: allow to compute prices of risky zero-coupon bonds
  - repo risk, squeeze risk, liquidity risk, recovery rate assumptions

- Computation of the P&L of a book of default swaps
  - Involves the computation of a P&L of a book of default swaps
  - The P&L is driven by changes in the credit spread curve and by the occurrence of default.
• Assessing the varieties of risks involved in credit derivatives
  – Specific risk or credit spread risk
    ➢ *prior to default*, the P&L of a book of credit derivatives is driven by changes in credit spreads.
  – Default risk
    ➢ *in case of default*, if unhedged,
    ➢ dramatic jumps in the P&L of a book of credit derivatives.
Modelling credit derivatives: Where do we stand?

The Noah’s arch of credit risk models

- **“firm-value” models:**
  - Modelling of firm’s assets
  - First time passage below a critical threshold

- **risk-intensity based models**
  - Default arrivals are no longer predictable
  - Model conditional local probabilities of default \( \lambda(t) \, dt \)
  - \( \tau \): default date, \( \lambda(t) \) risk intensity or hazard rate

\[
\lambda(t) \, dt = P\left[ \tau \in [t, t + dt] \mid \tau > t \right]
\]

- Lack of a **hedging based approach** to pricing.
  - Misunderstanding of hedging against default risk and credit spread risk
Trading credit risk: 
Closing the gap between supply and demand

- From stone age to the new millennium:
  - Several stages in the « equitization » of credit risk.
    - Financial intermediaries are more sophisticated.
  - Transferring risk from commercial banks to institutional investors:
    - Securitization.
    - Default Swaps
    - Dynamic Default Swaps, Basket Credit Derivatives.
    - Credit Spread Options
  - The previous means tend to be more integrated.
Trading credit risk: Closing the gap between supply and demand

• Securitization of credit risk:
  
  ![Diagram]

  - Credit risk seller
  - SPV
  - Investor 1
  - Investor 2

  - credits
  - senior debt
  - junior debt

• simplified scheme:
  - No residual risk remains within SPV.
  - All credit trades are simultaneous.
• Financial intermediaries provide structuring and arrangement advice.
  – Credit risk seller can transfer loans to SPV or instead use default swaps
• good news: low capital at risk for investment banks

• Good times for modelling credit derivatives
  – No need of hedging models
  – Credit pricing models are used to ease risk transfer
  – Need to assess the risks of various tranches
There is room for financial intermediation of credit risk

- The transfers of credit risk between commercial banks and investors may not be simultaneous.
- Since at one point in time, demand and offer of credit risk may not match.

  - Meanwhile, credit risk remains within the balance sheet of the financial intermediary.

- It is not further required to find customers with exact opposite interest at every new deal.

  - Residual risks remain within the balance sheet of the financial intermediary.
Credit risk management without hedging default risk

- **Emphasis on:**
  - portfolio effects: correlation between default events
  - posting collateral
  - computation of capital at risk, risk assessment

- **Main issues:**
  - capital at risk can be high
  - what is the competitive advantage of investment banks

Credit risk seller --> Default swap --> Credit derivatives trading book --> Default swap --> Bank

Bank --> Default swap --> Investor 1

Bank --> Default swap --> Investor 2
Credit risk management with hedging default risk

- Trading against other dealers enhances ability to transfer credit risk by lowering capital at risk
New ways to transfer credit risk: dynamic default swaps

- Anatomy of a general dynamic default swap
  - A dynamic default swap is like a standard default swap but with variable nominal (or exposure)
  - However the periodic premium paid for the credit protection remains fixed.
  - The protection payment arises at default of one given single risky counterparty.

- Examples
  - cancellable swaps
  - quanto default swaps
  - credit protection of vulnerable swaps, OTC options (stand-alone basis)
  - credit protection of a portfolio of contracts (full protection, excess of loss insurance, partial collateralization)
A new approach to credit derivatives modelling based on an hedging point of view

- **Rolling over the hedge:**
  - Short term default swaps v.s. long-term default swaps
  - Credit spread transformation risk
- **Dynamic Default Swaps, Basket Default Swaps**
  - Hedging default risk through dynamics holdings in standard default swaps
  - Hedging credit spread risk by choosing appropriate default swap maturities
  - Closing the gap between pricing and hedging
- **Practical hedging issues**
  - Uncertainty at default time
  - Managing net residual premiums
Long-term Default Swaps v.s. Short-term Default Swaps
Rolling over the hedge

- **Purpose:**
  - Introduction to dynamic trading of default swaps
  - Illustrates how default and credit spread risk arise

- **Arbitrage between long and short term default swap**
  - sell one long-term default swap
  - buy a series of short-term default swaps

- **Example:**
  - default swaps on a FRN issued by BBB counterparty
  - 5 years default swap premium : 50bp, recovery rate = 60%

[Diagram]

Credit derivatives
dealer

- Until default, 50 bp
- If default, 60%

Client
Long-term Default Swaps v.s. Short-term Default Swaps

Rolling over the hedge

- Rolling over short-term default swap
  - at inception, one year default swap premium: 33bp
  - cash-flows after one year:

  ![Diagram showing the flow of credit derivatives from dealer to market with a 33bp premium and 60% if default]

- Buy a one year default swap at the end of every yearly period, if no default:
  - Dynamic strategy,
  - future premiums depend on future credit quality
  - future premiums are unknown

  ![Diagram showing the flow of credit derivatives from dealer to market with an unknown premium and 60% if default]
Long-term Default Swaps v.s. Short-term Default Swaps
Rolling over the hedge

• *Risk analysis* of rolling over short term against long term default swaps

  - Exchanged cash-flows:
    - Dealer receives 5 years (fixed) credit spread,
    - Dealer pays 1 year (variable) credit spread.

• **Full one to one protection at default time**
  - the previous strategy has eliminated one source of risk, that is default risk
Long-term Default Swaps v.s. Short-term Default Swaps
Rolling over the hedge

- negative exposure to an increase in short-term default swap premiums
  - if short-term premiums increase from 33bp to 70bp
  - reflecting a lower (short-term) credit quality
  - and no default occurs before the fifth year

Credit derivatives dealer

70 bp

Market + Client

50 bp

- Loss due to negative carry
  - long position in long term credit spreads
  - short position in short term credit spreads
Hedging exotic default swaps: main features

- Exotic credit derivatives can be hedged against default:
  - Constrains the amount of underlying standard default swaps.
  - Variable amount of standard default swaps.
  - Full protection at default time by construction of the hedge.
  - No more discontinuity in the P&L at default time.
  - “Safety-first” criteria: main source of risk can be hedged.
  - Model-free approach.

- Credit spread exposure has to be hedged by other means:
  - Appropriate choice of maturity of underlying default swap
  - Computation of sensitivities with respect to changes in credit spreads are model dependent.
**Hedging Default Risk in Dynamic Default Swap**

- **Dynamic Default Swap**
  - client pays to dealer a periodic premium $p_T(C)$ until default time $\tau$, or maturity of the contract $T$.
  - dealer pays $C(\tau)$ to client at default time $\tau$, if $\tau \leq T$.

- **Hedging side:**
  - Dynamic strategy based on standard default swaps:
  - At time $t$, hold an amount $C(t)$ of standard default swaps
  - $\lambda(t)$ denotes the periodic premium at time $t$ for a short-term default swap
**Hedging side:**

Credit derivatives dealer $\lambda(t) C(t)$ until default $C(\tau)$ if default

- Amount of standard default swaps equals the (variable) credit exposure on the dynamic default swap.

**Net position is a “basis swap”:**

Credit derivatives dealer $\lambda(t) C(t)$ until default $p_T(C)$ until default

- The client transfers credit spread risk to the credit derivatives dealer.
Closing the gap between pricing and hedging

- Risky discount factors
  - Discount bond prices
  - Short term credit spreads
- PV of plain and dynamic default swaps
  - Default payment leg, premium payment leg
  - Default intensity and short term default swap premiums
- Cost of rolling over the hedge
- Dynamics of the PV of dynamic default swaps
  - Looking at theta effects
- Hedging credit spread risk
- Credit spread options
Closing the gap between pricing and hedging

Risky discount factors

- \( \tau \), default time, \( P_t(\tau \in [t, t+dt] \mid \tau>t) = \lambda(t)dt \), \( \lambda \) default intensity.

- \( I(t) = 1_{\{\tau>t\}} \) indicator function.
  - \( I(t) \) jumps from 1 to 0 at time \( \tau \).

- \( E_t[ I(t) - I(t+dt) ] = E_t[1_{\{\tau\in [t, t+dt]\}}] = P_t(\tau \in [t, t+dt]) = \lambda(t)I(t)dt \)

- Thus - \( \lambda(t) \) is the expected relative variation of \( I(t) \) and:
  \[
  E_t \left[ 1_{\{\tau>T\}} \right] = 1_{\{\tau>t\}} E_t \left[ \exp - \int_t^T \lambda(s)ds \right]
  \]

- Think of \( I(t) \) as a stochastic nominal amortizing at rate \( \lambda(t) \)
  - Parallels mortgages where \( \tau \) and \( \lambda \), prepayment date and rate.

- **Risky discount bond** with maturity \( T \): pays \( 1_{\{\tau>t\}} \) at time \( T \)
  - Denote by \( \overline{B}(t, T) \) its \( t \)-time price and by \( r() \) risk-free short rate
• Risky discount bond price:

\[ \bar{B}(t,T) = E_t \left[ 1_{\{\tau>T\}} \exp \left( - \int_t^T r(s) ds \right) \right] = 1_{\{\tau>t\}} E_t \left[ \exp \left( - \int_t^T (r + \lambda)(s) ds \right) \right] \]

- \( \lambda \) is the short term credit spread

• More generally let \( X_T \) be a payoff paid at \( T \), if \( \tau>T \):

\[ PV_{X}(t) = E_t \left[ X_T 1_{\{\tau>T\}} \exp \left( - \int_t^T r(s) ds \right) \right] = 1_{\{\tau>t\}} E_t \left[ X_T \exp \left( - \int_t^T (r + \lambda)(s) ds \right) \right] \]

• \( \exp \left( - \int_t^T (r + \lambda)(s) ds \right) \) stochastic risky discount factor
Closing the gap between pricing and hedging: 

**PV of plain default swaps**

- **Before default, time** $u$ -PV of a plain default swap:
  - Maturity $T$, *continuously* paid premium $p$, recovery rate $\delta$
  - Risk-free short rate $r$, default intensity $\lambda$
  - $E_u$ expectation conditional on information carried by financial prices.
    
    $$E_u\left[\int_u^T \exp\left(-\int_u^t (r + \lambda)(s)ds\right) \times (1 - \delta)\lambda(t) - p \right] dt$$
    
    - $r + \lambda$ is the « risky » short rate: payoffs discounted at a higher rate
    - Similar to an index amortizing swap (payments only if no prepayment).

- **PV of default payment leg**
  
  $$E_u\left[\int_u^T \exp\left(-\int_u^t (r + \lambda)(s)ds\right) \times (1 - \delta)\lambda(t)dt\right]$$

- **PV of premium payment leg**
  
  $$p \times E_u\left[\int_u^T \exp\left(-\int_u^t (r + \lambda)(s)ds\right)dt\right]$$
Closing the gap between pricing and hedging: 
**PV of plain default swaps**

- Current market premium $p_{u,T}$ is such that PV=0.

**Pricing equation:**

$$E_u \left[ \int_{u}^{T} \left( \exp \left( \int_{u}^{t} (r + \lambda) (s)ds \right) \times \left( (1 - \delta) \lambda(t) - p_{u,T} \right) dt \right) \right] = 0$$

- For short maturities $T=u+du$, pricing equation provides:

$$p_{u,T} = (1 - \delta) \lambda(u)$$

- And for digital default swaps ($\delta=0$), we get: $p_{u,T} = \lambda(u)$

- $\lambda$, **default intensity** = *short term default swap premium*
• **Before default**, time $t$ - PV of a dynamic default swap
  - Payment $C(\tau)$ at default time if $\tau<T$:

• **PV of default payment leg**

\[
P V (u) = E_u \left[ \int_u^T \left( \exp - \int_u^t (r + \lambda) (s) ds \right) C(t) \lambda(t) ds \right]
\]

  - This embeds the plain default swap case where $C(\tau)=1-\delta$

• **PV of premium payment leg**

\[
p \times E_u \left[ \int_u^T \left( \exp - \int_u^t (r + \lambda) (s) ds \right) dt \right]
\]

  - Same as in the case of plain default swap
What is the cost of hedging default risk?

PV of default payment leg:

\[ E \left[ \int_0^T \left( \exp - \int_0^t (r + \lambda(s)) ds \right) \lambda(t) C(t) dt \right] \]

- Discounting term
- Premium paid at time \( t \) on protection portfolio

equals PV of premiums paid on the hedging portfolio.

Pricing and rolling over the hedge approaches are consistent.
Consider a defaultable interest rate swap (with unit nominal)

- We are **default-free**, our counterparty is **defaultable** (default intensity \( \lambda(t) \)).
- We consider a (fixed-rate) **receiver** swap on a **standalone** basis.

Recovery assumption, payments in case of default:

- if default at time \( \tau \), compute the default-free value of the swap:
  \[
  PV_\tau = \delta (PV_\tau)^+ + (PV_\tau)^- = PV_\tau - (1-\delta)(PV_\tau)^+
  \]
- and get:
  - \( 0 \leq \delta \leq 1 \) recovery rate, \((PV_\tau)^+ = \text{Max}(PV_\tau, 0)\) , \((PV_\tau)^- = \text{Min}(PV_\tau, 0)\)
  - **In case of default**, 
    - we **receive** default-free value \( PV_\tau \)
    - **minus**
    - loss equal to \((1-\delta)(PV_\tau)^+\).
Exemple: defaultable interest rate swap

- Using a hedging instrument rather than a credit reserve
  - Consider a **dynamic default swap** paying \((1-\delta)(PV_\tau)^+\) at default time \(\tau\) (if \(\tau \leq T\)), where \(PV_\tau\) is the present value of a default-free swap with **same fixed rate** than defaultable swap.
  - At default, we receive \((1-\delta)(PV_\tau)^+ + PV_\tau - (1-\delta)(PV_\tau)^+ = PV_\tau\)
  - PV of **default payment leg** is equal to the Present Value of the **loss** \((1-\delta)(PV_\tau)^+\)
    \[
    E\left[ \int_0^T \left( \exp\left( \int_0^u (r + \lambda)(v)dv \right) \lambda(t)(1-\delta)(PV_\tau)^+ dt \right] \right.
    \]
  - Hedge against default by holding \((PV_\tau)^+\) ordinary default swaps at time \(t\).
Exemple: defaultable interest rate swap

- **Randomly exercised swaption:**
  - Assume for simplicity no recovery ($\delta=0$).
  - Interpret default time as a random time $\tau$ with intensity $\lambda(t)$.
  - At that time, defaulted counterparty “exercises” a swaption, i.e. decides whether to cancel the swap according to its present value.
  - PV of default-losses equals price of that *randomly exercised swaption*

- **American Swaption**
  - PV of *American swaption* equals the supremum over *all possible stopping times* of *randomly exercised swaptions*.
    - *But*, the upper bound can be reached for *special default arrival dates*:
    - $\lambda(t)=0$ above exercise boundary and $\lambda(t)=\infty$ on exercise boundary
    - Usually, PV of American swaption $>>$ PV of default payment leg.
Explaining theta effects with and without hedging

- **Different aspects** of “carrying” credit contracts through time.
  - Assume “historical” and “risk-neutral” intensities are equal.
- Consider a **short** position in a dynamic default swap.
- Present value of the deal provided by:
  \[
  PV(u) = E_u \left[ \int_u^T \left( \exp - \int_u^t (r + \lambda(s))ds \right) \times (p_T - \lambda(t)C(t))dt \right]
  \]
  
  (after computations) **Net expected capital gain:**
  \[
  E_u \left[ PV(u + du) - PV(u) \right] = \left( r(u) + \lambda(u) \right) PV(u)du + \left( \lambda(u)C(u) - p_T \right) du
  \]
  
  **Accrued cash-flows (received premiums):** \( p_T du \)
  - By summation, Incremental P&L (if no default between \( u \) and \( u+du \)):
    \[
    r(u)PV(u)du + \lambda(u)\left( C(u) + PV(u) \right)du
    \]
Explaining theta effects with and without hedging

**Apparent extra return effect**: \( \lambda(u)(C(u) + PV(u))du \)
- But, probability of default between \( u \) and \( u + du \): \( \lambda(u)du \).
- Losses in case of default:
  - Commitment to pay: \( C(u) \)
  - Loss of PV of the credit contract: \( PV(u) \)
  - \( PV(u) \) consists in **unrealised** capital gains or losses in the credit derivatives book that “disappear” in case of default.
- Expected loss charge: \( \lambda(u)(C(u) + PV(u))du \)

**Hedging aspects**:
- If we hold \( C(u) + PV(u) \) short-term digital default swaps, we are protected at default-time (no jump in the P&L).
- Premiums to be paid: \( \lambda(u)(C(u) + PV(u))du \)
- Same average rate of return, but smoother variations of the P&L.
Hedging default risk in dynamic default swaps

- **PV at time \( u \) of a digital default swap**

  \[
  PV(u) = 1_{\{\tau > u\}} E_u \left[ \int_u^T \left( \exp - \int_u^t (r + \lambda)(s) ds \right) \times (\lambda(t) - p) dt \right] + 1_{\{\tau \leq u\}} \exp \int_\tau^u r(t) dt
  \]

  - At default time \( \tau \), PV switches from

  \[
  E_u \left[ \int_u^T \left( \exp - \int_u^t (r + \lambda)(s) ds \right) \times (\lambda(t) - p) dt \right]
  \]

  - to one (default payment). If digital default swap at the money, \( dPV(\tau) = 1 \)

- **PV at time \( u \) of a dynamic default swap with payment \( C \): \( PV_C(u) \)**

  \[
  1_{\{\tau > u\}} E_u \left[ \int_u^T \left( \exp - \int_u^t (r + \lambda)(s) ds \right) \times (\lambda(t)C(t) - p_C) dt \right] + 1_{\{\tau \leq u\}} C(\tau) \exp \int_\tau^u r(t) dt
  \]

  - At default time \( \tau \), PV switches from predefault market value to \( C(\tau) \)

- **Rolling over the hedge**: we hold \( C(u) \) digital default swaps

  - Variation of PV on the hedging portfolio \( C(u) \) \( dPV(u) \)

  - At default time \( \tau \), PV hedging portfolio jumps of \( C(\tau) dPV(\tau) = C(\tau) \)

- **Complete hedge involves holding \( C(u) + PV_C(u) \) default swaps: model free.**
Hedging Default risk and credit spread risk in Dynamic Default Swaps

- **Purpose**: joint hedge of default risk and credit spread risk
- **Hedging default risk** only constrains the amount of underlying standard default swap.
  - Maturity of underlying default swap is arbitrary.
- Choose maturity to be protected against credit spread risk
  - PV of dynamic default swaps and standard default swaps are sensitive to the level of credit spreads
  - Sensitivity of standard default swaps to a shift in credit spreads increases with maturity
  - Choose maturity of underlying default swap in order to equate sensitivities.
Example:

- dependence of simple default swaps on defaultable forward rates.
- Consider a $T$-maturity default swap with continuously paid premium $p$. Assume zero-recovery (digital default swap).
- PV (at time 0) of a long position provided by:

\[ PV = E \left[ \int_0^T \left( \exp - \int_0^t (r + \lambda(s)) ds \right) \times (\lambda(t) - p) dt \right] \]

- where $r(t)$ is the short rate and $\lambda(t)$ the default intensity.
- Assume that $r(.)$ and $\lambda(.)$ are independent.
- $B(0,t)$: price at time 0 of a $t$-maturity default-free discount bond
- $f(0,t)$: corresponding forward rate

\[ B(0, t) = E \left[ \exp - \int_0^t r(u) du \right] = \exp - \int_0^t f(0, u) du \]
Hedging credit spread risk

- Let $\bar{B}(0, t)$ be the defaultable discount bond price and $\bar{f}(0, t)$ the corresponding instantaneous forward rate:

$$
\bar{B}(0, t) = E\left[\exp\left(-\int_{0}^{t}(r + \lambda(u))du\right)\right] = \exp\left(-\int_{0}^{t}\bar{f}(0, u)du\right)
$$

- Simple expression for the PV of the $T$-maturity default swap:

$$
PV(T) = \int_{0}^{T} \bar{B}(0, t)\left(\bar{f}(0, t) - f(0, t) - p\right)dt
$$

- The derivative of default swap present value with respect to a shift of defaultable forward rate $\bar{f}(0, t)$ is provided by:

$$
\frac{\partial PV}{\partial \bar{f}}(t) = PV(t) - PV(T) + \bar{B}(0, t)
$$

⇒ $PV(t)$-$PV(T)$ is usually small compared with $\bar{B}(0, t)$. 
Hedging credit spread risk

– Similarly, we can compute the sensitivities of plain default swaps with respect to default-free forward curves \( f(0,t) \).
– And thus to credit spreads.
– Same approach can be conducted with the dynamic default swap to be hedged.

➢ All the computations are model dependent.

– Several maturities of underlying default swaps can be used to match sensitivities.

➢ For example, in the case of defaultable interest rate swap, the nominal amount of default swaps \((PV_\tau)^+\) is usually small.

➢ Single default swap with nominal \((PV_\tau)^+\) has a smaller sensitivity to credit spreads than defaultable interest rate swap, even for long maturities.

➢ Short and long positions in default swaps are required to hedge credit spread risk.
Hedging credit spread risk

- Denote by $I(u) = 1_{\{\tau > u\}}$, $dI(u) = \text{variation of jump part}$.

- Digital default swap:
  - PV prior to default: $PV^b(u) = E_u \left[ \int_u^T \exp \left( - \int_u^t (r + \lambda(s)) ds \right) \times (\lambda(u) - p) dt \right]$
  - PV after default: $PV^b(u) = \exp \int_u^T r(t) dt$
  - PV whenever: $PV(u) = I(u) PV^b(u) + (1 - I(u)) PV^a(u)$

\[
dPV(u) = \left( PV^b(u) - PV^a(u) \right) dI(u) + I(u) dPV^b(u) + (1 - I(u)) dPV^a(u)
\]

  Discountinuous part default risk
  Continuous part (credit spread risk)

- Continuous part is hedged by usual delta, gamma analysis,

- Discontinuous part: constrains the amount of hedging default swaps
  - After hedging default risk, no jump in the PV at default time.
Hedging credit spread options

- Option to enter a given default swap with premium $p$, maturity $T'$ at exercise date $T$.
  - Call option provides positive payoff if credit spreads increase.
  - Credit spread risk
  - If default prior to $T$, cancellation of the option
  - Default risk

- The PV is of the form $PV(u) = 1_{\{\tau > u\}} PV^b(u)$
  - Hedge default risk by holding an amount of $PV^b(u)$ default swaps.
  - $PV^b(u)$ is usually small compared with payments involved in default swaps.
  - $PV^b(u)$ depends on risk-free and risky curves (mainly on credit spreads).
  - Credit spread risk is also hedged through default swaps.

- Our previous framework for hedging default risk and credit spread risk still holds.
Real World hedging and risk-management issues

• uncertainty at default time
  – illiquid default swaps
  – recovery risk
  – simultaneous default events

• Managing net premiums
  – Maturity of underlying default swaps
  – Lines of credit
  – Management of the carry
  – Finite maturity and discrete premiums
  – Correlation between hedging cash-flows and financial variables
New ways to transfer credit risk: Basket default swaps

- Consider a basket of $M$ risky bonds
  - multiple counterparties
- First to default swaps
  - protection against the first default
- $N$ out of $M$ default swaps ($N < M$)
  - protection against the first $N$ defaults
- Hedging and valuation of basket default swaps
  - involves the joint (multivariate) modelling of default arrivals of issuers in the basket of bonds.
  - Modelling accurately the dependence between default times is a critical issue.
Hedging Default Risk in Basket Default Swaps

• Example: first to default swap from a basket of two risky bonds.
  – If the first default time occurs before maturity,
  – The seller of the first to default swap pays the non recovered fraction of the defaulted bond.
  – Prior to that, he receives a periodic premium.

• Assume that the two bonds cannot default simultaneously
  – We moreover assume that default on one bond has no effect on the credit spread of the remaining bond.

• How can the seller be protected at default time?
  – The only way to be protected at default time is to hold two default swaps with the same nominal than the nominal of the bonds.
  – The maturity of underlying default swaps does not matter.
Consider a first to default swap associated with a basket of two defaultable loans.

- Hedging portfolios based on standard underlying default swaps
- Uncertain hedge ratios if:
  - simultaneous default events
  - Jumps of credit spreads at default times

Simultaneous default events:

- If counterparties default altogether, holding the complete set of default swaps is a conservative (and thus expensive) hedge.
- In the extreme case where default always occur altogether, we only need a single default swap on the loan with largest nominal.
- In other cases, holding a fraction of underlying default swaps does not hedge default risk (if only one counterparty defaults).
Real world hedging and risk-management issues
Case study: hedge ratios for first to default swaps

- What occurs if there is a *jump in the credit spread* of the second counterparty after default of the first?
  - default of first counterparty means *bad news* for the second.

- If hedging with short-term default swaps, *no capital gain* at default.
  - Since PV of short-term default swaps is not *sensitive* to credit spreads.

- This is not the case if hedging with long term default swaps.
  - If credit spreads *jump*, PV of long-term default swaps *jumps*.

- Then, the amount of hedging default swaps can be *reduced*.
  - This reduction is *model-dependent*. 
On the edge of completeness?

- **Firm-value** structural default models:
  - Stock prices follow a diffusion process (no jumps).
  - Default occurs at first time the stock value hits a barrier.

- **In this modelling**, default credit derivatives can be **completely** hedged by trading the stocks:
  - “Complete” pricing and hedging model:

- **Unrealistic features for hedging** *basket default swaps*:
  - Because default times are predictable, *hedge ratios are close to zero* except for the counterparty with the smallest “distance to default”.

On the edge of completeness?

**hazard rate based models**

- In **hazard rate** based models:
  - default is a sudden, *non predictable* event,
  - that causes a sharp *jump* in defaultable bond prices.
  - Most dynamic default swaps and basket default derivatives have payoffs that are *linear* (at default) in the prices of defaultable bonds.
  - Thus, good news: default risk can be *hedged*.
  - Credit spread risk can be *substantially reduced*, but model risk.
  - More *realistic* approach to default.
  - *Hedge ratios* are *robust* with respect to default risk.
On the edge of completeness

Conclusion

• Looking for a better understanding of credit derivatives
  – payments in case of default,
  – volatility of credit spreads.

• Bridge between risk-neutral valuation and the cost of the hedge approach.

• **dynamic** hedging strategy based on *standard default swaps*.
  – hedge ratios in order to get protection at default time.
  – hedging default risk is *model-independent*.
  – importance of quantitative models for a better management of the P&L and the residual premiums.