Applying Dynamic Hedging Techniques to Credit Derivatives

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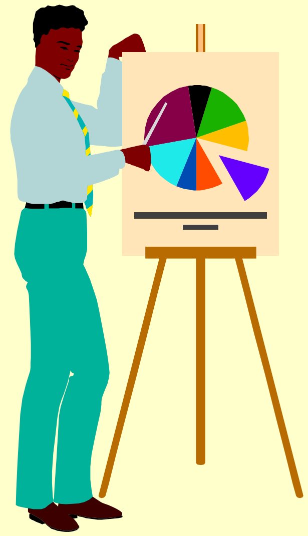
On the Edge of Completeness: Purpose and main ideas

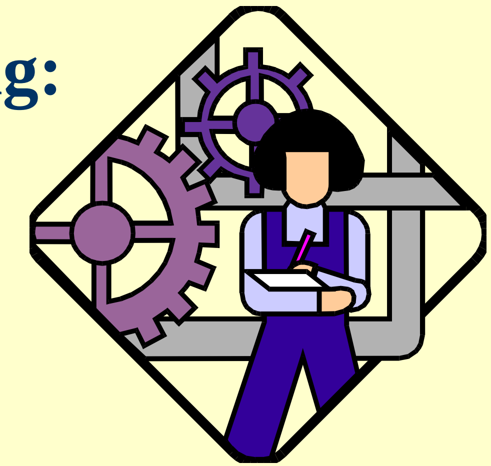
- **Purpose:**
  - **risk-analysis of exotic credit derivatives:**
    - dynamic default swaps, credit spread options, basket default swaps.
  - **pricing and hedging exotic credit derivatives.**
- **Main ideas:**
  - **distinguish between credit spread volatility and default risk.**
  - **dynamic hedge of exotic default swaps with standard default swaps.**
- **Reference paper: “On the edge of completeness”, RISK, October 1999.**

On the Edge of completeness: Overview



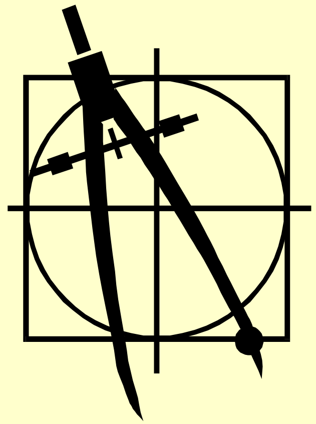
- **Modelling credit derivatives: the state of the art**
- **Trading credit risk : closing the gap between supply and demand**

- **A new approach to credit derivatives modelling:**
  - **closing the gap between pricing and hedging**
  - **disentangling default risk and credit spread risk**



Modelling credit derivatives: the state of the art

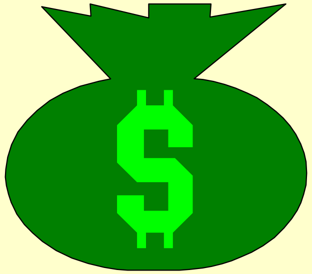
- **Modelling credit derivatives : Where do we stand ?**
- **Financial industry approaches**
  - **Plain default swaps and risky bonds**
  - **credit risk management approaches**



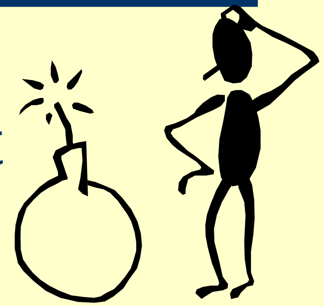
- **The Noah's arch of credit risk models**
  - **"firm-value" models**
  - **risk-intensity based models**
  - **Looking desperately for a hedging based approach to pricing.**

Modelling credit derivatives : Where do we stand ?
Plain default swaps

- **Static arbitrage of plain default swaps with short selling underlying bond**
  - **plain default swaps hedged using underlying risky bond**
  - **“bond strippers” : allow to compute prices of risky zero-coupon bonds**
  - **repo risk, squeeze risk, liquidity risk, recovery rate assumptions**
- **Computation of the P&L of a book of default swaps**
  - **Involves the computation of a P&L of a book of default swaps**
  - **The P&L is driven by changes in the credit spread curve and by the occurrence of default.**



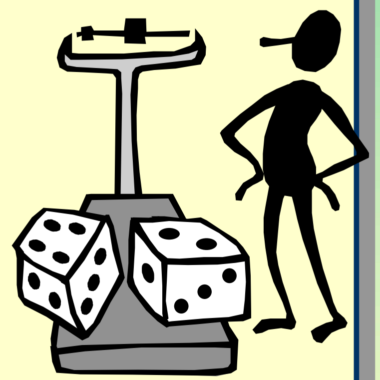
Modelling credit derivatives: Where do we stand ?
Credit risk management



- **Assessing the varieties of risks involved in credit derivatives**
  - **Specific risk or credit spread risk**
    - *prior to default*, the P&L of a book of credit derivatives is driven by changes in credit spreads.
  - **Default risk**
    - *in case of default*, if unhedged,
    - dramatic jumps in the P&L of a book of credit derivatives.

Modelling credit derivatives: Where do we stand ?
The Noah's arch of credit risk models

- **"firm-value" models :**
  - **Modelling of firm's assets**
  - **First time passage below a critical threshold**
- **risk-intensity based models**
  - **Default arrivals are no longer <u>predictable</u>**
  - **Model conditional local probabilities of default $\lambda(t)\,dt$**
  - **τ : default date, $\lambda(t)$ *risk intensity* or *hazard rate***



$$\lambda(t)dt = P\left[\tau \in \left[t, t + dt\right[\,\middle|\, \tau > t\right]$$

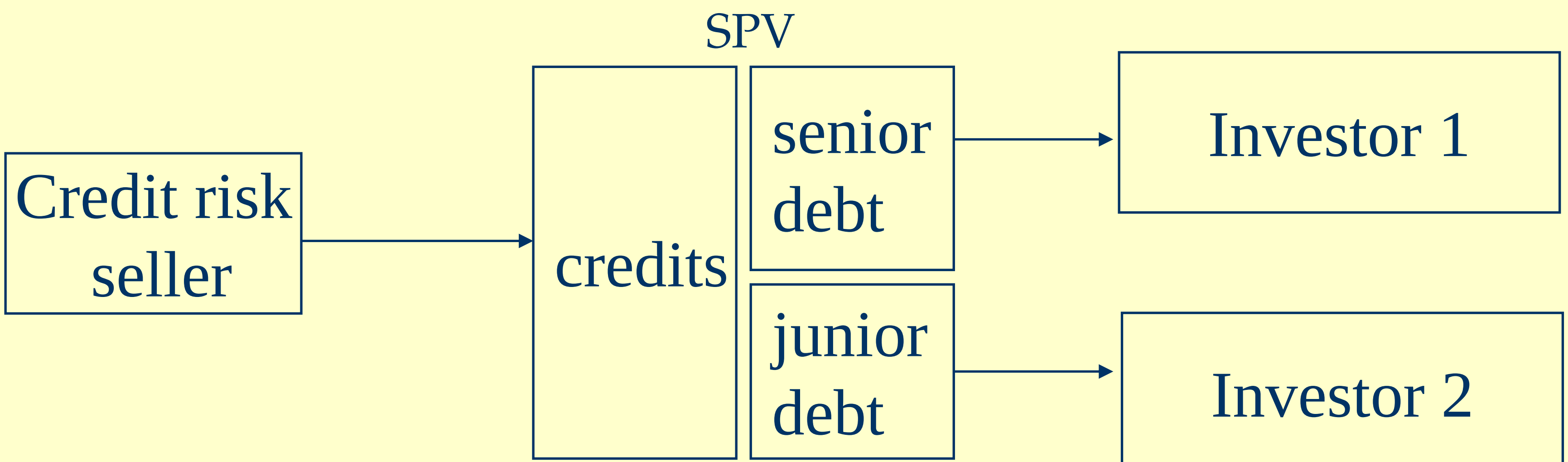
- **Lack of a <u>hedging based approach</u> to pricing.**
  - **Misunderstanding of hedging against default risk and credit spread risk**

Trading credit risk: Closing the gap between supply and demand

- **From stone age to the new millennium:**
  - **Several stages in the « equitization » of credit risk.**
    - Financial intermediaries are more sophisticated.
  - **Transferring risk from commercial banks to institutional investors:**
    - Securitization.
    - Default Swaps
    - Dynamic Default Swaps, Basket Credit Derivatives.
    - Credit Spread Options
  - **The previous means tend to be more integrated.**

Trading credit risk: Closing the gap between supply and demand

- **Securitization of credit risk:**



- **simplified scheme:**
  - **No residual risk remains within SPV.**
  - **All credit trades are simultaneous.**

Trading Credit Risk:
Closing the gap between supply and demand

- **Financial intermediaries provide structuring and arrangement advice.**
  - **Credit risk seller can transfer loans to SPV or instead use default swaps**
- **good news : low capital at risk for investment banks**

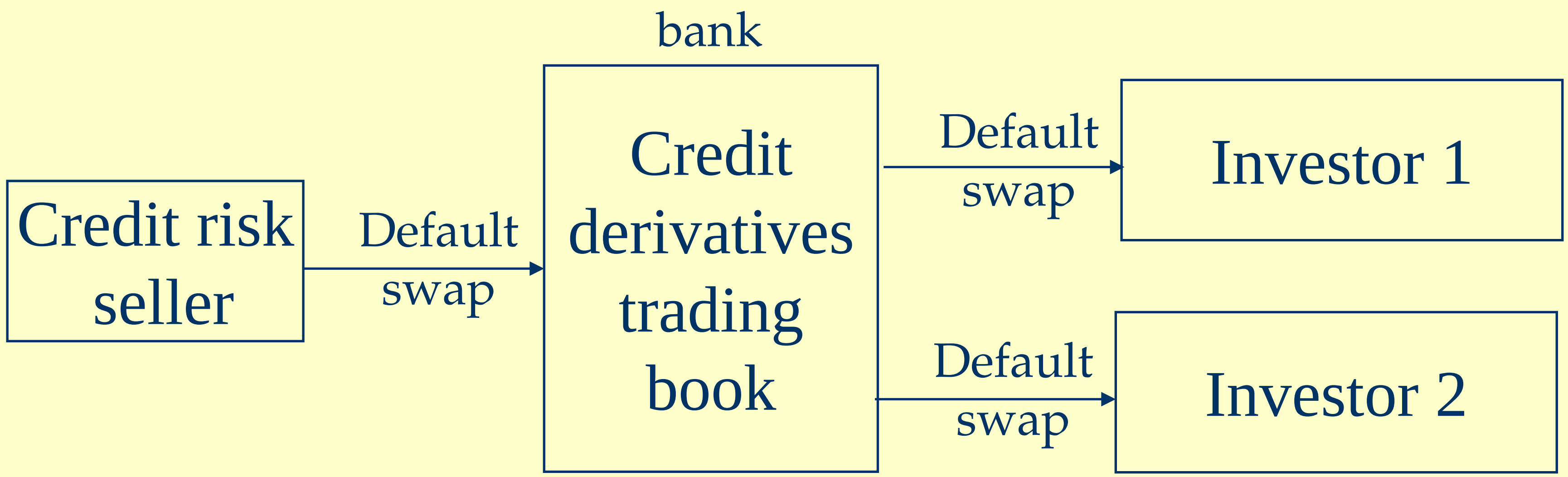
- **Good times for modelling credit derivatives**
  - **No need of <u>hedging</u> models**
  - **credit pricing models are used to ease risk transfer**
  - **need to assess the risks of various tranches**

Trading Credit Risk: Closing the gap between supply and demand

- **There is room for financial intermediation of credit risk**
  - **The transfers of credit risk between commercial banks and investors may not be <u>simultaneous</u>.**
  - **Since at one point in time, demand and offer of credit risk may not match.**
    - ➢ <u>Meanwhile</u>, credit risk remains within the balance sheet of the financial intermediary.
  - **It is not further required to find customers with exact opposite interest at every new deal.**
    - ➢ <u>Residual risks</u> remain within the balance sheet of the financial intermediary.

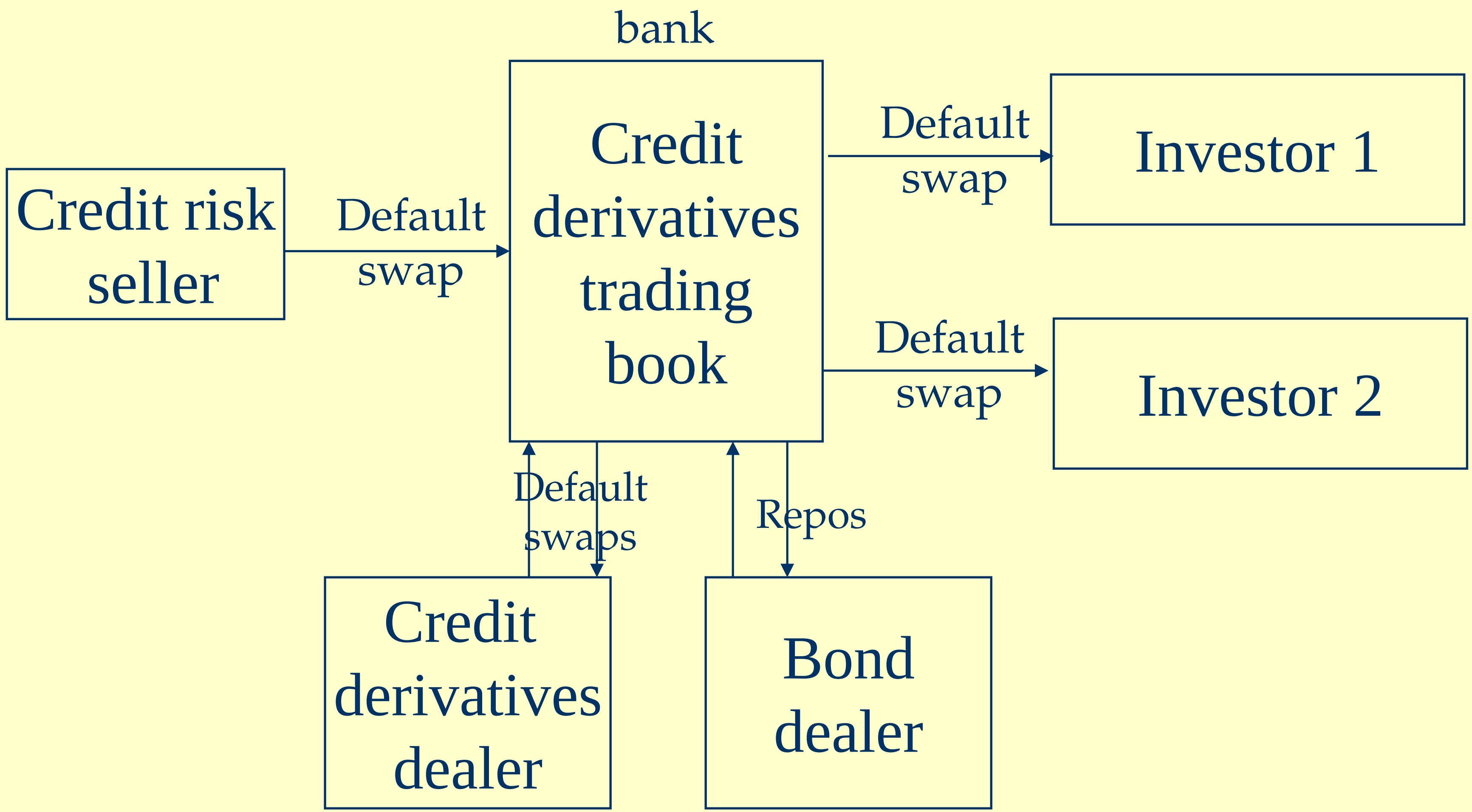
Credit risk management without hedging default risk

- **Emphasis on:**
  - **portfolio effects: correlation between default events**
  - **posting collateral**
  - **computation of capital at risk, risk assessment**
- **Main issues:**
  - **capital at risk can be high**
  - **what is the competitive advantage of investment banks**



Credit risk management with hedging default risk

- **Trading against other dealers enhances ability to transfer credit risk by lowering capital at risk**



New ways to transfer credit risk :
dynamic default swaps

- **Anatomy of a general dynamic default swap**
  - **A dynamic default swap is like a standard default swap but with variable nominal (or exposure)**
  - **However the periodic premium paid for the credit protection remains fixed.**
  - **The protection payment arises at default of one given single risky counterparty.**
- **Examples**
  - **cancellable swaps**
  - quanto default swaps
  - credit protection of vulnerable swaps, OTC options (stand-alone basis)
  - credit protection of a portfolio of contracts (full protection, excess of loss insurance, partial collateralization)

A new approach to credit derivatives modelling based on an hedging point of view

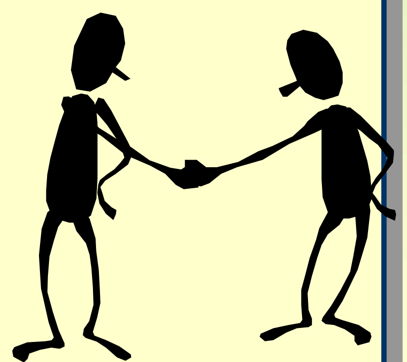
- *Rolling over* the hedge:
  - Short term default swaps v.s. long-term default swaps
  - Credit spread transformation risk
- *Dynamic Default Swaps*, *Basket Default Swaps*
  - Hedging **default risk** through dynamics holdings in standard default swaps
  - Hedging **credit spread risk** by choosing appropriate default swap maturities
  - Closing the gap between pricing and hedging
- Practical hedging issues
  - Uncertainty at default time
  - Managing net residual premiums

Long-term Default Swaps v.s. Short-term Default Swaps
Rolling over the hedge

- **Purpose:**
  - **Introduction to dynamic trading of default swaps**
  - **Illustrates how default and credit spread risk arise**
- **Arbitrage between long and short term default swap**
  - **sell one long-term default swap**
  - **buy a series of short-term default swaps**
- **Example:**
  - **default swaps on a FRN issued by BBB counterparty**
  - **5 years default swap premium : 50bp, recovery rate = 60%**

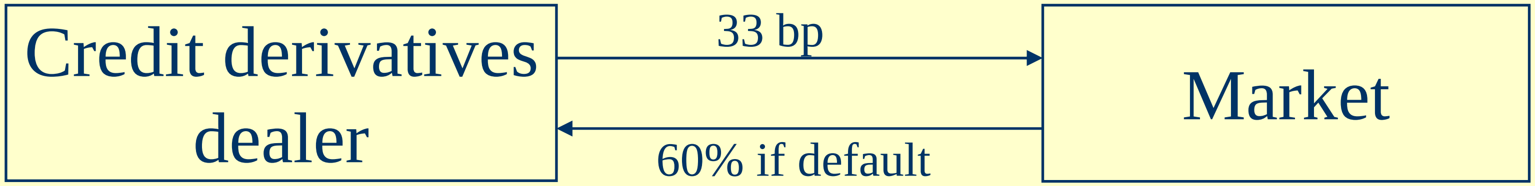
Credit derivatives dealer —(If default, 60%)→ Client

Client —(Until default, 50 bp)→ Credit derivatives dealer

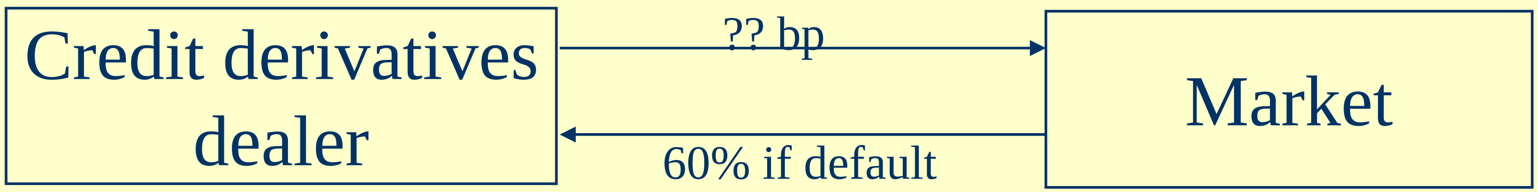


Long-term Default Swaps v.s. Short-term Default Swaps
Rolling over the hedge

- **Rolling over short-term default swap**
  - **at inception, one year default swap premium : 33bp**
  - **cash-flows after one year:**

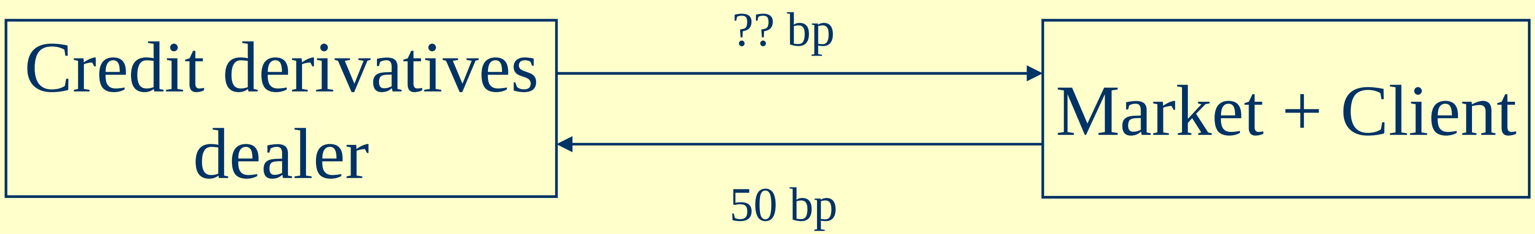


- **Buy a one year default swap at the end of every yearly period, if no default:**
  - **Dynamic strategy,**
  - **<u>future</u> premiums depend on <u>future</u> credit quality**
  - **future premiums are <u>unknown</u>**



Long-term Default Swaps v.s. Short-term Default Swaps
Rolling over the hedge

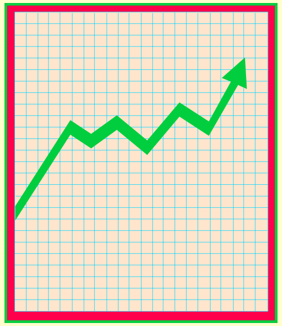
- ***Risk analysis* of rolling over short term against long term default swaps**



- **Exchanged cash-flows :**
  - **Dealer receives 5 years (fixed) credit spread,**
  - **Dealer pays 1 year (variable) credit spread.**
- **Full one to one protection at default time**
  - **the previous strategy has eliminated one source of risk, that is default risk**

Long-term Default Swaps v.s. Short-term Default Swaps
Rolling over the hedge

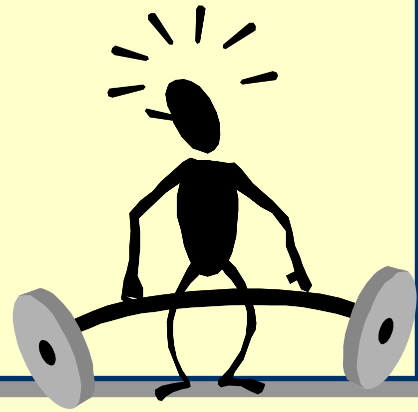
- **negative exposure to an increase in short-term default swap premiums**
  - **if short-term premiums increase from 33bp to 70bp**
  - **reflecting a lower (short-term) credit quality**
  - **and no default occurs before the fifth year**



Credit derivatives dealer → 70 bp → Market + Client

Market + Client → 50 bp → Credit derivatives dealer

- **Loss due to negative carry**
  - **long position in long term credit spreads**
  - **short position in short term credit spreads**

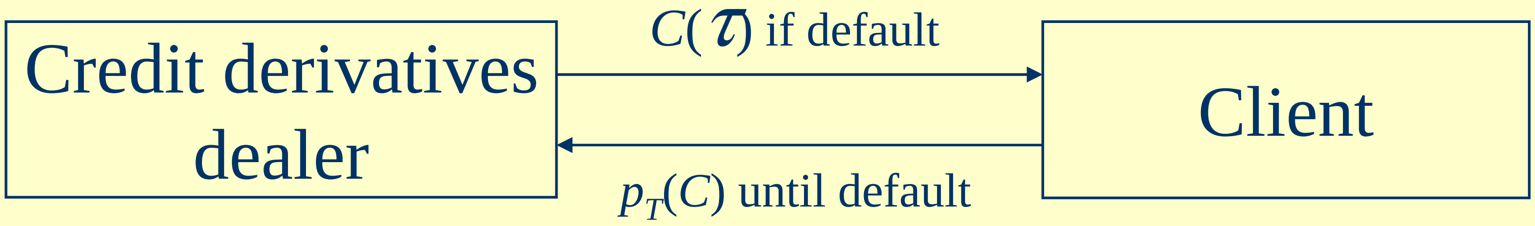


Hedging exotic default swaps : main features

- **Exotic credit derivatives can be *hedged* against <u>default</u>:**
  - **Constrains the <u>amount</u> of underlying <u>standard</u> default swaps.**
  - **<u>Variable</u> amount of standard default swaps.**
  - **<u>Full protection</u> at default time by construction of the hedge.**
  - **No more *<u>discontinuity</u>* in the P&L at default time.**
  - **"Safety-first" criteria: *main source of risk* can be hedged.**
  - **<u>Model-free</u> <u>approach.</u>**
- **<u>Credit spread exposure</u> has to be hedged by *other means*:**
  - **Appropriate *choice of maturity* of underlying default swap**
  - **Computation of sensitivities with respect to changes in credit spreads are <u>model</u> <u>dependent</u>.**

Hedging Default Risk in Dynamic Default Swap

- ***Dynamic Default Swap***
  - **client pays to dealer a periodic premium $p_T(C)$ until default time τ, or maturity of the contract T.**
  - **dealer pays $C(\tau)$ to client at default time τ, if $\tau \leq T$.**



- **Hedging side:**
  - **<u>Dynamic</u> strategy based on <u>standard</u> default swaps:**
  - **At time t, hold an amount $C(t)$ of standard default swaps**
  - **$\lambda(t)$ denotes the periodic premium at time t for a short-term default swap**

Hedging Default Risk in Dynamic Default Swaps

- **<u>Hedging side</u>:**

Credit derivatives dealer $\xrightarrow{\lambda(t)\, C(t) \text{ until default}}$ Market

Market $\xrightarrow{C(\tau) \text{ if default}}$ Credit derivatives dealer

  - **Amount of standard default swaps equals the (variable) <u>credit exposure</u> on the dynamic default swap.**

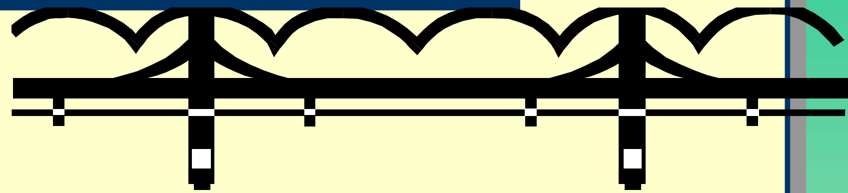
- **Net position is a "*basis swap*":**

Credit derivatives dealer $\xrightarrow{\lambda(t)\, C(t) \text{ until default}}$ Market+Client

Market+Client $\xrightarrow{p_T(C) \text{ until default}}$ Credit derivatives dealer

- **The client transfers credit spread risk to the credit derivatives dealer**

Closing the gap between pricing and hedging



- **Risky discount factors**
  - **Discount bond prices**
  - **Short term credit spreads**
- **PV of plain and dynamic default swaps**
  - **Default payment leg, premium payment leg**
  - **Default intensity and short term default swap premiums**
- **Cost of rolling over the hedge**
- **Dynamics of the PV of dynamic default swaps**
  - **Looking at theta effects**
- **Hedging credit spread risk**
- **Credit spread options**

Closing the gap between pricing and hedging
Risky discount factors

- τ, default time, $P_t(\tau \in [t,t+dt[\mid \tau > t) = \lambda(t)dt$, λ *default intensity*.
- $I(t)=1_{\{\tau>t\}}$ indicator function.
  - $I(t)$ jumps from 1 to 0 at time τ.
- $E_t[I(t)-I(t+dt)]=E_t[1_{\{\tau\in[t,t+dt[\}}]=P_t(\tau \in [t,t+dt[) = \lambda(t)I(t)dt$
- Thus $-\lambda(t)$ is the expected relative variation of $I(t)$ and:

$$E_t\left[1_{\{\tau>T\}}\right]=1_{\{\tau>t\}}E_t\left[\exp-\int_t^T\lambda(s)ds\right]$$

- Think of $I(t)$ as a stochastic nominal amortizing at rate $\lambda(t)$
  - Parallels mortgages where τ and λ, prepayment date and rate.
- **Risky discount bond** with maturity T : pays $1_{\{\tau>t\}}$ at time T
  - Denote by $\bar{B}(t,T)$ its t-time price and by $r()$ risk-free short rate

Closing the gap between pricing and hedging
Risky discount factors

- **Risky discount bond price:**

$$\bar{B}(t,T) = E_t\left[1_{\{\tau>T\}} \exp - \int_t^T r(s)ds\right] = 1_{\{\tau>t\}} E_t\left[\exp - \int_t^T (r+\lambda)(s)ds\right]$$

  - **λ is the short term credit spread**

- **More generally let X_T be a payoff paid at T, if $\tau>T$:**

$$PV_X(t) = E_t\left[X_T 1_{\{\tau>T\}} \exp - \int_t^T r(s)ds\right] = 1_{\{\tau>t\}} E_t\left[X_T \exp - \int_t^T (r+\lambda)(s)ds\right]$$

- $\exp - \int_t^T (r+\lambda)(s)ds$ ***stochastic risky discount factor***

Closing the gap between pricing and hedging: PV of plain default swaps

- ***Before default***, **time *u* -PV of a plain default swap:**
  - **Maturity T, *continuously* paid premium p, recovery rate δ**
  - **Risk-free short rate r, default intensity λ**
  - **E_u expectation conditional on information carried by financial prices.**

$$E_u\left[\int_u^T \left(\exp - \int_u^t (r+\lambda)(s)ds\right) \times \big((1-\delta)\lambda(t) - p\big)\,dt\right]$$

  - **$r+\lambda$ is the « risky » short rate : payoffs discounted at a higher rate**
  - **Similar to an index amortizing swap (payments only if no prepayment).**
- **PV of *default payment leg*** $E_u\left[\int_u^T \left(\exp - \int_u^t (r+\lambda)(s)ds\right) \times (1-\delta)\lambda(t)\,dt\right]$
- **PV of *premium payment leg*** $p \times E_u\left[\int_u^T \left(\exp - \int_u^t (r+\lambda)(s)ds\right) dt\right]$

Closing the gap between pricing and hedging: PV of plain default swaps

- **Current market premium $p_{u,T}$ is such that PV=0.**
- ***Pricing equation*:**

$$E_u\left[\int_u^T \left(\exp - \int_u^t (r+\lambda)(s)ds\right) \times \left((1-\delta)\lambda(t) - p_{u,T}\right)dt\right] = 0$$

- **For short maturities *T*=*u*+d*u*, pricing equation provides:**

$$p_{u,T} = (1-\delta)\lambda(u)$$

- **And for digital default swaps (δ=0), we get:** $p_{u,T} = \lambda(u)$

- **λ, <u>default intensity</u> = *short term default swap premium***

Closing the gap between pricing and hedging: PV of dynamic default swaps

- ***Before default***, **time t -PV of a dynamic default swap**
  - **Payment $C(\tau)$ at default time if $\tau<T$:**
- **PV of *default payment leg***

$$PV(u) = E_u\left[\int_u^T \left(\exp - \int_u^t (r+\lambda)(s)ds\right) C(t)\lambda(t)ds\right]$$

  - **This embeds the plain default swap case where $C(\tau)=1-\delta$**
- **PV of *premium payment leg*** $p \times E_u\left[\int_u^T \left(\exp - \int_u^t (r+\lambda)(s)ds\right) dt\right]$
  - **Same as in the case of plain default swap**

Closing the gap between pricing and hedging
Cost of rolling over the hedge

- **What is the cost of hedging default risk ?**
- **PV of default payment leg:**

$$E\left[\int_0^T \left(\exp - \int_0^t (r+\lambda)(s)\,ds\right)\lambda(t)C(t)\,dt\right]$$

Discounting term

Premium paid at time t
on protection portfolio

- **equals PV of premiums paid on the hedging portfolio.**
- **Pricing and rolling over the hedge approaches are <u>consistent</u>.**

Exemple: defaultable interest rate swap

- **Consider a defaultable interest rate swap (with unit nominal)**
  - **We are <u>default-free</u>, our counterparty is <u>defaultable</u> (default intensity $\lambda(t)$).**
  - **We consider a (fixed-rate) *receiver* swap on a <u>standalone</u> basis.**
- **Recovery assumption, payments in case of default:**
  - **if default at time τ, compute the <u>default-free</u> value of the swap: PV_τ**

$$\delta(PV_\tau)^+ + (PV_\tau)^- = PV_\tau - (1-\delta)(PV_\tau)^+$$

  - **and get:**
  - **$0 \leq \delta \leq 1$ recovery rate, $(PV_\tau)^+ = \mathrm{Max}(PV_\tau, 0)$, $(PV_\tau)^- = \mathrm{Min}(PV_\tau, 0)$**
  - ***In case of default,***
    - **we <u>receive</u> default-free value PV_τ**
    - ***minus***
    - **<u>loss</u> equal to $(1-\delta)(PV_\tau)^+$.**

Exemple: defaultable interest rate swap

- **Using a hedging instrument rather than a credit reserve**
  - **Consider a <u>dynamic default swap</u> paying $(1-\delta)(PV_\tau)^+$ at default time τ (if $\tau \leq T$), where PV_τ is the present value of a default-free swap with *same fixed rate* than defaultable swap.**
  - **At default, we receive $(1-\delta)(PV_\tau)^+ + PV_\tau - (1-\delta)(PV_\tau)^+ = PV_\tau$**
  - **PV of <u>default payment leg</u> is equal to the Present Value of the <u>loss</u> $(1-\delta)(PV_\tau)^+$**

$$E\left[\int_0^T \left(\exp - \int_0^t (r+\lambda)(u)\,du\right)\lambda(t)(1-\delta)\left(PV_t\right)^+ dt\right]$$

  - **Hedge against default by holding $(PV_t)^+$ ordinary default swaps at time *t*.**

Exemple: defaultable interest rate swap

- **Randomly exercised swaption:**
  - **Assume for simplicity no recovery (δ=0).**
  - **Interpret default time as a *random time* τ with *intensity* $\lambda(t)$.**
  - **At that time, defaulted counterparty "exercises" a swaption, i.e. decides whether to cancel the swap according to its present value.**
  - **PV of default-losses equals price of that *randomly exercised swaption***
- **American Swaption**
  - **PV of American swaption equals the supremum over *all possible stopping times* of *randomly exercised swaptions*.**
    - ➢ *But*, the upper bound can be reached for *special default arrival dates*:
    - ➢ $\lambda(t)$=0 above exercise boundary and $\lambda(t)=\infty$ on exercise boundary
    - ➢ Usually, PV of American swaption >> PV of default payment leg.

Explaining theta effects with and without hedging

- ***Different*** ***aspects*** **of "carrying" credit contracts through time.**
  - **Assume "historical" and "risk-neutral" intensities are equal.**
- **Consider a *short* position in a dynamic default swap.**
- **Present value of the deal provided by:**

$$PV(u) = E_u\left[\int_u^T \left(\exp - \int_u^t (r+\lambda)(s)ds\right) \times \left(p_T - \lambda(t)C(t)\right)dt\right]$$

- **(after computations) *Net expected capital gain*:**

$$E_u\left[PV(u+du) - PV(u)\right] = \left(r(u) + \lambda(u)\right)PV(u)du + \left(\lambda(u)C(u) - p_T\right)du$$

- ***Accrued cash-flows*** **(received premiums):** $p_T du$
  - **By summation, Incremental P&L (if no default between *u* and *u+du*):**

$$r(u)PV(u)du + \lambda(u)\left(C(u) + PV(u)\right)du$$

Explaining theta effects with and without hedging

- **Apparent** ***extra return*** **effect :** $\lambda(u)\big(C(u)+PV(u)\big)du$
  - **But, probability of default between *u* and *u+du*: $\lambda(u)du$.**
  - **Losses in case of default:**
    - Commitment to pay: $C(u)$
    - Loss of PV of the credit contract: PV(u)
    - PV(u) consists in **unrealised** capital gains or losses in the credit derivatives book that "disappear" in case of default.
  - **Expected loss charge:** $\lambda(u)\big(C(u)+PV(u)\big)du$
- **Hedging aspects:**
  - **If we hold** $C(u)+PV(u)$ **short-term digital default swaps, we are protected at default-time (no jump in the P&L).**
  - **Premiums to be paid:** $\lambda(u)\big(C(u)+PV(u)\big)du$
  - **Same average rate of return, but smoother variations of the P&L.**

Hedging default risk in dynamic default swaps

- **PV at time *u* of a digital default swap**

$$PV(u) = 1_{\{\tau>u\}} E_u\left[\int_u^T \left(\exp - \int_u^t (r+\lambda)(s)ds\right) \times \left(\lambda(t) - p\right) dt\right] + 1_{\{\tau\le u\}} \exp \int_\tau^u r(t)dt$$

  - **At default time τ, PV switches from** $E_u\left[\int_u^T \left(\exp - \int_u^t (r+\lambda)(s)ds\right) \times \left(\lambda(t) - p\right) dt\right]$
  - **to one (default payment). If digital default swap at the money, d*PV*(τ)=1**
- **PV at time *u* of a dynamic default swap with payment *C*: $PV_C(u)$**

$$1_{\{\tau>u\}} E_u\left[\int_u^T \left(\exp - \int_u^t (r+\lambda)(s)ds\right) \times \left(\lambda(t)C(t) - p_C\right) dt\right] + 1_{\{\tau\le u\}} C(\tau) \exp \int_\tau^u r(t)dt$$

  - **At default time τ, PV switches from predefault market value to *C*(τ)**
- **Rolling over the hedge : we hold *C*(*u*) digital default swaps**
  - **Variation of PV on the hedging portolio *C*(*u*) d*PV*(*u*)**
  - **At default time τ, PV hedging portfolio jumps of C(τ)d*PV*(τ)=C(τ)**
- **Complete hedge involves holding *C*(*u*)+$PV_C(u)$ default swaps: model free.**

Hedging Default risk <u>and</u> credit spread risk in Dynamic Default Swaps

- **Purpose : joint hedge of default risk and credit spread risk**
- **Hedging *default risk* only constrains the <u>amount</u> of underlying standard default swap.**
  - **<u>Maturity</u> of underlying default swap is arbitrary.**
- **Choose maturity to be protected against credit spread risk**
  - **PV of dynamic default swaps and standard default swaps are sensitive to the level of credit spreads**
  - **Sensitivity of standard default swaps to a shift in credit spreads increases with maturity**
  - **Choose maturity of underlying default swap in order to <u>equate sensitivities</u>.**

Hedging credit spread risk

- **Example:**
  - **dependence of simple default swaps on defaultable forward rates.**
  - **Consider a *T*-maturity default swap with continuously paid premium *p*. Assume zero-recovery (digital default swap).**
  - **PV (at time 0) of a long position provided by:**

$$PV = E\left[\int_0^T \left(\exp - \int_0^t (r+\lambda)(s)ds\right) \times (\lambda(t) - p)dt\right]$$

  - **where *r*(*t*) is the short rate and λ(*t*) the default intensity.**
  - **Assume that *r*(.) and λ(.) are independent.**
  - ***B*(0,*t*): price at time 0 of a *t*-maturity default-free discount bond**
  - ***f*(0,*t*): corresponding forward rate**

$$B(0,t) = E\left[\exp - \int_0^t r(u)du\right] = \exp - \int_0^t f(0,u)du$$

Hedging credit spread risk

– **Let $\overline{B}(0,t)$ be the *defaultable discount bond price* and $\bar{f}(0,t)$ the corresponding instantaneous forward rate:**

$$\overline{B}(0,t) = E\left[\exp - \int_0^t (r+\lambda)(u)\,du\right] = \exp - \int_0^t \bar{f}(0,u)\,du$$

– **Simple expression for the PV of the *T*-maturity default swap:**

$$PV(T) = \int_0^T \overline{B}(0,t)\left(\bar{f}(0,t) - f(0,t) - p\right)dt$$

– **The derivative of default swap present value with respect to a shift of defaultable forward rate $\bar{f}(0,t)$ is provided by:**

$$\frac{\partial PV}{\partial \bar{f}}(t) = PV(t) - PV(T) + \overline{B}(0,t)$$

- PV(t)-PV(T) is usually small compared with $\overline{B}(0,t)$.

Hedging credit spread risk

- **Similarly, we can compute the sensitivities of plain default swaps with respect to *default-free forward curves* $f(0,t)$.**
- **And thus to credit spreads.**
- **Same approach can be conducted with the *dynamic default swap* to be hedged.**
  - All the computations are ***model dependent***.
- ***Several maturities* of underlying default swaps can be used to match sensitivities.**
  - For example, in the case of **defaultable** interest rate swap, the nominal amount of default swaps $(PV_\tau)^+$ is usually small.
  - ***Single*** default swap with nominal $(PV_\tau)^+$ has a ***smaller sensitivity*** to credit spreads than ***defaultable interest rate swap***, even for long maturities.
  - **Short** and **long** positions in default swaps are required to hedge ***credit spread risk***.

Hedging credit spread risk

- **Denote by $I(u)=1_{\{\tau>u\}}$, d$I(u)$ = variation of jump part.**
- **Digital default swap:**
  - **PV prior to default:** $PV^b(u) = E_u\left[\int_u^T \left(\exp - \int_u^t (r+\lambda)(s)ds\right) \times \left(\lambda(u) - p\right) dt\right]$
  - **PV after default:** $PV^b(u) = \exp\int_\tau^u r(t)dt$
  - **PV whenever:** $PV(u) = I(u)PV^b(u) + \left(1 - I(u)\right)PV^a(u)$

$$dPV(u) = \left(PV^b(u) - PV^a(u)\right)dI(u) + I(u)dPV^b(u) + \left(1 - I(u)\right)dPV^a(u)$$

Discountinuous part
default risk

Continuous part (credit spread risk)

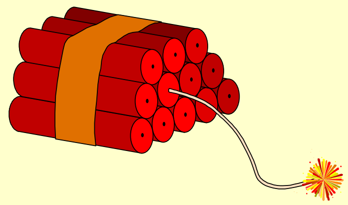
- **Continuous part is hedged by usual delta, gamma analysis,**
- **Discontinuous part : constrains the amount of hedging default swaps**
  - **After hedging default risk, no jump in the PV at default time.**

Hedging credit spread options

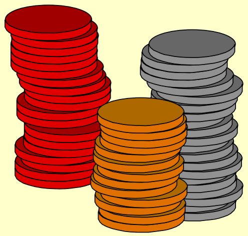
- **Option to enter a given default swap with premium *p*, maturity *T'* at exercise date *T*.**
  - **Call option provides positive payoff if credit spreads increase.**
    - Credit spread risk
  - **If default prior to *T*, cancellation of the option**
    - Default risk
- **The PV is of the form** $PV(u) = 1_{\{\tau > u\}} PV^b(u)$
  - **Hedge default risk by holding an amount of $PV^b(u)$ default swaps.**
  - **$PV^b(u)$ is usually small compared with payments involved in default swaps.**
  - **$PV^b(u)$ depends on risk-free and risky curves (mainly on credit spreads).**
  - **Credit spread risk is also hedged through default swaps.**
- **Our previous framework for hedging default risk and credit spread risk still holds.**

Real World hedging and risk-management issues

- **uncertainty at default time**
  - illiquid default swaps
  - recovery risk
  - simultaneous default events



- **Managing net premiums**
  - Maturity of underlying default swaps
  - Lines of credit
  - Management of the carry
  - Finite maturity and discrete premiums
  - Correlation between hedging cash-flows and financial variables



New ways to transfer credit risk : <u>Basket default swaps</u>

- Consider a basket of *M* risky bonds
  - <u>multiple</u> counterparties
- First to default swaps
  - protection against the first default
- *N* out of *M* default swaps (*N* < *M*)
  - protection against the first *N* defaults
- Hedging and valuation of basket default swaps
  - involves the joint (<u>multivariate</u>) modelling of default arrivals of issuers in the basket of bonds.
  - Modelling accurately the <u>dependence</u> between default times is a critical issue.

Hedging Default Risk in Basket Default Swaps

- **Example: first to default swap from a basket of two risky bonds.**
  - **If the first default time occurs before maturity,**
  - **The seller of the first to default swap pays the non recovered fraction of the defaulted bond.**
  - **Prior to that, he receives a periodic premium.**
- **Assume that the two bonds cannot default simultaneously**
  - **We moreover assume that default on one bond has *no effect* on the credit spread of the remaining bond.**
- **How can the seller be protected *at default time* ?**
  - **The only way to be protected at default time is to hold two default swaps with the *same nominal* than the *nominal* of the bonds.**
  - **The *maturity* of underlying default swaps does not matter.**

Real world hedging and risk-management issues
Case study : hedge ratios for first to default swaps

- **Consider a first to default swap associated with a basket of two defaultable loans.**
  - **Hedging portfolios based on standard underlying default swaps**
  - **Uncertain hedge ratios if:**
    - <u>simultaneous</u> default events
    - *Jumps* of <u>credit spreads</u> at default times
- **Simultaneous default events:**
  - **If counterparties default *altogether*, holding the *complete* set of default swaps is a <u>conservative</u> (and thus <u>expensive</u>) hedge.**
  - **In the *extreme* case where default *always* occur altogether, we only need a <u>single</u> default swap on the loan with largest nominal.**
  - **In other cases, holding a *fraction* of underlying default swaps <u>does not hedge default risk</u> (if *only one* counterparty defaults).**

Real world hedging and risk-management issues
Case study : hedge ratios for first to default swaps

- **What occurs if there is a *jump in the credit spread* of the second counterparty after default of the first ?**
  - **default of first counterparty means *bad news* for the second.**
- **If hedging with short-term default swaps, no capital gain at default.**
  - **Since PV of short-term default swaps is not *sensitive* to credit spreads.**
- **This is not the case if hedging with long term default swaps.**
  - **If credit spreads jump, PV of long-term default swaps jumps.**
- **Then, the amount of hedging default swaps can be reduced.**
  - **This reduction is *model-dependent*.**

On the edge of completeness ?

- **Firm-value structural default models:**
  - **Stock prices follow a diffusion processes (no jumps).**
  - **Default occurs at first time the stock value hits a barrier**
- ***In this modelling*, default credit derivatives can be completely hedged by trading the stocks:**
  - **"*Complete*" pricing and hedging model:**
- **Unrealistic features for hedging *basket default swaps*:**
  - **Because default times are predictable, *hedge ratios are close to zero* except for the counterparty with the smallest "distance to default".**

On the edge of completeness ?
hazard rate based models

- **In hazard rate based models :**
  - **default is a sudden, *non predictable* event,**
  - **that causes a sharp jump in defaultable bond prices.**
  - **Most dynamic default swaps and basket default derivatives have payoffs that are *linear* (at default) in the prices of defaultable bonds.**
  - **Thus, good news: default risk can be *hedged*.**
  - **Credit spread risk can be *substantially reduced*, but model risk.**
  - **More realistic approach to default.**
  - ***Hedge ratios* are robust with respect to default risk.**

On the edge of completeness
Conclusion

- **Looking for a better understanding of credit derivatives**
  - **payments in case of default,**
  - **volatility of credit spreads.**
- **Bridge between risk-neutral valuation and the cost of the hedge approach.**
- **<u>dynamic</u> hedging strategy based on *standard default swaps*.**
  - **hedge ratios in order to get protection at default time.**
  - **hedging default risk is *model-independent.***
  - **importance of quantitative models for a better management of the P&L and the <u>residual</u> <u>premiums</u>.**