Modélisation comportementale en ALM bancaire : application aux dépôts à vue

Conférence scientifique PRMIA Paris et AFGAP
Finance comportementale et risques
29 avril 2009

Jean-Paul LAURENT
http://laurent.jeanpaul.free.fr

Alexandre ADAM, BNP Paribas Asset and Liability Management

Mohamed HOUKARI, BNP Paribas ALM and ISFA, Université de Lyon, Université Lyon 1

Jean-Paul LAURENT, ISFA, Université de Lyon, Université Lyon 1
Presentation related to:

- *Hedging Interest Rate Margins on Demand Deposits*
  - Working paper available on SSRN (to be updated soon)

Presentation Outlook

- Modeling framework
  - *customer rates*
  - *deposit amounts*
  - *Interest rate margins*

- Optimal strategies
  - *The blinkered investor*
  - *Integrated risk management*

- Conclusion
Prolegomena

Demand Deposits involve huge amounts

**Bank of America Annual Report – Dec. 2007**

(Dollars in millions)

<table>
<thead>
<tr>
<th>Assets</th>
<th>2007</th>
<th>2006</th>
</tr>
</thead>
<tbody>
<tr>
<td>Federal funds sold and securities purchased under agreements to resell</td>
<td>$155,828</td>
<td>$175,334</td>
</tr>
<tr>
<td>Trading account assets</td>
<td>187,287</td>
<td>145,321</td>
</tr>
<tr>
<td>Debt securities</td>
<td>186,466</td>
<td>225,219</td>
</tr>
<tr>
<td>Loans and leases, net of allowance for loan and lease losses</td>
<td>766,329</td>
<td>643,259</td>
</tr>
<tr>
<td>All other assets</td>
<td>306,163</td>
<td>277,548</td>
</tr>
<tr>
<td><strong>Total assets</strong></td>
<td>$1,602,073</td>
<td>$1,466,681</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Liabilities</th>
<th>2007</th>
<th>2006</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deposits</td>
<td>$717,182</td>
<td>$672,995</td>
</tr>
<tr>
<td>Federal funds purchased and securities sold under agreements to repurchase</td>
<td>253,481</td>
<td>286,903</td>
</tr>
<tr>
<td>Trading account liabilities</td>
<td>82,721</td>
<td>64,689</td>
</tr>
<tr>
<td>Commercial paper and other short-term borrowings</td>
<td>171,333</td>
<td>124,229</td>
</tr>
<tr>
<td>Long-term debt</td>
<td>169,855</td>
<td>130,124</td>
</tr>
<tr>
<td>All other liabilities</td>
<td>70,839</td>
<td>57,278</td>
</tr>
<tr>
<td><strong>Total liabilities</strong></td>
<td>1,465,411</td>
<td>1,336,218</td>
</tr>
<tr>
<td>Shareholders’ equity</td>
<td>136,662</td>
<td>130,463</td>
</tr>
<tr>
<td><strong>Total liabilities and shareholders’ equity</strong></td>
<td>$1,602,073</td>
<td>$1,466,681</td>
</tr>
</tbody>
</table>

Demand deposits involve both interest rate and liquidity risks
Modeling Deposit Rate – Examples

- We assume the customer rate to be a function of the market rate.
  - Affine in general (US) / Sometimes more complex (Japan)

\[ g(L_T) = \alpha + \beta \cdot L_T \]

United States

\[ g(L_T) = (\alpha + \beta \cdot L_T) \cdot 1\{L_T \geq R\} \]

Japan

![Graph showing Affine Dependence and Quasi Zero Rates](image-url)
Dynamics for Market Rate $L_t$ : forward Libor rate

- Market Model for forward Libor rate(s)

$$\frac{dL_t}{L_t} = \mu_L dt + \sigma_L dW_L(t)$$

$\mu_L \neq 0$ Long-Term Investment Risk Premium

- Coefficient specification assumptions:
  - Our model: $\mu_L, \sigma_L$ constant
  - Assumptions can be relaxed:
    - Time dependent coefficients
    - CEV type Libor models
Deposit Amount Dynamics

- Diffusion process for Deposit Amount

\[ dK_t = K_t \left[ \mu_K dt + \sigma_K d\bar{W}_K(t) \right] \]

- Sensitivity of deposit amount to market rates
  - Money transfers between deposits and other accounts
- Interest Rate partial contingency.
  - Business risk, …
  - Incomplete market framework

\[ d\bar{W}_K(t) = \rho dW_L(t) + \sqrt{1 - \rho^2} dW_K(t) \quad -1 < \rho < 0 \]
Deposit Amount Dynamics – Examples

\[ dK_t = K_t (\mu_K dt + \sigma_K dW_K(t)) \]

EuroZone – \( \hat{\mu}_K = 10.19\%, \hat{\sigma}_K = 6.56\% \)

Turkey – \( \hat{\mu}_K = 51.74\%, \hat{\sigma}_K = 37.38\% \)
Demand Deposit Interest Rate Margin

- For a given quarter $T$
- Income generated by:
  - Investing Demand Deposit Amount on interbank markets
  - while paying a deposit rate to customers

**Interest Rate Margin**

$$IRM_g(K_T, L_T) = K_T(L_T - g(L_T)) \cdot \Delta T$$

- Deposit Amount at $T$
- Investment Market Rate during time interval $[T, T+\Delta T]$
- Customer rate at $T$
We need to focus on Interest Rate Margins

\[ \text{IRM}_g(K_T, L_T) = K_T(L_T - g(L_T)) \cdot \Delta T \]

- According to the IFRS (International accounting standards):
  - The IFRS recommend the accounting of non maturing assets and liabilities at Amortized Cost / Historical Cost
- Recognition of related hedging strategies from the accounting viewpoint
  - *Interest Margin Hedge* (IMH).
- The fair value approach does not apply to demand deposits
Risks in interest rate margins

\[ IRM_g (K_T, L_T) = K_T \left( L_T - g(L_T) \right) \cdot \Delta T \]

Interest rate risk

- Direct interest rate risk on unit margins \( L_T - g(L_T) \)
- Indirect interest rate risk due to correlation between \( K_T \) and \( L_T \)

Business risk

- Deposit amounts are not fully correlated to interest rates

Hedging tools

- Interest rate swaps (FRA’s)
- \( L_t \): three months forward Libor at date \( t \) for quarter \( T \)
- \( dL_t \): incremental cash-flow at time \( T \) associated with a unit FRA
Sets of Hedging Strategies

- Hedging strategies based on FRAs
  - Amount held in FRA’s varies with available information
- 1st case: (myopic) self-financed strategies taking into account the evolution of market rates only
  \[ H_{S2} = \left\{ S = \int_0^T \theta_t^L dL_t ; \theta^L \in \Theta^L \right\} \]
  Set of admissible investment strategies adapted to \( F_{WL} \)

- Management of interest rate risk achieved by trading desk far-off ALM
- 2nd case: self-financed strategies taking into account the evolution of the deposit amount
  \[ H_D = \left\{ S = \int_0^T \theta_t dL_t ; \theta \in \Theta \right\} \]
  Set of admissible investment strategies adapted to \( F_{WL} \vee F_{WK} \)

- Integrated risk management
**Interest Rate Margin**

\[ IRM_g (K_T, L_T) = K_T \left( L_T - g(L_T) \right) \cdot \Delta T \]

- Deposit Amount at \( T \)
- Investment Market Rate during time interval \([T, T + \Delta T]\)
- Customer rate at \( T \)

**Mean-variance framework:**

- Including a return constraint – due to the interest rate risk premium.

\[
\min_S \mathbb{E} \left[ IRM_g (K_T, L_T) - S \right]^2 \quad \text{under constraint} \quad \mathbb{E} \left[ IRM_g (K_T, L_T) - S \right] \geq r
\]

- Incomplete markets: perfect hedge cannot be achieved with interest rate swaps
Useful mathematical finance concepts

- **Martingale Minimal Measure:**
  \[
  \frac{d\bar{P}}{dP} = \exp \left( -\frac{1}{2} \int_0^T \lambda^2 dt - \int_0^T \lambda dW_L(t) \right)
  \]
  - \( \lambda = \mu_L / \sigma_L \) risk premium associated with holding long term assets
  - Risk-neutral with respect to traded risks (interest rates)
  - Historical with respect to non hedgeable risks (business risk)

- In our framework, coincides with the **variance minimal measure:**

  \[
  \bar{P} = \text{Arg min}_{\mathcal{Q} \in \Pi_{RN}} \mathbb{E}^P \left[ \frac{d\mathcal{Q}}{dP} \right]^2
  \]

- Here, the Variance Minimal Measure density is a power function of the Libor rate:

  \[
  \frac{d\bar{P}}{dP} = \left( \frac{L_T}{L_0} \right)^{-\lambda / \sigma_L} \exp \left( \frac{1}{2} \left( \lambda^2 - \lambda \sigma_L \right) T \right)
  \]
Optimal Hedging Strategy – blinkers’ case

- Payoff of optimal hedging strategy:
  \[ \varphi^{S2} (L_T) = \mathbb{E}^P \left[ \text{IRM}_g (K_T, L_T) | L_T \right] - \mathbb{E}^P \left[ \text{IRM}_g (K_T, L_T) \right] \]

  - Only depends upon terminal Libor!
  - European option type payoff / Analytical computations
  - Optimal hedging strategy in FRAs consists in replicating \( \varphi^{S2} (L_T) \)

Japanese case
Non linear customer rates
Optimal Hedging Strategy – Integrated Risk Management

The optimal investment in FRA’s is determined as follows:

\[
\theta_t^{**} = \frac{\partial E_P^t \left[ IRM_g \left( K_T, L_T \right) \right]}{\partial L_t} + \frac{\lambda}{\sigma_L L_t} \left[ E_P^t \left[ IRM_g \left( K_T, L_T \right) \right] - V_t (x^{**}, \theta^{**}) \right]
\]

- **Delta term**
- **Hedging Numéraire**
- **Feedback term**

This portfolio aims at some fixed return while minimizing the final quadratic dispersion.

\[
E_P^t \left[ \int_0^T \frac{\lambda}{\sigma_L L_t} dL_t - (-1) \right]^2 = \min_{\theta \in \Theta} E_P^t \left[ \int_0^T \theta dL_t - (-1) \right]^2
\]

- Computations are fully explicit
- Case of No Deposit Rate: \( g(L_T) = 0 \)
  - Optimal strategy reduces to earlier results of Duffie & Richardson
Dealing with **Massive Bank Run**

- Introducing a Poisson Jump component in the deposit amount:
  \[
  dK_t = K_t \left[ \mu_K dt + \sigma_K d\bar{W}_K(t) - dN(t) \right]
  \]
  \[
  (N(t))_{0 \leq t \leq T} \text{ is assumed to be independent from } W_K \text{ and } W_L
  \]

- Same hedging numéraire and variance minimal measure as before.
  - **Computations are still explicit:**
    \[
    \theta^{**}_t = \frac{\partial E_t^P[IRM(K_T, L_T)]}{\partial L_t} + \frac{\lambda}{\sigma_L L_t} \left[ E_t^P[IRM_g(K_T, L_T)] - V_t(x^{**}, \theta^{**}) \right]
    \]
  - Due to independence, the jump element can be put out the conditional expectations:
    \[
    E_t^P \left[ IRM_g(K_T, L_T) \right] = e^{-\gamma(T-t)} \times (\text{Previous conditional expectation term})
    \]

- **Previous technique has a wide range of applications**
  - Changing the deposit amount dynamics
  - Changing the customer rate specification
Relevance of Integrated Risk Management?

- Optimal strategy based on market rates only (blue) and the one knowing both rates and deposits (pink):
  - At minimum variance point (*risk minimization*)
  - Taking into account deposit amounts leads to higher accuracy when correlation between interest rates and deposit amounts is low
  - $\rho = 1$ corresponds to a complete market case

![Risk Reduction and Correlation](image)

- Total Deposit Volatility = 6.5% - $K(0) = 100$
- Hedged Margin Standard Deviation
- Optimal Dynamic Hedge (Rates)
- Optimal Dynamic Hedge (rates + deposits)
Choice of Risk Criterion

- The mean-variance optimal dynamic strategy (following deposits and rates) behaves quite well under other risk criteria.
- Example of Expected Shortfall (99.5%) and VaR (99.95%).

<table>
<thead>
<tr>
<th>Barrier Deposit Rate</th>
<th>Expected Return</th>
<th>Standard Deviation</th>
<th>ES (99.5%)</th>
<th>VaR (99.95%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Level</td>
<td>Risk Reduction</td>
<td>Level</td>
</tr>
<tr>
<td>Unhedged Margin</td>
<td>3.16</td>
<td>0.39</td>
<td>-2.02</td>
<td>-1.90</td>
</tr>
<tr>
<td>Static Hedge Case 1</td>
<td>3.04</td>
<td>0.28</td>
<td>-0.11</td>
<td>-2.34</td>
</tr>
<tr>
<td>Static Hedge Case 2</td>
<td>3.01</td>
<td>0.23</td>
<td>-0.16</td>
<td>-2.26</td>
</tr>
<tr>
<td>Jarrow and van Deventer</td>
<td>3.01</td>
<td>0.24</td>
<td>-0.15</td>
<td>-2.35</td>
</tr>
<tr>
<td>Optimal Dynamic Hedge</td>
<td>3.01</td>
<td>0.22</td>
<td>-0.17</td>
<td>-2.38</td>
</tr>
</tbody>
</table>

- Mean-variance optimal dynamic strategy are additive with respect to different items of the balance sheet.
  - One can deal separately with demand deposits and mortgages (say)
  - Which is not the case with ES or VaR.
Conclusions

- Abstract mathematical finance concepts lead to analytical and easy to implement optimal hedging strategies for demand deposits:
  - Taking both into account interest rate risk and business risk
  - Sheds new light on risk management architecture
  - Consistent with standard accounting principles
  - Robust with respect to choice of risk criteria
  - Can cope with a wide range of specifications
  - Applicable to various balance sheet items
  - That can be dealt with separately (additivity)

- But…
  - Lack of stability towards deposit rate’s specification
  - Growth and volatility of deposit amounts
  - As usual, significant model risk