Modélisation comportementale en ALM bancaire : application aux dépôts à vue

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- **Presentation related to:**
  - ***Hedging Interest Rate Margins on Demand Deposits***
    - **Working paper available on SSRN (to be updated soon)**
- **Presentation Outlook**
  - **Modeling framework**
    - ***customer rates***
    - ***deposit amounts***
    - ***Interest rate margins***
  - **Optimal strategies**
    - ***The blinkered investor***
    - ***Integrated risk management***
  - **Conclusion**

Prolegomena

- Demand Deposits involve huge amounts
  - **Bank of America Annual Report – Dec. 2007**

(Dollars in millions)	Average Balance 2007	Average Balance 2006
Assets		
Federal funds sold and securities purchased under agreements to resell	**$ 155,828**	$ 175,334
Trading account assets	**187,287**	145,321
Debt securities	**186,466**	225,219
Loans and leases, net of allowance for loan and lease losses	**766,329**	643,259
All other assets	**306,163**	277,548
Total assets	**$ 1,602,073**	$ 1,466,681
Liabilities		
Deposits	**$ 717,182**	$ 672,995
Federal funds purchased and securities sold under agreements to repurchase	**253,481**	286,903
Trading account liabilities	**82,721**	64,689
Commercial paper and other short-term borrowings	**171,333**	124,229
Long-term debt	**169,855**	130,124
All other liabilities	**70,839**	57,278
Total liabilities	**1,465,411**	1,336,218
Shareholders' equity	**136,662**	130,463
Total liabilities and shareholders' equity	**$ 1,602,073**	$ 1,466,681

- ***Demand deposits involve both interest rate and liquidity risks***

Modeling Deposit Rate – Examples

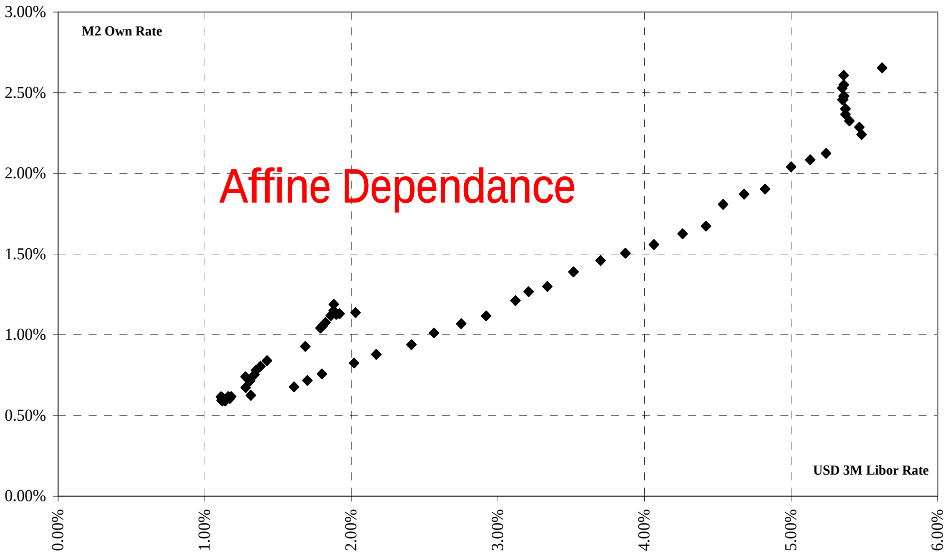
- **We assume the customer rate to be a function of the market rate.**
  - ***Affine in general (US) / Sometimes more complex (Japan)***

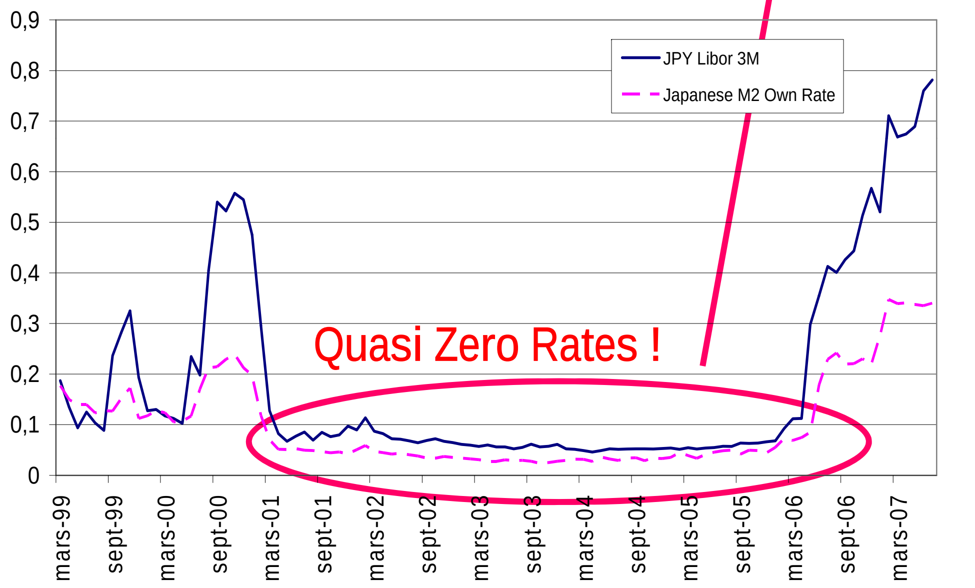
$$g(L_T) = \alpha + \beta \cdot L_T$$

United States

$$g(L_T) = (\alpha + \beta \cdot L_T) \cdot \mathbf{1}\{L_T \geq R\}$$

Japan





Dynamics for Market Rate L_t : forward Libor rate

- **Market Model for forward Libor rate(s)**

$$\frac{dL_t}{L_t} = \mu_L dt + \sigma_L dW_L(t)$$

$$\mu_L \neq 0$$ Long-Term Investment Risk Premium

- **Coefficient specification assumptions:**
  - **Our model: μ_L, σ_L constant**
  - **Assumptions can be relaxed:**
    - *Time dependent coefficients*
    - *CEV type Libor models*

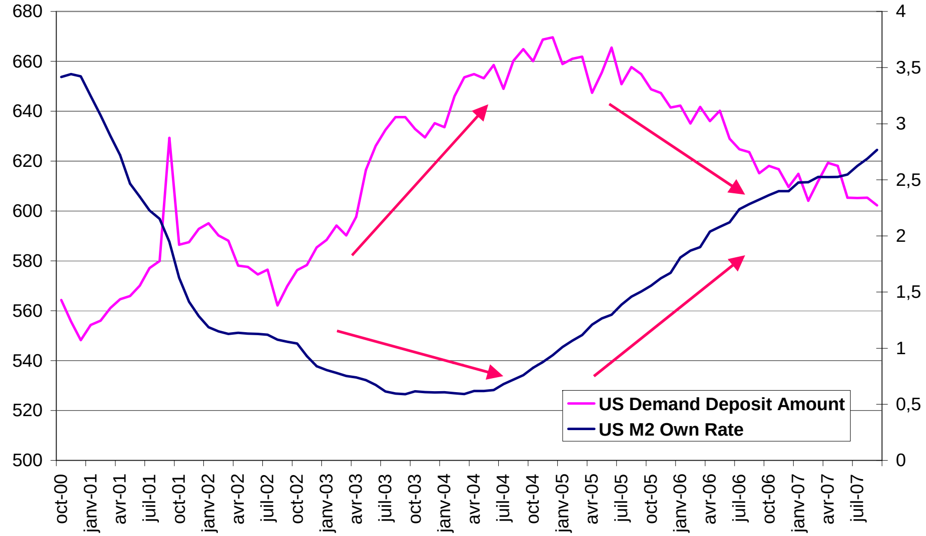
Deposit Amount Dynamics

- Diffusion process for Deposit Amount

$$dK_t = K_t\left[\mu_K dt + \sigma_K d\overline{W}_K(t)\right]$$

- Sensitivity of deposit amount to market rates
  - Money transfers between deposits and other accounts
- Interest Rate partial contingence.
  - Business risk, …
  - **Incomplete market framework**

(US marketplace)

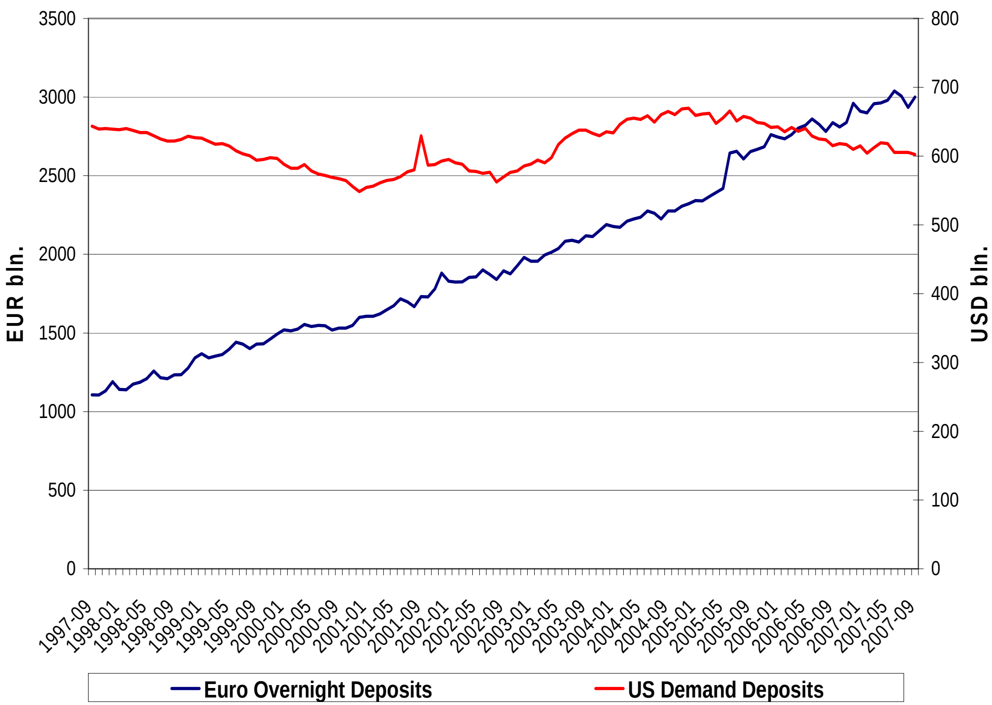


$$d\overline{W}_K(t) = \rho dW_L(t) + \sqrt{1-\rho^2}\, dW_K(t) \qquad -1 < \rho < 0$$

Deposit Amount Dynamics – Examples

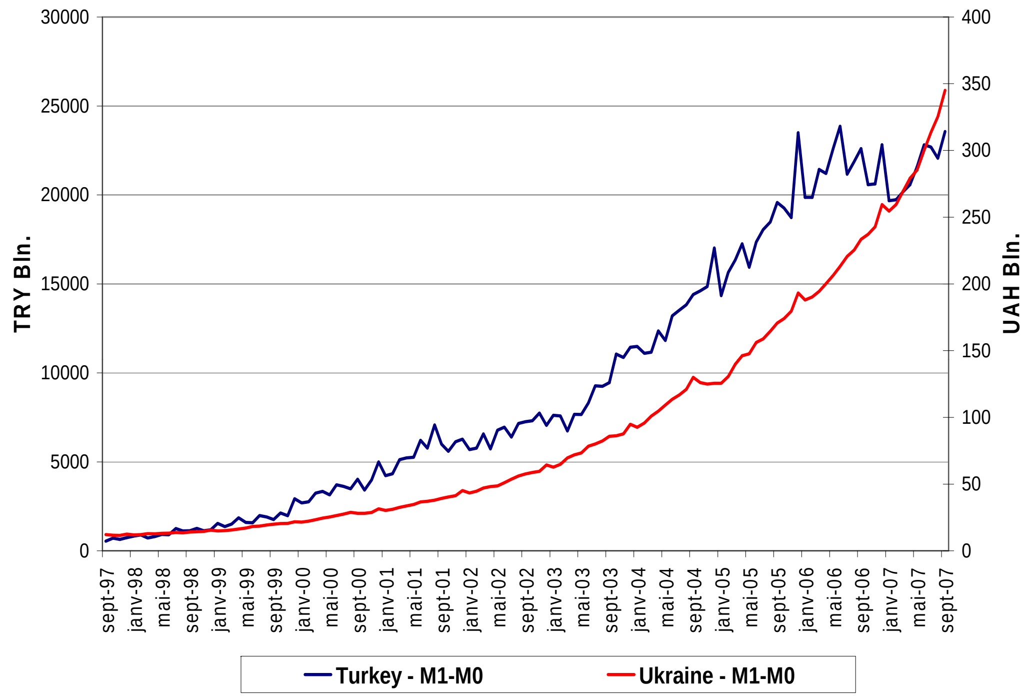
$$dK_t = K_t\left(\mu_K dt + \sigma_K d\overline{W}_K(t)\right)$$

US and Euro Zone



$$\textbf{EuroZone} - \hat{\mu}_K = 10.19\%, \hat{\sigma}_K = 6.56\%$$

Emerging Markets



$$\textbf{Turkey} - \hat{\mu}_K = 51.74\%, \hat{\sigma}_K = 37.38\%$$

- **Demand Deposit Interest Rate Margin**
  - *For a given quarter T*
  - *Income generated by:*
    - *Investing Demand Deposit Amount on interbank markets*
    - *while paying a deposit rate to customers*

<u>Interest Rate Margin</u> $$IRM_g\left(K_T, L_T\right) = K_T\left(L_T - g\left(L_T\right)\right)\cdot \Delta T$$

Deposit Amount at T ← K_T

Investment Market Rate during time interval [***T,T+ΔT***] ← L_T

Customer rate at T ← $g\left(L_T\right)$

We need to focus on Interest Rate Margins

$$IRM_g(K_T, L_T) = K_T(L_T - g(L_T)) \cdot \Delta T$$

- According to the IFRS (International accounting standards) :
  - The IFRS recommend the accounting of non maturing assets and liabilities at Amortized Cost / Historical Cost
- Recognition of related hedging strategies from the accounting viewpoint
  - *Interest Margin Hedge* (IMH).
- The fair value approach does not apply to demand deposits

- Risks in interest rate margins $IRM_g\left(K_T, L_T\right) = K_T\left(L_T - g\left(L_T\right)\right)\cdot\Delta T$
- Interest rate risk
  - Direct interest rate risk on unit margins $L_T - g\left(L_T\right)$
  - Indirect interest rate risk due to correlation between K_T and L_T
- Business risk
  - Deposit amounts are not fully correlated to interest rates
- Hedging tools
  - Interest rate swaps (FRA's)
  - L_t three months forward Libor at date *t* for quarter *T*
  - dL_t: incremental cash-flow at time *T* associated with a unit FRA

Sets of Hedging Strategies

- Hedging strategies based on FRAs
  - *Amount held in FRA's varies with available information*
- 1st case: (myopic) self-financed strategies taking into account the evolution of market rates only

$$H_{S2} = \left\{ S = \int_0^T \theta_t^L dL_t \; ; \; \theta^L \in \Theta^L \right\}$$

Set of admissible investment strategies adapted to F^{W_L}

  - *Management of interest rate risk achieved by trading desk far-off ALM*

... is contained in ...

- 2nd case: self-financed strategies taking into account the evolution of the deposit amount

$$H_D = \left\{ S = \int_0^T \theta_t dL_t \; ; \; \theta \in \Theta \right\}$$

Set of admissible investment strategies adapted to $F^{W_L} \vee F^{W_K}$

  - *Integrated risk management*

Interest Rate Margin $IRM_g(K_T, L_T) = K_T(L_T - g(L_T)) \cdot \Delta T$

Deposit Amount at T ← K_T

Investment Market Rate during time interval [*T,T+ΔT*] ← L_T

Customer rate at T ← $g(L_T)$

- **Mean-variance framework:**
  - Including a return constraint – due to the interest rate risk premium.

$$\min_S \mathbf{E}\left[IRM_g(K_T, L_T) - S\right]^2 \text{ under constraint } \mathbf{E}\left[IRM_g(K_T, L_T) - S\right] \geq r$$

  - Incomplete markets: perfect hedge cannot be achieved with interest rate swaps

Useful mathematical finance concepts

- Martingale Minimal Measure: $$\frac{d\overline{\mathbf{P}}}{d\mathbf{P}} = \exp\left(-\frac{1}{2}\int_0^T \lambda^2 dt - \int_0^T \lambda dW_L(t) \right)$$
  - $\lambda = \mu_L / \sigma_L$ risk premium associated with holding long term assets
  - Risk-neutral with respect to traded risks (interest rates)
  - Historical with respect to non hedgeable risks (business risk)
- In our framework, coincides with the variance minimal measure:

$$\overline{\mathbf{P}} = \operatorname{Arg}\min_{\mathbf{Q}\in\Pi_{RN}} \mathbf{E}^{\mathbf{P}}\left[\frac{d\mathbf{Q}}{d\mathbf{P}}\right]^2$$

- Here, the Variance Minimal Measure density is a power function of the Libor rate :

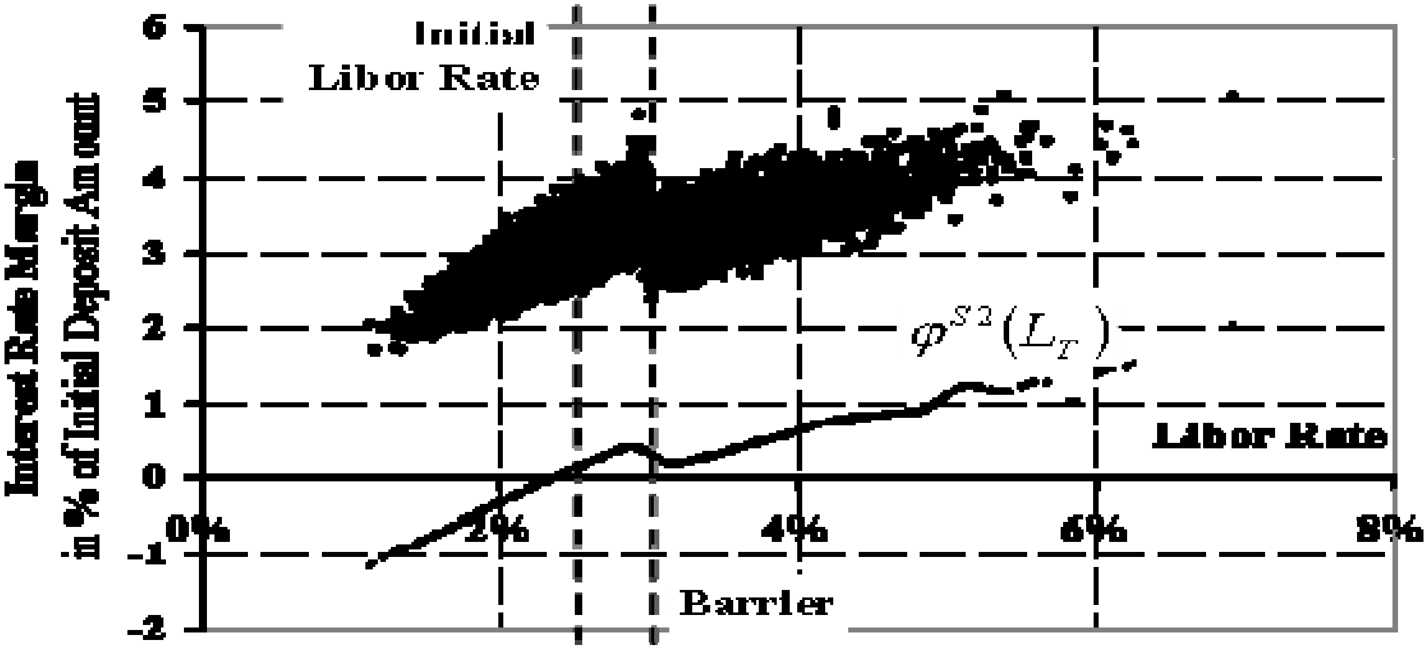
$$\frac{d\overline{\mathbf{P}}}{d\mathbf{P}} = \left(\frac{L_T}{L_0}\right)^{-\frac{\lambda}{\sigma_L}} \exp\left(\frac{1}{2}\left(\lambda^2 - \lambda\sigma_L\right)T\right)$$

Optimal Hedging Strategy – blinkers' case

- **Payoff of optimal hedging strategy:**

$$\varphi^{S2}(L_T) = \mathbf{E}^{\mathbf{P}}\left[IRM_g(K_T, L_T)\middle| L_T\right] - \mathbf{E}^{\bar{\mathbf{P}}}\left[IRM_g(K_T, L_T)\right]$$

  - **Only depends upon terminal Libor!**
    - ***European option type payoff / Analytical computations***
  - **Optimal hedging strategy in FRAs consists in replicating** $\varphi^{S2}(L_T)$



Japanese case

Non linear customer rates

Optimal Hedging Strategy – Integrated Risk Management

- **The optimal investment in FRA's is determined as follows:**

$$\theta_t^{**} = \underbrace{\frac{\partial \mathbf{E}_t^{\bar{\mathbf{P}}}\left[IRM_g\left(K_T, L_T\right)\right]}{\partial L_t}}_{\text{Delta term}} + \underbrace{\frac{\lambda}{\sigma_L L_t}}_{\text{Hedging Numéraire}} \underbrace{\left[\mathbf{E}_t^{\bar{\mathbf{P}}}\left[IRM_g\left(K_T, L_T\right)\right] - V_t\left(x^{**}, \theta^{**}\right)\right]}_{\text{Feedback term}}$$

Delta term + ***Hedging Numéraire*** × Feedback term

- Shift between the RN anticipation of the *margin* and the present value of the hedging portfolio

Investment in some ***Elementary Portfolio*** which verifies

This portfolio aims at some fixed return while minimizing the final quadratic dispersion.

$$\mathbf{E}^{\mathbf{P}}\left[\int_0^T \frac{\lambda}{\sigma_L L_t} dL_t - (-1)\right]^2 = \min_{\theta \in \Theta} \mathbf{E}^{\mathbf{P}}\left[\int_0^T \theta_t dL_t - (-1)\right]^2$$

- Computations are fully explicit
- Case of No Deposit Rate: $g\left(L_T\right) = 0$
  - *Optimal strategy reduces to earlier results of Duffie & Richardson*

Dealing with Massive Bank Run

- **Introducing a Poisson Jump component in the deposit amount:**

$$dK_t = K_t\left[\mu_K dt + \sigma_K d\overline{W}_K(t) - dN(t)\right]$$

$\left(N(t)\right)_{0\leq t\leq T}$ is assumed to be independent from W_K and W_L

- **Same hedging numéraire and variance minimal measure as before.**
  - *Computations are still explicit :*

$$\theta_t^{**} = \frac{\partial \mathbf{E}_t^{\overline{\mathbf{P}}}\left[IRM\left(K_T, L_T\right)\right]}{\partial L_t} + \frac{\lambda}{\sigma_L L_t}\left[\mathbf{E}_t^{\overline{\mathbf{P}}}\left[IRM_g\left(K_T, L_T\right)\right] - V_t\left(x^{**}, \theta^{**}\right)\right]$$

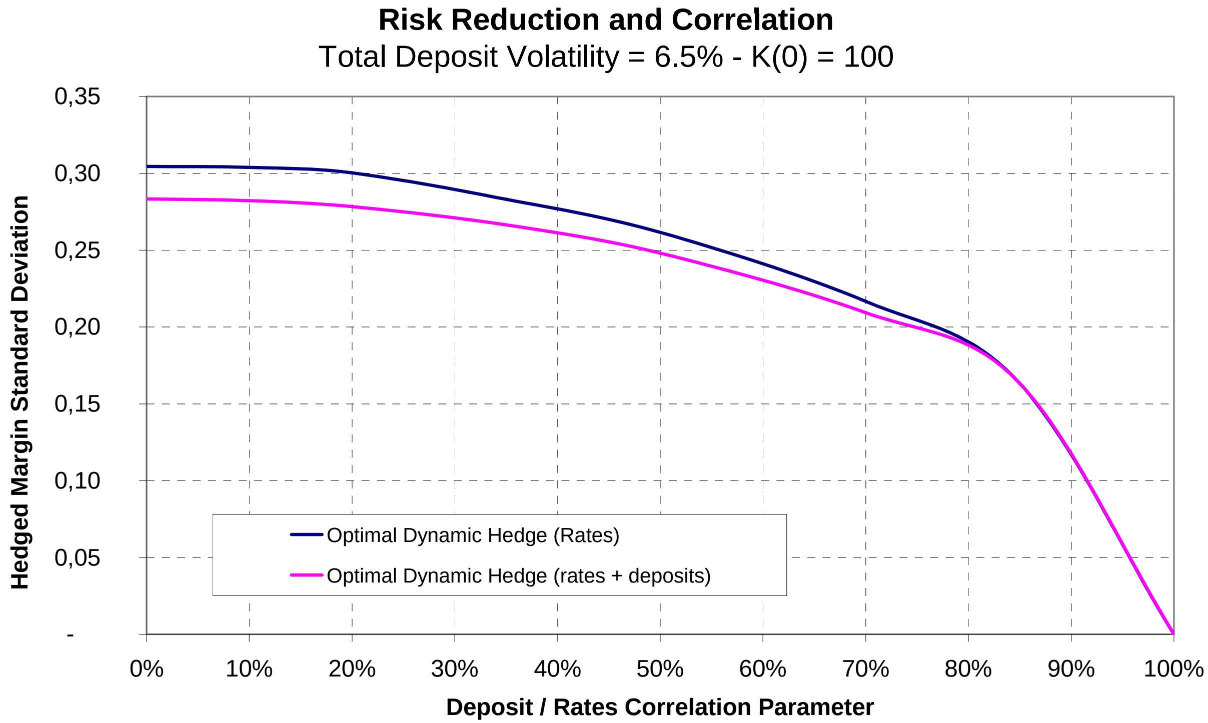
  - *Due to independence, the jump element can be put out the conditional expectations:*

$$\mathbf{E}_t^{\overline{\mathbf{P}}}\left[IRM_g\left(K_T, L_T\right)\right] = e^{-\gamma(T-t)} \times \text{ (Previous conditional expectation term)}$$

- **Previous technique has a wide range of applications**
  - *Changing the deposit amount dynamics*
  - *Changing the customer rate specification*

Relevance of Integrated Risk Management ?

- **Optimal strategy based on market rates only (blue) and the one knowing both rates and deposits (pink):**
  - At minimum variance point (*risk minimization*)
- **Taking into account deposit amounts leads to higher accuracy when correlation between interest rates and deposit amounts is low**
- $\rho = 1$ corresponds to a complete market case



Choice of Risk Criterion

- **The mean-variance optimal dynamic strategy (following deposits and rates) behaves quite well under other risk criteria**
  - Example of Expected Shortfall (99.5%) and VaR (99.95%).

Barrier Deposit Rate	**Expected Return**	**Standard Deviation**		**ES** (99.5%)		**VaR** (99.95%)	
		Level	*Risk Reduction*	Level	*Risk Reduction*	Level	*Risk Reduction*
Unhedged Margin	3.16	0.39		-2.02		-1.90	
Static Hedge Case 1	3.04	0.28	*-0.11*	-2.34	*-0.32*	-2.26	*-0.36*
Static Hedge Case 2	3.01	0.23	*-0.16*	-2.26	*-0.24*	-2.04	*-0.14*
Jarrow and van Deventer	3.01	0.24	*-0.15*	-2.35	*-0.33*	-2.25	*-0.35*
Optimal Dynamic Hedge	3.01	0.22	***-0.17***	-2.38	***-0.36***	-2.29	***-0.39***

- **Mean-variance optimal dynamic strategy are additive with respect to different items of the balance sheet**
  - One can deal separately with demand deposits and mortgages (say)
  - Which is not the case with ES or VaR

Conclusions

- Abstract mathematical finance concepts lead to analytical and easy to implement optimal hedging strategies for demand deposits:
  - *Taking both into account interest rate risk and business risk*
  - *Sheds new light on risk management architecture*
  - *Consistent with standard accounting principles*
  - *Robust with respect to choice of risk criteria*
  - *Can cope with a wide range of specifications*
  - *Applicable to various balance sheet items*
  - *That can be dealt with separately (additivity)*
- But…
  - *Lack of stability towards deposit rate's specification*
  - *Growth and volatility of deposit amounts*
  - *As usual, significant model risk*