On the Edge of Completeness

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Jean-Paul LAURENT
Professor, ISFA Actuarial School, University of Lyon,
Scientific Advisor, BNP Paribas

Correspondence
laurent.jeanpaul@online.fr
On the Edge of Completeness:  
Purpose and main ideas

- **Purpose:**
  - risk-analysis of exotic credit derivatives:
    - credit contingent contracts, basket default swaps.
  - pricing and hedging exotic credit derivatives.

- **Main ideas:**
  - distinguish between credit spread volatility and default risk.
  - dynamic hedge of exotic default swaps with standard default swaps.

On the Edge of completeness: Overview

- Trading credit risk: closing the gap between supply and demand
- Modelling credit derivatives: the state of the art
- A new approach to credit derivatives modelling:
  - closing the gap between pricing and hedging
  - disentangling default risk and credit spread risk
Trading credit risk:  
Closing the gap between supply and demand

- From stone age to the new millennium:
  - Technical innovations in credit derivatives are driven by economic forces.
  - Transferring risk from commercial banks to institutional investors:
    - Securitization.
    - Default Swaps: portfolio and hedging issues.
    - Credit Contingent Contracts, Basket Credit Derivatives.
  - The previous means tend to be more integrated.
Securitization of credit risk:

- Trading credit risk: Closing the gap between supply and demand

- Simplified scheme:
  - No residual risk remains within SPV.
  - All credit trades are simultaneous.
Trading Credit Risk: 
Closing the gap between supply and demand

• Financial intermediaries provide structuring and arrangement advice.
  – Credit risk seller can transfer loans to SPV or instead use default swaps

• good news: low capital at risk for investment banks

• Good times for modelling credit derivatives
  – No need of hedging models
  – credit pricing models are used to ease risk transfer
  – need to assess the risks of various tranches
Trading Credit Risk: Closing the gap between supply and demand

• There is room for financial intermediation of credit risk
  – The transfers of credit risk between commercial banks and investors may not be simultaneous.
  – Since at one point in time, demand and offer of credit risk may not match.
    ➢ Meanwhile, credit risk remains within the balance sheet of the financial intermediary.
  – It is not further required to find customers with exact opposite interest at every new deal.
    ➢ Residual risks remain within the balance sheet of the financial intermediary.
Credit risk management without hedging default risk

- **Emphasis on:**
  - portfolio effects: correlation between default events
  - posting collateral
  - computation of capital at risk, risk assessment

- **Main issues:**
  - capital at risk can be high
  - what is the competitive advantage of investment banks
Credit risk management with hedging default risk

- Trading against other dealers enhances ability to transfer credit risk by lowering capital at risk

```
Credit risk seller  Default swap  Credit derivatives trading book
                  bank                      
                          Default swap
                          Default swaps
                           Repos
                         Credit derivatives dealer
                           
                          
                          Bond dealer

Investor 1

Investor 2

Bond dealer

```
New ways to transfer credit risk:

credit contingent contracts

• Anatomy of a general credit contingent contract
  – A credit contingent contract is like a standard default swap but with variable nominal (or exposure)
  – However the periodic premium paid for the credit protection remains fixed.
  – The protection payment arises at default of one given single risky counterparty.

• Examples
  ➢ cancellable swaps
  ➢ quanto default swaps
  ➢ credit protection of vulnerable swaps, OTC options (stand-alone basis)
  ➢ credit protection of a portfolio of contracts (full protection, excess of loss insurance, partial collateralization)
New ways to transfer credit risk: 
**Basket default derivatives**

- Consider a basket of $M$ risky bonds
  - multiple counterparties
- First to default swaps
  - protection against the first default
- $N$ out of $M$ default swaps ($N < M$)
  - protection against the first $N$ defaults
- Hedging and valuation of basket default derivatives
  - involves the joint (multivariate) modelling of default arrivals of issuers in the basket of bonds.
  - Modelling accurately the dependence between default times is a critical issue.
Modelling credit derivatives: the state of the art

- Modelling credit derivatives: Where do we stand?
- Financial industry approaches
  - Plain default swaps and risky bonds
  - Credit risk management approaches

- The Noah's arch of credit risk models
  - "firm-value" models
  - Risk-intensity based models
  - Looking desperately for a hedging based approach to pricing.
Modelling credit derivatives: Where do we stand?

**Plain default swaps**

- Static arbitrage of plain default swaps with short selling underlying bond
  - plain default swaps hedged using underlying risky bond
  - “bond strippers”: allow to compute prices of risky zero-coupon bonds
  - repo risk, squeeze risk, liquidity risk, recovery rate assumptions

- Computation of the P&L of a book of default swaps
  - Involves the computation of a P&L of a book of default swaps
  - The P&L is driven by changes in the credit spread curve and by the occurrence of default.
Modelling credit derivatives: Where do we stand?

Credit risk management

• Assessing the varieties of risks involved in credit derivatives
  – Specific risk or credit spread risk
    ➢ *prior to default*, the P&L of a book of credit derivatives is driven by changes in credit spreads.
  – Default risk
    ➢ *in case of default*, if unhedged,
    ➢ dramatic jumps in the P&L of a book of credit derivatives.
Modelling credit derivatives: Where do we stand?
The Noah’s arch of credit risk models

• “firm-value” models:
  – Modelling of firm’s assets
  – First time passage below a critical threshold

• risk-intensity based models
  – Default arrivals are no longer predictable
  – Model conditional local probabilities of default $\lambda(t) \, dt$
  – $\tau$: default date, $\lambda(t)$ risk intensity or hazard rate

$$\lambda(t) \, dt = P[\tau \in [t, t + dt] | \tau > t]$$

• Lack of a hedging based approach to pricing.
  – Misunderstanding of hedging against default risk and credit spread risk
A new approach to credit derivatives modelling based on an hedging point of view

• **Rolling over** the hedge:
  – Short term default swaps v.s. long-term default swaps
  – Credit spread transformation risk

• Credit contingent contracts, basket default swaps
  – Hedging default risk through dynamics holdings in standard default swaps
  – Hedging credit spread risk by choosing appropriate default swap maturities
  – Closing the gap between pricing and hedging

• **Practical hedging issues**
  – Uncertainty at default time
  – Managing net residual premiums
Long-term Default Swaps v.s. Short-term Default Swaps
Rolling over the hedge

• Purpose:
  – Introduction to dynamic trading of default swaps
  – Illustrates how default and credit spread risk arise

• Arbitrage between long and short term default swap
  – sell one long-term default swap
  – buy a series of short-term default swaps

• Example:
  – default swaps on a FRN issued by BBB counterparty
  – 5 years default swap premium : 50bp, recovery rate = 60%

Credit derivatives dealer

If default, 60%

Until default, 50 bp

Client
Long-term Default Swaps v.s. Short-term Default Swaps
Rolling over the hedge

- Rolling over short-term default swap
  - at inception, one year default swap premium: 33bp
  - cash-flows after one year:

    ![Diagram showing credit derivatives dealer to market 33bp with 60% if default]

- Buy a one year default swap at the end of every yearly period, if no default:
  - Dynamic strategy,
  - future premiums depend on future credit quality
  - future premiums are unknown

![Diagram showing credit derivatives dealer to market ??bp with 60% if default]
• **Risk analysis** of rolling over short term against long term default swaps

**Diagram:**

- Credit derivatives dealer ➡️ Market + Client
  - ?? bp
  - 50 bp

**Exchanged cash-flows:**
  - Dealer receives 5 years (fixed) credit spread,
  - Dealer pays 1 year (variable) credit spread.

**Full one to one protection at default time**
  - The previous strategy has eliminated one source of risk, that is *default risk*
Long-term Default Swaps v.s. Short-term Default Swaps
Rolling over the hedge

• negative exposure to an increase in short-term default swap premiums
  – if short-term premiums increase from 33bp to 70bp
  – reflecting a lower (short-term) credit quality
  – and no default occurs before the fifth year

Credit derivatives dealer 70 bp Market + Client
  50 bp

• Loss due to negative carry
  – long position in long term credit spreads
  – short position in short term credit spreads
Consider a portfolio of homogeneous loans

- same unit nominal, non amortising
- $\tau_i$: default time of counterparty $i$
- same default time distribution (same hazard rate $\lambda(t)$):
  \[ P[\tau_i \in [t, t + dt \mid \tau_i > t] = \lambda(t)dt \]
- $F_t$: available information at time $t$
- Conditional independence between default events $\{\tau_i \in [t, t + dt]\}$
  \[ P[\tau_i, \tau_j \in [t, t + dt \mid F_t] = P[\tau_i \in [t, t + dt \mid F_t] \times P[\tau_j \in [t, t + dt \mid F_t] \]
  \]
  - equal to zero or to $\lambda^2(t)(dt)^2$, i.e no simultaneous defaults.
  - Remark that indicator default variables $1_{\{\tau_i \in [t, t+dt]\}}$ are
    (conditionally) independent and equally distributed.
Denote by $N(t)$ the outstanding amount of the portfolio (i.e. the number of non defaulted loans) at time $t$.

By law of large numbers, 

$$\frac{1}{N(t)} \sum 1_{\{\tau_i \in [t, t+dt]\}} \rightarrow \lambda(t) dt$$

Since 

$$N(t + dt) - N(t) = - \sum 1_{\{\tau_i \in [t, t+dt]\}}$$

we get, 

$$\frac{N(t + dt) - N(t)}{N(t)} = - \lambda(t) dt$$

The outstanding nominal decays as 

$$N(t) = N(0) \exp\left(- \int_0^t \lambda(s) ds \right)$$

Assume zero recovery; Total default loss $t$ and $t+dt$: $N(t)-N(t+dt)$

Cost of default per outstanding loan: 

$$\frac{N(t) - N(t + dt)}{N(t)} = \lambda(t) dt$$
Rolling over the hedge: portfolio of homogeneous loans

- Cost of default per outstanding loan = $\lambda(t)dt$ is known at time $t$.
- **Insurance** diversification approach holds
- *Fair premium* for a short term insurance contract on a single loan (i.e. a short term default swap) has to be equal to $\lambda(t)dt$.
- Relates **hazard rate** and short term default swap premiums.

- Expanding on rolling over the hedge
  - Let us be short in 5 years (say) default swaps written on all individual loans.
    - $p_{5Y} dt$, periodic premium per loan.
  - Let us buy the short term default swaps on the outstanding loans.
    - Corresponding premium per loan: $\lambda(t)dt$.
  - Cash-flows related to default events $N(t) - N(t+dt)$ perfectly offset
**Rolling over the hedge: portfolio of homogeneous loans**

- **Net (premium) cash-flows** between $t$ and $t+dt$: \( N(t) \left[ p_{5Y} - \lambda(t) \right] dt \)
- **Where** \( N(t) = N(0) \exp{\int_{0}^{t} \lambda(s) ds} \)
  - Payoff similar to an "index amortising swap".
- At inception, \( p_{5Y} \) must be such that the risk-neutral expectation of the discounted net premiums equals zero:
- Pricing equation for the long-term default swap premium \( p_{5Y} \):
  \[
  E \left[ \int_{0}^{T} \left( \exp{\int_{0}^{t} r(s) ds} \right) \times N(t)(p_{5Y} - \lambda(t)) dt \right] = 0
  \]
  - where \( r(t) \) is the short rate at time \( t \).
- **Premiums received when selling long-term default swaps**: \( N(t)p_{5Y} dt \)
- **Premiums paid on “hedging portfolio”**: \( N(t)\lambda(t) dt \)
• Convexity effects and the cost of the hedge
  – Net premiums paid \( N(t)[p_{5Y} - \lambda(t)]dt \)

• What happens if short term premiums \( \lambda(t) \) become more volatile?
  - Net premiums become negative when \( \lambda(t) \) is high.
  - Meanwhile, the outstanding amount \( N(t) \) tends to be small, mitigating the losses.
  - Contrarily when \( \lambda(t) \) is small, the dealer experiments positive cash-flows \( p_{5Y} - \lambda(t) \) on a larger amount \( N(t) \).

• The more volatile \( \lambda(t) \), the smaller the average cost of the hedge and thus the long term premium \( p_{5Y} \).
Hedging exotic default swaps: main features

• Exotic credit derivatives can be *hedged* against default:
  – Constrains the *amount* of underlying *standard* default swaps.
  – *Variable* amount of standard default swaps.
  – *Full protection* at default time by construction of the hedge.
  – No more *discontinuity* in the P&L at default time.
  – “Safety-first” criteria: *main source of risk* can be hedged.
  – *Model-free* approach.

• *Credit spread exposure* has to be hedged by *other means*:
  – Appropriate *choice of maturity* of underlying default swap
  – Computation of sensitivities with respect to changes in credit spreads are *model dependent*.
Hedging Default Risk in Credit Contingent Contracts

• Credit contingent contracts
  – client pays to dealer a periodic premium $p_T(C)$ until default time $\tau$, or maturity of the contract $T$.
  – dealer pays $C(\tau)$ to client at default time $\tau$, if $\tau \leq T$.

• Hedging side:
  – Dynamic strategy based on standard default swaps:
  – At time $t$, hold an amount $C(t)$ of standard default swaps
  – $\lambda(t)$ denotes the periodic premium at time $t$ for a short-term default swap
Hedging Default Risk in Credit Contingent Contracts

- **Hedging side:**
  
  Credit derivatives dealer \( \lambda(t) C(t) \) until default \( C(\tau) \) if default
  
  - Amount of standard default swaps equals the (variable) credit exposure on the credit contingent contract.

- **Net position is a “basis swap”:**
  
  Credit derivatives dealer \( \lambda(t) C(t) \) until default \( p_T(C) \) until default
  
  - The client transfers credit spread risk to the credit derivatives dealer
• **What is the cost of hedging default risk?**

• **Discounted value of hedging default swap premiums:**

\[
E \left[ \int_0^T \left( \exp - \int_0^t (r + \lambda(s))ds \right) \lambda(t)C(t)dt \right]
\]

- Discounting term
- Premium paid at time \(t\) on protection portfolio

• **Equals the discounted value of premiums received by the seller:**

\[
E \left[ \int_0^T \left( \exp - \int_0^t (r + \lambda(s))ds \right) p_Tdt \right]
\]
Case study: defaultable interest rate swap

- Consider a defaultable interest rate swap (with unit nominal)
  - We are default-free, our counterparty is defaultable (default intensity $\lambda(t)$).
  - We consider a (fixed-rate) receiver swap on a standalone basis.
- Recovery assumption, payments in case of default.
  - if default at time $\tau$, compute the default-free value of the swap: $PV_\tau$
  - and get: $\delta((PV_\tau)^+ + (PV_\tau)^-) = PV_\tau - (1 - \delta)(PV_\tau)^+$
  - $0 \leq \delta \leq 1$ recovery rate, $(PV_\tau)^+ = \max(PV_\tau, 0)$, $(PV_\tau)^- = \min(PV_\tau, 0)$
  - In case of default,
    - we receive default-free value $PV_\tau$
    - minus
    - loss equal to $(1 - \delta)(PV_\tau)^+$. 
Case study: defaultable interest rate swap

- **Defaultable and default-free swap**
  - Present value of *defaultable* swap = Present value of *default-free* swap (with same fixed rate) – Present value of the loss.
  - To compensate for default, fixed rate of defaultable swap (with given market value) is *greater* than fixed rate of default-free swap *(with same market value).*
  - Let us remark, that default immediately after negotiating a defaultable swap results in a *positive* jump in the P&L, because recovery is based on default-free value.

- To account for the possibility of default, we may constitute a *credit reserve.*
  - Amount of credit reserve equals expected Present Value of the loss
  - This accounts for the *expected* loss but does not hedge against realized loss.
Case study: defaultable interest rate swap

- Using a hedging instrument rather than a credit reserve
  - Consider a credit contingent contract that pays \((1-\delta)(PV_\tau)^+\) at default time \(\tau\) (if \(\tau \leq T\)), where \(PV_\tau\) is the present value of a default-free swap with same fixed rate than defaultable swap.
  - Such a credit contract + a defaultable swap synthesises a default-free swap (at a fixed rate equal to the initial fixed rate):
  - At default, we receive \((1-\delta)(PV_\tau)^+ + PV_\tau - (1-\delta)(PV_\tau)^+ = PV_\tau\)
  - The upfront premium for this credit protection is equal to the Present Value of the loss \((1-\delta)(PV_\tau)^+\) given default:

\[
E\left[\int_0^T \left( \exp - \int_0^t (r + \lambda(u)du) \right) \lambda(t)(1-\delta)(PV_t)^+ dt \right]
\]
**Case study: defaultable interest rate swap**
**Interpreting the cost of the hedge**

- **Average cost of default on a large portfolio of swaps**
  - Large number of *homogeneous defaultable* receiver swaps:
    - Same fixed rate and maturity; initial nominal value $N(0)=1$
    - Independent default dates and same default intensity $\lambda(t)$.
  - **Outstanding nominal amount:** $N(t) = \exp \int_0^t \lambda(s)ds$
  - Nominal amount defaulted in $[t, t+dt[: N(t)−N(t+dt)=\lambda(t)dt\exp\int_0^t \lambda(s)ds$
  - **Cost of default in $[t, t+dt[: (N(t)−N(t+dt)) (1-\delta)(PV_t)^+$
  - Where $PV_t$: present value of receiver swap with unit nominal.
  - Aggregate cash-flows do not depend on default risk.
  - Aggregate cash-flows are those of an index amortising swap
  - Standard discounting provides previous slide pricing equation
**Case study: defaultable interest rate swap**  
**Interpreting the cost of the hedge**

- **Randomly exercised swaption:**
  - Assume for simplicity no recovery ($\delta=0$).
  - Interpret default time as a random time $\tau$ with intensity $\lambda(t)$.
  - At that time, defaulted counterparty “exercises” a swaption, i.e. decides whether to cancel the swap according to its present value.
  - PV of default-losses equals price of that *randomly exercised swaption*.

- **American Swaption**
  - PV of *American swaption* equals the supremum over *all possible stopping times of randomly exercised swaptions*.
    - The upper bound can be reached for special default arrival dates:
      - $\lambda(t)=0$ above exercise boundary and $\lambda(t)=\infty$ on exercise boundary.
Case study: defaultable interest rate swap

• Previous hedge leads to (small) jumps in the P&L:
  – Consider a 5.1% fixed rate defaultable receiver swap with PV=3%.
  – Buy previous credit contingent contract at market price.
    ➢ Due to credit protection, we hold a synthetic default-free 5.1% swap.
    ➢ Total PV remains equal to 3%.
  – Assume that default immediate default: \( \tau=0^+ \).
  – Clearly a 5.1% default free swap has PV>3%, thus occurring a positive jump in P&L.

• Jumps in the P&L due to extra default insurance:
  – To hedge the previous credit contingent contract:
  – At time 0, we hold an amount of short term default swap that is equal to the Present Value of a default-free 5.1% swap
  – This amount is greater than 3%, the current Present Value.
• Alternative hedging approach:
  ➢ Fixed rate of default-free swap with 3% PV = 5% (say)
    – Consider a credit contingent contract that pays at default time:
    – Present value of a default free 5% swap minus recovered value on the 5,1% defaultable swap.
    – at default time, holder of defaultable swap + credit contract receives:
      ➢ recovery value on 5,1% defaultable swap + PV of default free 5% swap - recovered value on 5,1% defaultable swap
        ➢ = PV of default free 5% swap
    – Assume credit contract has a periodic annual premium denoted by p.
    – Prior to default time, defaultable swap + credit contract pays:
      ➢ Default-free swap cash-flows with fixed rate = 5,1% - p
    – p must be equal to 10bp = 5,1%-5%, otherwise arbitrage with 5% default-free swap.
Case study: defaultable interest rate swap

- Credit contingent contract transforms 5.1% defaultable swap into a 5% default free swap with the same PV.
  - If default occurs immediately, no jump in the hedged P&L.
  - To hedge the default payment on the credit contingent contract, we must hold default swaps providing payments of:
    - PV of default free 5% swap - recovery on 5.1% defaultable swap:
      \[ PV_\tau(5\%) - \delta PV_\tau(5.1\%)^+ - PV_\tau(5.1\%)^- \]
      - \( PV_\tau(5.1\%) \) is close to \( PV_\tau(5\%) \) (here 3\% = PV of defaultable swap).
      - Required payment on hedging default swap close to \((1- \delta) PV_\tau(5.1\%)^+\)
        \[ \text{Plain default swap pays } 1- \delta \text{ at default time.} \]
- Nominal amount of hedging default swap almost equal to \( PV_\tau(5.1\%)^+ \)
Hedging Default risk and credit spread risk in Credit Contingent Contracts

- Purpose: joint hedge of default risk and credit spread risk
- Hedging default risk only constrains the amount of underlying standard default swap.
  - Maturity of underlying default swap is arbitrary.
- Choose maturity to be protected against credit spread risk
  - PV of credit contingent contracts and standard default swaps are sensitive to the level of credit spreads
  - Sensitivity of standard default swaps to a shift in credit spreads increases with maturity
  - Choose maturity of underlying default swap in order to equate sensitivities.
Hedging credit spread risk

- Example:
  - dependence of simple default swaps on defaultable forward rates.
  - Consider a $T$-maturity default swap with continuously paid premium $p$. Assume zero-recovery (digital default swap).
  - PV (at time 0) of a long position provided by:
    \[
    PV = E \left[ \int_0^T \exp \left( - \int_0^t (r(s) + \lambda(s)) ds \right) \times (\lambda(t) - p) dt \right]
    \]
    where $r(t)$ is the short rate and $\lambda(t)$ the default intensity.
  - Assume that $r(.)$ and $\lambda(.)$ are independent.
  - $B(0,t)$: price at time 0 of a $t$-maturity default-free discount bond
  - $f(0,t)$: corresponding forward rate
    \[
    B(0,t) = E \left[ \exp - \int_0^t r(u) du \right] = \exp - \int_0^t f(0,u) du
    \]
Hedging credit spread risk

− Let \( \overline{B}(0,t) \) be the \textit{defaultable discount bond price} and \( \overline{f}(0,t) \) the corresponding instantaneous forward rate:

\[
\overline{B}(0,t) = E\left[ \exp - \int_{0}^{t} (r + \lambda)(u)du \right] = \exp - \int_{0}^{t} \overline{f}(0,u)du
\]

− Simple expression for the PV of the \( T \)-maturity default swap:

\[
PV(T) = \int_{0}^{T} \overline{B}(0,t) \left( \overline{f}(0,t) - f(0,t) - p \right) dt
\]

− The derivative of default swap present value with respect to a shift of defaultable forward rate \( \overline{f}(0,t) \) is provided by:

\[
\frac{\partial PV}{\partial \overline{f}}(t) = PV(t) - PV(T) + \overline{B}(0,t)
\]

\( \triangleright \) \( PV(t) - PV(T) \) is usually small compared with \( \overline{B}(0,t) \).
Hedging credit spread risk

- Similarly, we can compute the sensitivities of plain default swaps with respect to default-free forward curves \( f(0,t) \).
- And thus to credit spreads.
- Same approach can be conducted with the credit contingent contract to be hedged.
  
  ➢ All the computations are model dependent.
- Several maturities of underlying default swaps can be used to match sensitivities.
  
  ➢ For example, in the case of defaultable interest rate swap, the nominal amount of default swaps \( (PV_\tau)^+ \) is usually small.
  
  ➢ Single default swap with nominal \( (PV_\tau)^+ \) has a smaller sensitivity to credit spreads than defaultable interest rate swap, even for long maturities.

➢ Short and long positions in default swaps are required to hedge credit spread risk.
Explaining theta effects with and without hedging

- **Different aspects** of “carrying” credit contracts through time.
  - Assume “historical” and “risk-neutral” intensities are equal.
- Consider a *short* position in a credit contingent contract.
- Present value of the deal provided by:

\[
PV(u) = E_u \left[ \int_u^T \left( \exp - \int_u^t (r + \lambda(s)) ds \right) \times (p_T - \lambda(t)C(t)) dt \right]
\]

- (after computations) **Net expected capital gain**:

\[
E_u \left[ PV(u + du) - PV(u) \right] = \left( r(u) + \lambda(u) \right) PV(u) du + \left( \lambda(u)C(u) - p_T \right) du
\]

- **Accrued cash-flows** (received premiums): \( p_T du \)
  - By summation, Incremental P&L (if no default between \( u \) and \( u + du \)):

\[
r(u)PV(u) du + \lambda(u) \left( C(u) + PV(u) \right) du
\]
Explaining theta effects with and without hedging

- **Apparent extra return effect**: \( \lambda(u)(C(u) + PV(u))du \)
  - But, probability of default between \( u \) and \( u + du \): \( \lambda(u)du \).
  - Losses in case of default:
    - Commitment to pay: \( C(u) \)
    - Loss of PV of the credit contract: \( PV(u) \)
    - \( PV(u) \) consists in **unrealised** capital gains or losses in the credit derivatives book that “disappear” in case of default.
  - Expected loss charge: \( \lambda(u)(C(u) + PV(u))du \)

- **Hedging aspects**:
  - If we hold \( C(u) + PV(u) \) short-term digital default swaps, we are protected at default-time (no jump in the P&L).
  - Premiums to be paid: \( \lambda(u)(C(u) + PV(u))du \)
  - Same average rate of return, but smoother variations of the P&L.
Example: first to default swap from a basket of two risky bonds.
- If the first default time occurs before maturity,
- The seller of the first to default swap pays the non recovered fraction of the defaulted bond.
- Prior to that, he receives a periodic premium.

Assume that the two bonds cannot default simultaneously
- We moreover assume that default on one bond has no effect on the credit spread of the remaining bond.

How can the seller be protected at default time?
- The only way to be protected at default time is to hold two default swaps with the same nominal than the nominal of the bonds.
- The maturity of underlying default swaps does not matter.
Real World hedging and risk-management issues

• uncertainty at default time
  – illiquid default swaps
  – recovery risk
  – simultaneous default events

• Managing net premiums
  – Maturity of underlying default swaps
  – Lines of credit
  – Management of the carry
  – Finite maturity and discrete premiums
  – Correlation between hedging cash-flows and financial variables
Consider a first to default swap associated with a basket of two defaultable loans.

- Hedging portfolios based on standard underlying default swaps
- Uncertain hedge ratios if:
  - simultaneous default events
  - Jumps of credit spreads at default times

Simultaneous default events:

- If counterparties default altogether, holding the complete set of default swaps is a conservative (and thus expensive) hedge.
- In the extreme case where default always occur altogether, we only need a single default swap on the loan with largest nominal.
- In other cases, holding a fraction of underlying default swaps does not hedge default risk (if only one counterparty defaults).
Real world hedging and risk-management issues
Case study: hedge ratios for first to default swaps

• What occurs if there is a *jump in the credit spread* of the second counterparty after default of the first?
  – default of first counterparty means *bad news* for the second.

• If hedging with short-term default swaps, *no capital gain* at default.
  – Since PV of short-term default swaps is not *sensitive* to credit spreads.

• This is not the case if hedging with long term default swaps.
  – If credit spreads *jump*, PV of long-term default swaps *jumps*.

• Then, the amount of hedging default swaps can be *reduced*.
  – This reduction is *model-dependent*. 
On the edge of completeness?

- **Firm-value structural default models:**
  - Stock prices follow a diffusion processes (no jumps).
  - Default occurs at first time the stock value hits a barrier

- **In this modelling,** default credit derivatives can be **completely** hedged by trading the stocks:
  - “Complete” pricing and hedging model:

- **Unrealistic features for hedging basket default swaps:**
  - Because default times are predictable, *hedge ratios are close to zero* except for the counterparty with the smallest “distance to default”.
In hazard rate based models:

− default is a sudden, non predictable event,
− that causes a sharp jump in defaultable bond prices.
− Most credit contingent contracts and basket default default derivatives have payoffs that are linear in the prices of defaultable bonds.
− Thus, good news: default risk can be hedged.
− Credit spread risk can be substantially reduced but not completely eliminated.
− More realistic approach to default.
− Hedge ratios are robust with respect to default risk.
On the edge of completeness

Conclusion

- Looking for a better understanding of credit derivatives
  - payments in case of default,
  - volatility of credit spreads.
- Bridge between risk-neutral valuation and the cost of the hedge approach.
- **dynamic** hedging strategy based on *standard default swaps*.
  - hedge ratios in order to get protection at default time.
  - hedging default risk is *model-independent*.
  - importance of quantitative models for a better management of the P&L and the *residual premiums*. 