An overview of the valuation of collateralized derivatives contracts

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An overview of the valuation of collateralized derivatives contracts

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An overview of the valuation of collateralized derivatives contracts

- Overview of the presentation
  - *Financial context*
  - *Variation margins paid on a collateral account*
  - *Settlement prices and collateralization schemes*
  - *Pricing equation : perfect collateralization*
    - OIS discounting
    - Futures pricing
    - Costless collateral
    - Posting bonds
  - *Overcollateralization, haircuts, run on repos*
  - *Pricing equation for unilateral collateral agreement*
    - CVA and funding cost adjustments
    - Trade contribution

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An overview of the valuation of collateralized derivatives contracts

- Due to new regulations and counterparty risk management, a large amount of derivatives contracts is collateralized
- OTC transactions
  - *Swaps ISDA (CSA: Credit Support Annex)*
  - *2012 Margin Survey report 137,869 active collateral agreements*
- Central clearing
  - *LCH Clearnet, ICE, Eurex,*
  - *Futures exchanges (CME, ...)*
  - *SEF (Swap Exchange Facilities)*
- Need of a unifying pricing framework
  - *Variation margins, initial margins,*
  - *Bilateral, unilateral agreements*
  - *Collateral type: Cash, bonds, currencies*
  - *We remain in the standard mathematical finance approach*

eurex clearing

LCH.CLEARNET

CME Group

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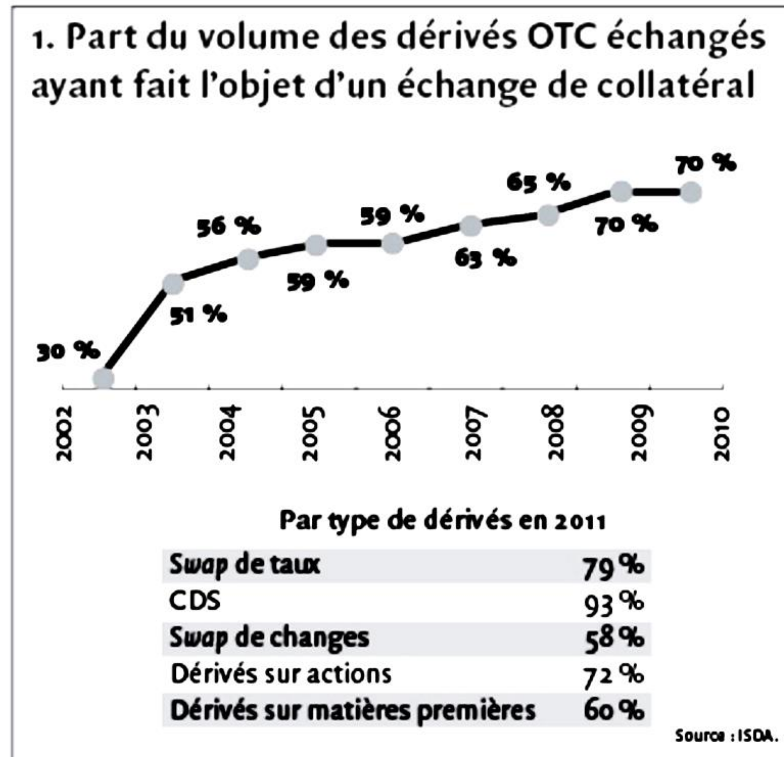
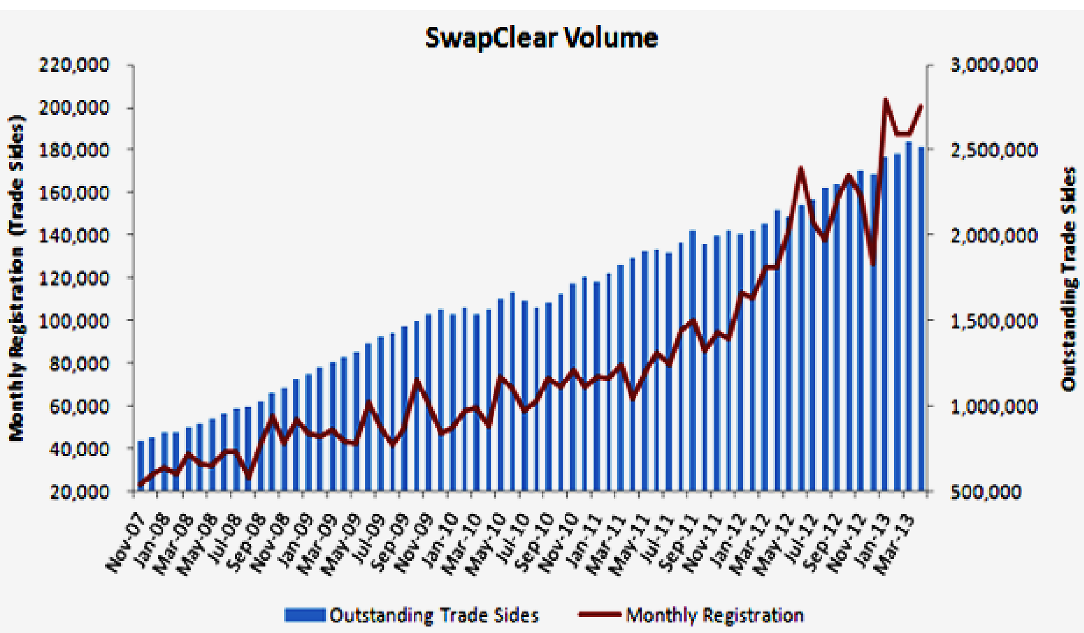
An overview of the valuation of collateralized derivatives contracts

- Most ISDA trades are associated with collateral agreements (right figure)
- Swapclear (LCH.Clearnet) prominent CCP for IRS outstanding notional US $ 380 trillion (figure below)





An overview of the valuation of collateralized derivatives contracts

- Variation margins paid on a collateral account
  - $V(s)$ *amount on collateral account in a bilateral agreement*
    - Collateral accounts recorded by ISDA evaluated at US $ 3.6 trillion end of year 2011
  - $A(s)$ *price of posted security*
    - Approximately 80% cash, 20% government bonds
  - $V(s)/A(s)$ *number of posted securities*
- $dVM(s)$ Variation margin: net inflow on collateral account
  - $dVM(s) = V(s+ds) - \frac{V(s)}{A(s)} A(s+ds)$
  - *Equivalently written as:* $dVM(s) = dV(s) - V(s)\frac{dA(s)}{A(s)}$
  - $dA(s)/A(s)$ *realized return on collateral*

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An overview of the valuation of collateralized derivatives contracts

- The dynamics of collateral account $V(s)$ can equivalently be written as:
- $dV(s) = V(s) \times \frac{dA(s)}{A(s)} + dVM(s)$
  - $V(s) \times \frac{dA(s)}{A(s)}$ *self-financed part*
  - $dVM(s)$ *inflow in collateral account*
- Case of cash-collateral $A \equiv C$
  - $dC(s) = c(s)C(s)ds$
  - $c(s)$: *EONIA, fed fund rate, …*
  - $dV(s) = c(s)V(s)ds + dVM(s)$

An overview of the valuation of collateralized derivatives contracts

- Collateralization schemes
  - $h(t)$ *settlement price (credit risk exposure)*
  - $V(t) = h(t)$ *Bilateral collateral agreement*
    - Interdealer contracts
  - *Unilateral collateral agreement* $V(t) = \min(h(t), 0)$
    - Unilateral: sovereign entities

Unilateral			**Bilateral**			**Total Active**
ISDA collateral agreements	**Non-ISDA agreements**	**Total number Unilateral**	**ISDA collateral agreements**	**Non-ISDA agreements**	**Total number Bilateral**	
14,212	8,001	22,213	103,398	12,258	115,656	137,869
10.3%	5.9%	16.1%	74.9%	9.0%	83.9%	100%

  - $V(s) = \alpha_A \times h(s), \alpha_A > 1$ *overcollateralization*
    - $1/\alpha_A$ haircut ratio

An overview of the valuation of collateralized derivatives contracts

- The basic pricing equation (bilateral, no haircut)
  - *Does not account for possible price impact of initial margin*
  - *Bilateral case, no haircut* $V(t) = h(t)$
  - *Enter a collateralized derivatives contract at* t
    - Buy collateralized security: outflow $h(t)$
    - Receive collateral to secure credit exposure : inflow $V(t)$
    - Since $V(t) = h(t)$ net cash-flow at trade inception $t = 0$
  - *Exit trade at* $t + dt$ *after paying variation margin* $dVM(t)$
    - $dVM(t) = dV(t) - V(t)\frac{dA(t)}{A(t)} = dh(t) - h(t)\frac{dA(t)}{A(t)}$
    - Since $V(t + dt) = h(t + dt)$ no extra cash-flow at $t + dt$
    - Q^{β} usual risk-neutral pricing measure, assumed to be given
  - $E_t^{Q^{\beta}}[dVM(t)] = 0 \Rightarrow \boldsymbol{E_t^{Q^{\beta}}\left[\frac{dh(t)}{h(t)}\right] = E_t^{Q^{\beta}}\left[\frac{dA(t)}{A(t)}\right]}$

An overview of the valuation of collateralized derivatives contracts

- $E_t^{Q^\beta}\left[\frac{dh(t)}{h(t)}\right] = E_t^{Q^\beta}\left[\frac{dA(t)}{A(t)}\right]$
- $E_t^{Q^\beta}\left[\frac{dA(t)}{A(t)}\right] = r_A(t)dt$ $r_A(t)$ *:expected rate of return on collateral*

- $\Rightarrow E_t^{Q^\beta}\left[\frac{dh(t)}{h(t)}\right] = r_A(t)dt$ $h(t)$ settlement price
- Plus terminal condition: $h(T)$ payment at maturity date T
- $h(t) = E_t^{Q^\beta}\left[h(T)\exp\left(-\int_t^T r_A(s)ds\right)\right]$
  - *Discount with expected rate of return on collateral*
  - $t \rightarrow T$ *: liquidity assumptions*
    - Assumes collateral is available from t to T
    - No haircut on collateral being introduced after t
    - No gap risks: possible exit at settlement and collateral price

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An overview of the valuation of collateralized derivatives contracts

- Cash-collateral
  - $dC(t) = c(t)C(t)dt$, $c(t)$ *EONIA, effective fed fund rate*
  - $r_A(t) = c(t)$
  - $h(t) = E_t^{Q^\beta}\left[h(T)\exp\left(-\int_t^T c(s)ds\right)\right]$ "OIS *discounting*"
- $c(t) = 0$, futures market (Duffie, 1989)
  - $dVM(t) = dh(t) - \cancel{c(t)h(t)dt} = dh(t)$
  - *variation margin* $dVM(t)$ = *change in settlement price* $dh(t)$
  - $h(t) = E_t^{Q^\beta}[h(T)]$, $h(t)$ Q^β*-martingale,*
- $c(t) = r(t)$, ($r(t)$ default-free short term rate)
  - *"costless collateral", Johannes & Sundaresan (2007)*
  - *Then, collateralized prices = uncollateralized prices*
  - *Usually* $c(t) > r(t)$ *!*

An overview of the valuation of collateralized derivatives contracts

- Stylized collateralized OIS (Overnight Index Swap)
  - *Payment of* $h(T) = \exp(r_f T) - \exp\left(\int_0^T c(s)ds\right)$
    - $c(s)$ EONIA, r_f fixed rate
    - Cash-collateral with collateral rate $c(s)$
  - *Settlement price* $h(0) = E^{Q^\beta}\left[h(T)\exp\left(-\int_0^T c(s)ds\right)\right]$
  - *Newly traded contract:* $r_f(T)$ *is such that* $h(0) = 0$
    - Thus, $\exp(-r_f(T)T) = E^{Q^\beta}\left[\exp\left(-\int_0^T c(s)ds\right)\right]$
    - Observable market input $r_f(T)$ directly provides collateralized discount factor $E^{Q^\beta}\left[\exp\left(-\int_0^T c(s)ds\right)\right]$
  - <u>*Market observables on collat. markets drive PV computations*</u>
    - This extends to collateralized Libor contracts
    - PV of collateralized Libor = forward (collat.) Libor × collateralized discount factor

An overview of the valuation of collateralized derivatives contracts

- Swapclear (LCH.Clearnet)
  - *OIS and Euribor swaps*
  - *Cash-collateral remunerated at Eonia*
  - *Different swap rates*

EUR

IRS | OIS

	Anl Mny / EONIA
3 month	0.06650
6 month	0.05720
1 year	0.05240
2 year	0.10770
3 year	0.23605
5 year	0.58408
10 year	1.40660
30 year	2.18460

EUR

IRS | OIS

	Anl Bnd / 1MEURIBOR	Anl Bnd / 3MEURIBOR
2 year	0.18288	0.28688
3 year	0.29955	0.40705
5 year	0.64708	0.75308
10 year	1.45860	1.55260
30 year	2.21610	2.27860

An overview of the valuation of collateralized derivatives contracts

- Bonds posted as collateral: bond price $A(t)$
  - *Short-term repo contract* $t \rightarrow t + dt$,
    - no haircut
  - *At* t, *buy the bond, deliver it, receive cash-collateral*
    - no haircut ⇒ no net payment at t: $-A(t) + A(t) = 0$
  - *At* $t + dt$ *receive bond, sell it, reimburse cash + interest*
    - Net payment at $t + dt$: $A(t + dt) - (1 + \mathrm{repo}_A(t)dt)A(t)$
    - $\mathrm{repo}_A(t)$: repo rate, does not account for day count conventions
  - $E_t^{Q^\beta}[A(t + dt) - (1 + \mathrm{repo}_A(t)dt)A(t)] = 0$
- $\Rightarrow E_t^{Q^\beta}\left[\frac{dA(t)}{A(t)}\right] = \mathrm{repo}_A(t)dt$

An overview of the valuation of collateralized derivatives contracts

- Bonds posted as collateral: bond price $A(t)$
  - $E_t^{Q^\beta}\left[\frac{dA(t)}{A(t)}\right] = \text{repo}_A(t)dt$
    - Expected rate of return on posted bond = repo rate $\text{repo}_A(t)$
    - Repo rate: market observable
  - $h(t) = E_t^{Q^\beta}\left[h(T)\exp\left(-\int_t^T \text{repo}_A(s)\, ds\right)\right]$
    - Parallels OIS discounting
  - *Under perfect collateralization, no CVA/DVA counterparty risk is involved*
  - *Unobserved default free short rate $r(t)$ is not involved*
    - Deriving pricing equation only involves cash-flows at $t + dt$
    - No lending/borrowing between t and $t + dt$

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An overview of the valuation of collateralized derivatives contracts

- Overcollateralization, haircuts, run on repos
  - $V(s) = \alpha_A \times h(s)$
  - *$V(s)$ collateral account, $h(s)$ settlement price*
    - $\alpha_A > 1$ overcollateralization, $\alpha_A < 1$ partial collateralization
    - $1/\alpha_A$ haircut ratio
  - *Enter a collateralized derivatives contract at t*
    - Buy collateralized security: outflow $h(t)$
    - Receive collateral to secure credit exposure : inflow $\alpha_A h(t)$
    - Net inflow at $t = (\alpha_A - 1)h(t)$ is not anymore equal to zero
  - *Taking into account cash-flows at $t + dt$*
    - Variation margin & unwinding collateralized contracts
  - *Leads to* $E_t^{Q^\beta}\left[\frac{dh(t)}{h(t)}\right] = \alpha_A \times r_A(t) + (1 - \alpha_A)r(t))$

An overview of the valuation of collateralized derivatives contracts

- Overcollateralization, haircuts, run on repos
  - $E_t^{Q^\beta}\left[\frac{dh(t)}{h(t)}\right] = \alpha_A(t) \times r_A(t) + \left(1 - \alpha_A(t)\right)r(t)$
    - Perfect collateralization $\alpha_A = 1$, $E_t^{Q^\beta}\left[\frac{dh(t)}{h(t)}\right] = r_A(t)$
    - No collateralization $\alpha_A = 0$, $E_t^{Q^\beta}\left[\frac{dh(t)}{h(t)}\right] = r(t)$
    - Time dependent $\alpha_A(t)$ allows to account for increase of collateral requirement during periods of market stress
    - Departure from discounting at repo rate.
  - *Previous equation accounts for settlement price between two default-free counterparties*
    - Does not account for counterparty risks and funding costs
    - Only accounts for adjustments due to collateral cash-flows
    - Involves unobserved default free short rate $r(t)$

An overview of the valuation of collateralized derivatives contracts

- Unilateral collateral agreement
  - $\alpha_A = 1$, *if* $h(t) \leq 0$, $\alpha_A = 0$, *if* $h(t) > 0$
  - $V(t) = \min\big(0, h(t)\big)$
  - *Applicable discount rate:* $r(t)1_{\{h(t)>0\}} + r_A(t)1_{\{h(t)\leq 0\}}$
  - *Settlement price* $h(t)$ *solves for*
  - $E_t^{Q^\beta}[dh(t)] = \Big(r(t)\max\big(0, h(t)\big) + r_A(t)\min\big(0, h(t)\big)\Big)\,dt$
  - *BSDE with generator* $g(h) = rh^+ + r_A h^-$
  - $h(t)$ *conditional* $g -$ *expectation of* $h(T)$ *(Peng, 2004)*
  - g *degree-one homogeneous* $\Rightarrow h(t)$ *degree one homogeneous with respect to* $h(T)$
  - $r \geq r_A \Rightarrow$ *concave price functional (portfolio effects)*

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An overview of the valuation of collateralized derivatives contracts

- Previous equation can be extended: funding rate and CVA
  - $\alpha_A = 1$, *if* $h(t) \leq 0$ *applicable discount rate is* $r_A(t)$ *expected rate of return on collateral*
    - $c(t)$ usually EONIA in case of cash collateral
    - $\text{repo}_A(t)$ repo rate in case of posted bonds
    - No extra funding term (deal is funded through the collateral account)
    - No DVA term, counterparty is fully protected against own default
  - $\alpha_A = 0$, *if* $h(t) > 0$ *applicable discount rate is* $r_b + \lambda(1-\delta)$
    - λ default intensity of counterparty, δ recovery rate
    - Recovery of Market Value, $\lambda(1-\delta)$ CVA cost
    - r_b : default-free short term borrowing rate
    - No collateral is being posted, deal is funded on the market
  - $\boldsymbol{E_t^{Q^\beta}[dh(t)] = \Big((r_b + \lambda(1-\delta))(s)1_{\{h(s)>0\}} + r_A(s)1_{\{h(s)\leq 0\}}\Big)h(t)dt}$

An overview of the valuation of collateralized derivatives contracts

- Previous equation can be extended: funding rate and CVA
  - $E_t^{Q^\beta}[dh(t)] = \Big(\big(r_b + \lambda(1-\delta)\big)(s)1_{\{h(s)>0\}} + r_A(s)1_{\{h(s)\leq 0\}}\Big)h(t)dt$
    - r_b is free of DVA. r_b is not a market observable
    - $\lambda(1-\delta)$ related to short term CDS of counterparty
    - Recovery of Market Value: proxy for "risky close-out convention"
- Trade contributions
    - $\exp\Big(-\int_t^T\big((r_b + \lambda(1-\delta))(s)1_{\{h(s)>0\}} + r_A(s)1_{\{h(s)\leq 0\}}\big)ds\Big)$ pricing kernel or Gâteaux derivative of price functional
    - Trade contribution of deal $h_j(T)$ with $h(T) = h_1(T) + \cdots + h_n(T)$
    - $E_t^{Q^\beta}\Big[h_j(T)\exp\Big(-\int_t^T\big((r_b + \lambda(1-\delta))(s)1_{\{h(s)>0\}} + r_A(s)1_{\{h(s)\leq 0\}}\big)ds\Big)\Big]$
    - Complies with marginal pricing and Euler's allocation rules
    - Linear trade contributions, CSA/portfolio dependent change of measure

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