Alternative Risk Measures for Alternative Investments

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Outline

- **Optimizing under VaR constraints**
  - **Estimation techniques**
  - **VaR analytics and efficient portfolios comparison**
- **Optimizing under alternative risk constraints**
  - **Expected Shortfall, Downside Risk measure,…**
  - **Risk measures analytics and efficient portfolios comparison**

Main result

Decomposition of risk measures

- **VaR**
- **Expected Shortfall**
- **Downside Risk**

A way to understand optimal portfolios structure

Data set

- 16 hedge funds

Hedge funds summary statistics

Fund	Style	mean	annual std	skewness	kurtosis
Axa Rosenberg	Equity Market Neutral	5.61%	8.01%	0.82	13.65
Discovery MasterFund	Equity Market Neutral	6.24%	14.91%	-0.27	0.25
Aetos Corp	Event Driven	12.52%	8.13%	-1.69	7.78
Bennet Restructuring	Event Driven	16.02%	7.48%	-0.74	7.37
Calamos Convertible	Convertible Arbitrage	10.72%	0.71	-9.35%	2.59
Sage Capital	Convertible Arbitrage	9.81%	2.45%	-3.19	3.00
Genesis Emerging Markets	Emerging Markets	10.54%	20.03%	-3.34	6.40
RXR Secured Note	Fixed Income Arbitrage	12.29%	6.45%	2.33	4.84
Arrowsmith Fund	Funds of Funds	26.91%	27.08%	14.51	60.7
Blue Rock Captial	Funds of Funds	8.65%	3.47%	1.66	7.51
Dean Witter Cornerstone	Global Macro	13.95%	23.19%	7.42	9.17
GAMut Investments	Global Macro	24.73%	14.43%	3.38	4.61
Aquila International	Long Short Equity	9.86%	16.88%	-1.22	2.32
Bay Capital Management	Long Short Equity	10.12%	19.31%	1.94	0.70
Blenheim Investments LP	Managed Futures	16.51%	29.59%	3.07	10.25
Red Oak Commododity	Managed Futures	19.80%	29.08%	1.94	3.52

- Data structure
  - monthly data
  - 139 observations
- High skewness and kurtosis
- Low (or negative) correlations
  - diversification potentiality

Value at Risk estimation techniques

- **Empirical quantile**
- **Weighted average of quantiles: “L-estimator” (Granger & Silvapulle (2001))**
- **Kernel smoothing: (Gourieroux, Laurent & Scaillet (2000))**
- **Gaussian VaR : computed under Gaussian assumption**

Contribution of rank statistics (1)

- We denote by $(a'r)_{1:n} \leq \ldots \leq (a'r)_{n:n}$ the rank statistics of the portfolio allocation a
- VaR estimators depend only on the rank statistics
- VaR estimators are differentiable and positively homogeneous of degree one (with respect to the rank statistics)

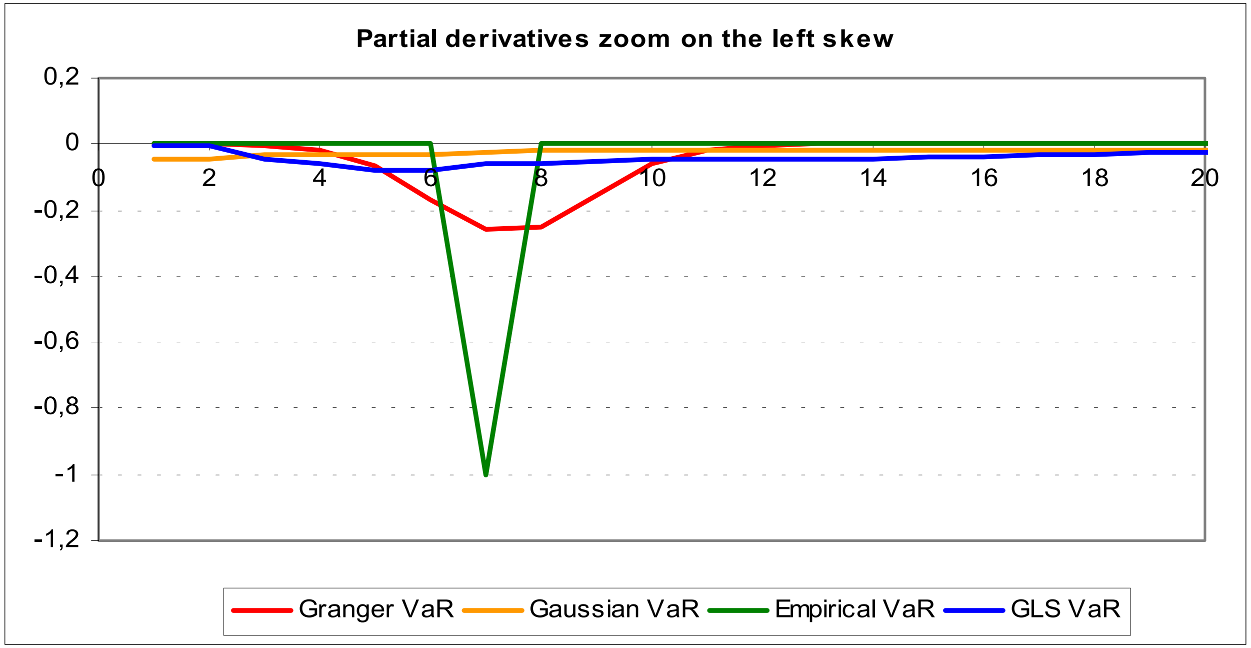
Thus, we can decompose VaR using Euler 's equality :

$$VaR(a'R) = \sum_{i=1}^{n} \frac{\partial VaR(a'R)}{\partial (a'r)_{i:n}} (a'r)_{i:n}$$

see J-P. Laurent [2003]

Contribution of rank statistics (2)

- Weights associated with the rank statistics for the different VaR estimators

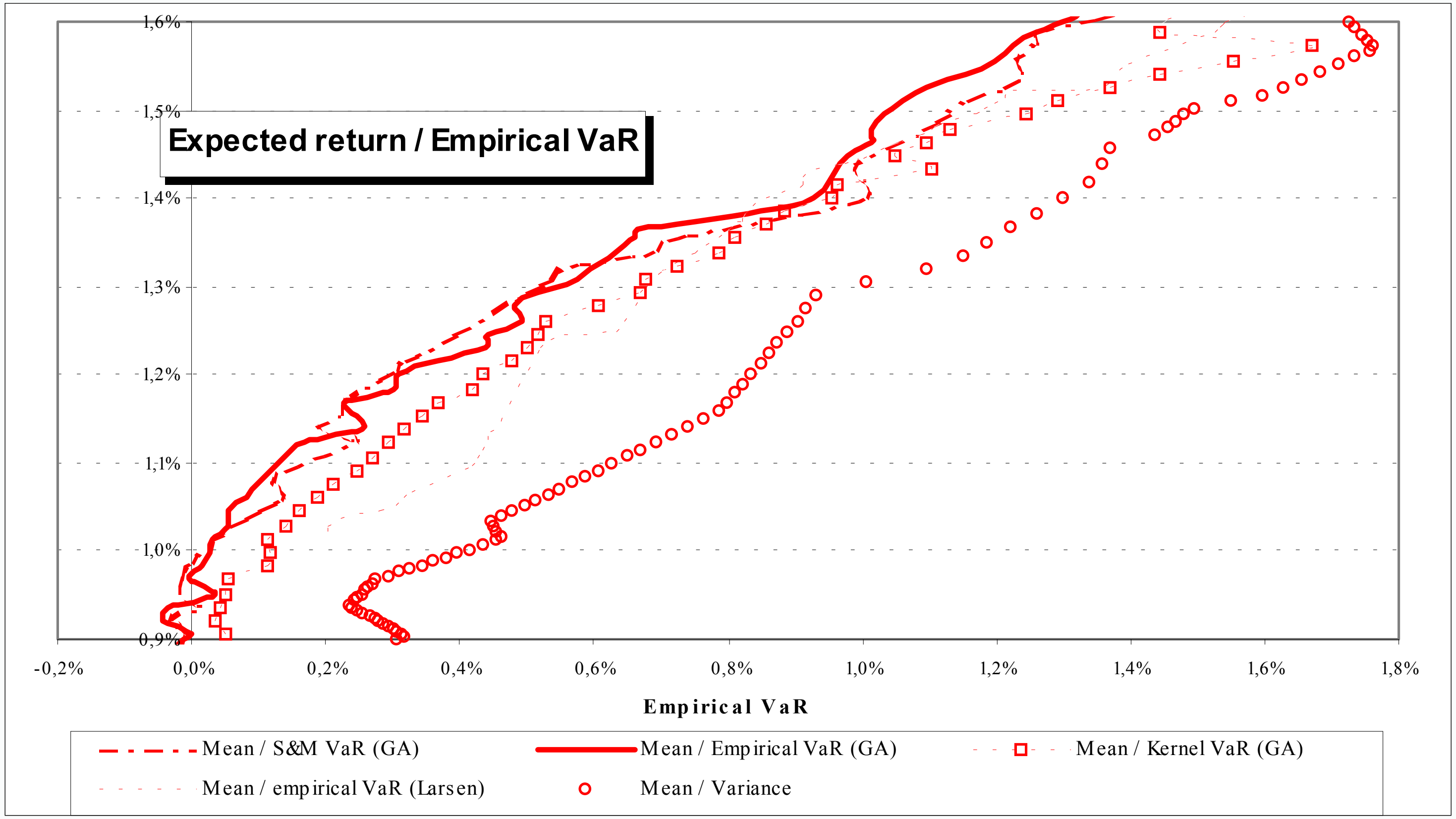


- Empirical VaR is concentrated on a single point
- Granger VaR is distributed around empirical VaR
- GLS VaR : smoother weighting scheme
- Gaussian VaR involves an even smoother pattern

Mean VaR optimization

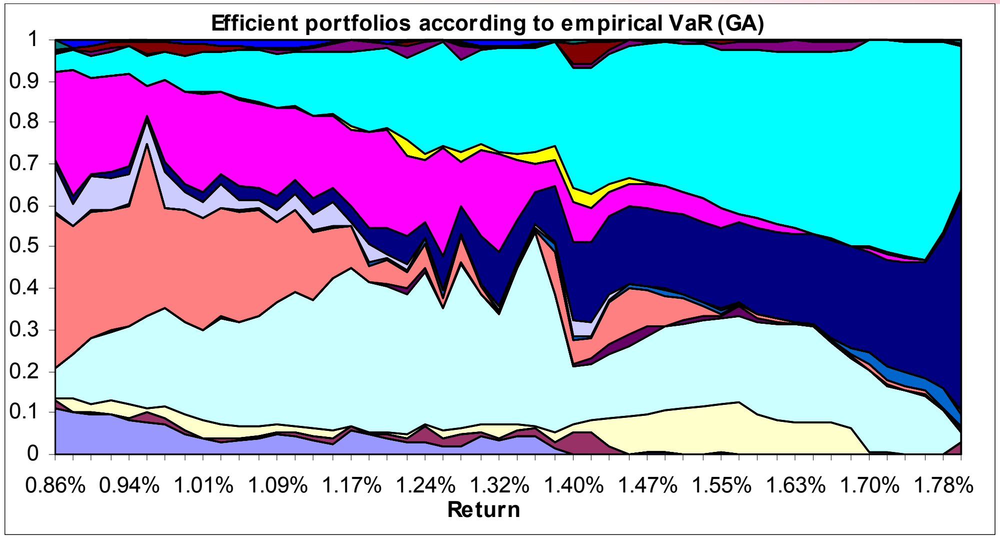
- **A non-standard optimization program**
  - **VaR is not a convex function**
  - **VaR is not differentiable in the general case (with respect to allocation)**
  - **Local minima are often encountered**
- **Genetic algorithms** (see P.-A. Barès, R.Gibson and S. Gyger [2002])
  - **Derived from the natural selection process**
  - **Time consuming: slow convergence**
- **Approximating algorithm Larsen, Mausser & Uryasev**
  - **Based on Expected Shortfall (see below) optimization program**
  - **We get a sub-optimal solution**

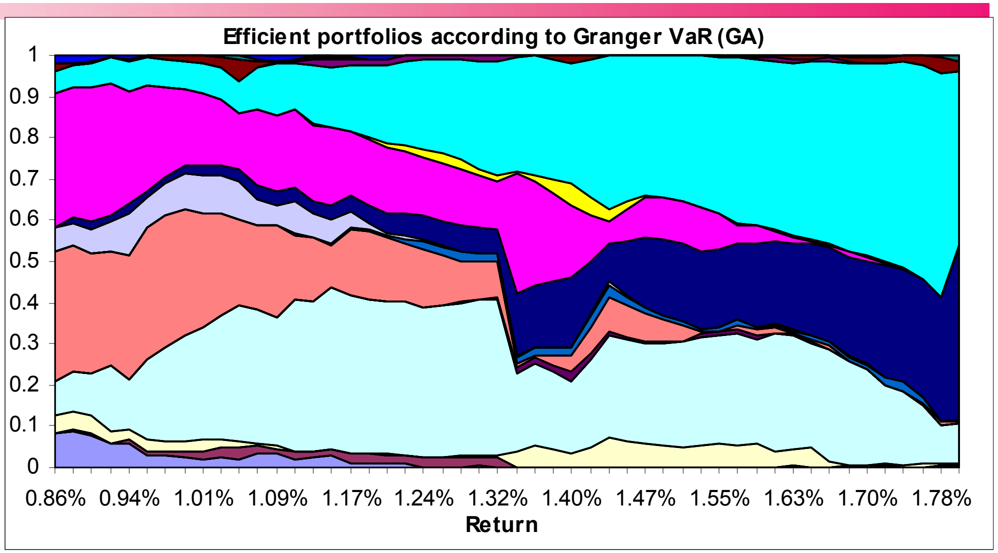
Mean VaR efficient frontier

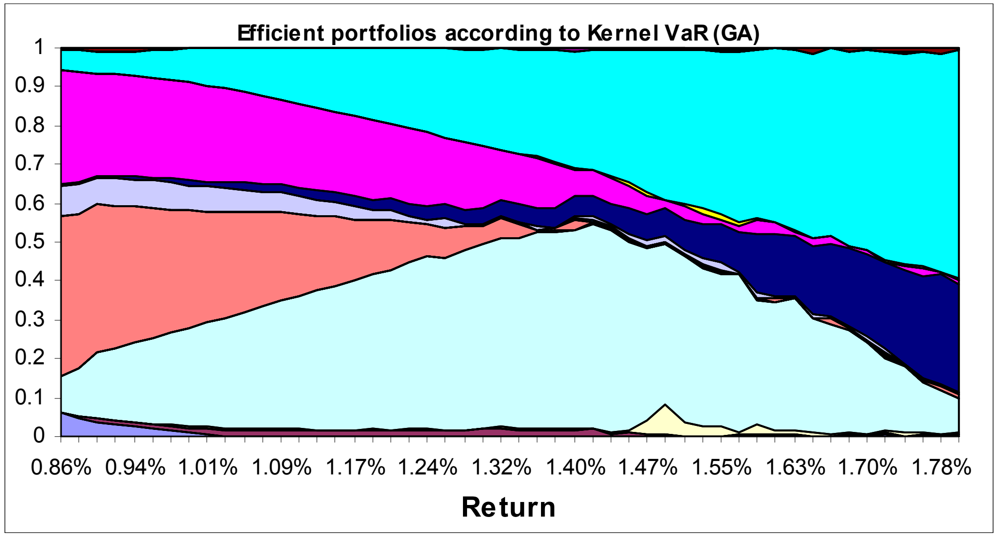


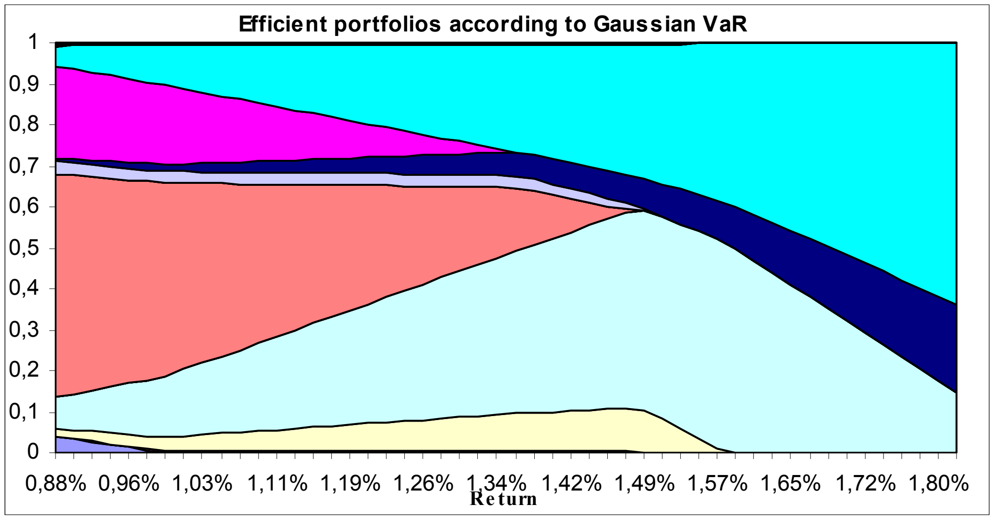
- VaR efficient frontiers are close
- Far from the mean-Gaussian VaR efficient frontier
- Larsen & al. approximating algorithm performs poorly

Mean VaR efficient portfolios (1)





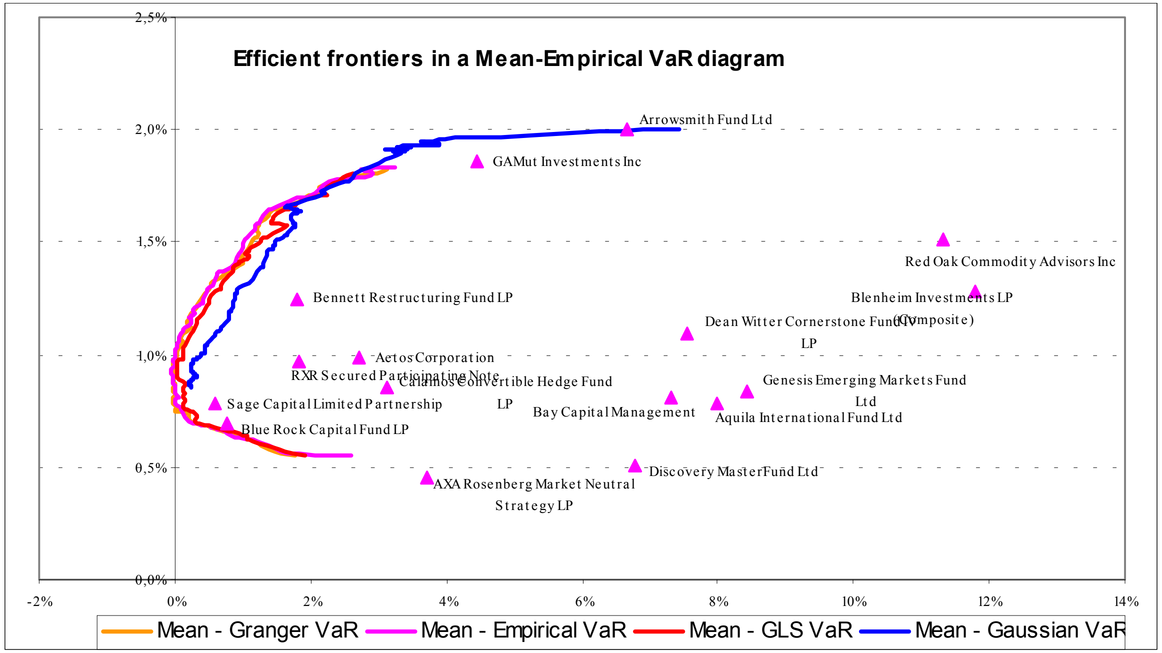




- AXA Rosenberg Market Neutral Strategy LP
- Bennett Restructuring Fund LP
- Genesis Emerging Markets Fund Ltd
- Blue Rock Capital Fund LP
- Aquila International Fund Ltd
- Red Oak Commodity Advisors Inc
- Discovery MasterFund Ltd
- Calamos Convertible Hedge Fund LP
- RXR Secured Participating Note
- Dean Witter Cornerstone Fund IV LP
- Bay Capital Management
- Aetos Corporation
- Sage Capital Limited Partnership
- Arrowsmith Fund Ltd
- GAMut Investments Inc
- Blenheim Investments LP (Composite)

Mean VaR optimal portfolios (2)

- **Interpretations of the previous graphs**
  - **Empirical VaR leads to portfolio allocations that change quickly with the return objectives since it is based on a single rank statistic**
  - **As expected (according to the decomposition) GLS VaR leads to smooth changes in the efficient allocations**



- **VaR is not sub-additive but…**

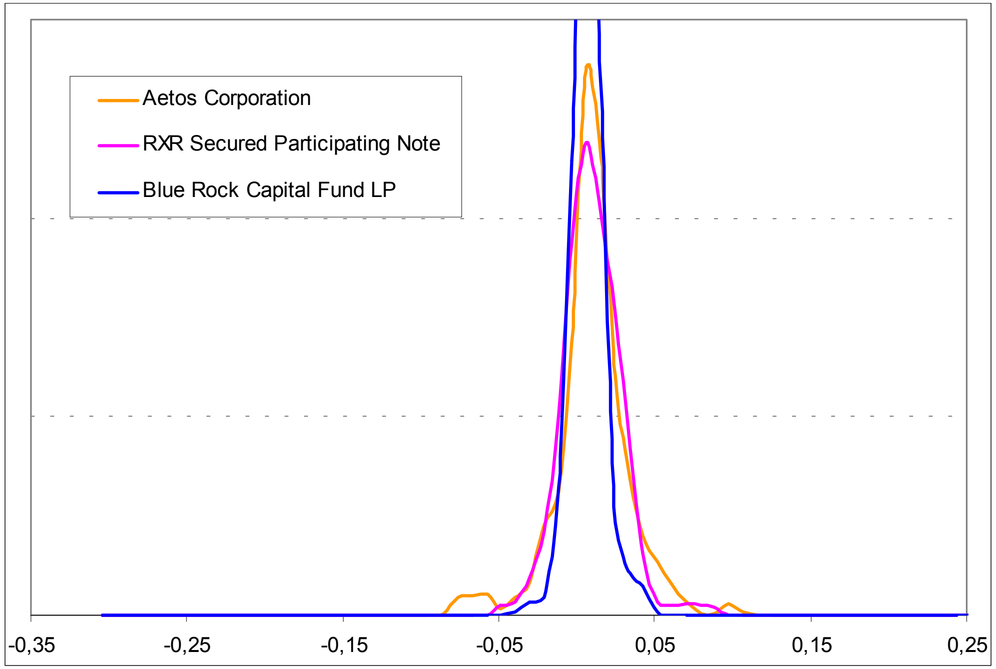
  …we find a surprisingly strong diversification effect
- **Taking into account Hedge Funds indexes leads to different result**

  (see Y. Malevergne and D. Sornette [2002], H. Geman and C. Kharoubi [2003])

Comparison of efficient portfolios

- **Comparison of efficient portfolios under VaR constraints for a given 1.2% level of expected return**

	Optimal portfolios for 1% level of return		
	Emprical VaR constrainted	Granger VaR constrainted	GLS VaR constrainted
AXA Rosenberg Market Neutral Strategy LP	3,9%	2,2%	1,0%
Discovery MasterFund Ltd	0,3%	2,0%	1,2%
Aetos Corporation	4,5%	2,4%	0,6%
Bennett Restructuring Fund LP	21,0%	26,0%	25,4%
Calamos Convertible Hedge Fund LP	0,1%	0,0%	0,0%
Sage Capital Limited Partnership	28,6%	30,4%	29,8%
Genesis Emerging Markets Fund Ltd	0,2%	0,0%	0,0%
RXR Secured Participating Note	2,1%	8,8%	6,5%
Arrowsmith Fund Ltd	2,0%	2,2%	1,2%
Blue Rock Capital Fund LP	23,6%	17,6%	25,1%
Dean Witter Cornerstone Fund IV LP	0,1%	0,0%	0,0%
GAMut Investments Inc	9,9%	6,9%	9,1%
Aquila International Fund Ltd	0,0%	0,0%	0,0%
Bay Capital Management	2,6%	1,6%	0,0%
Blenheim Investments LP (Composite)	0,3%	0,0%	0,0%
Red Oak Commodity Advisors Inc	0,9%	0,0%	0,0%



Behavior of left tails

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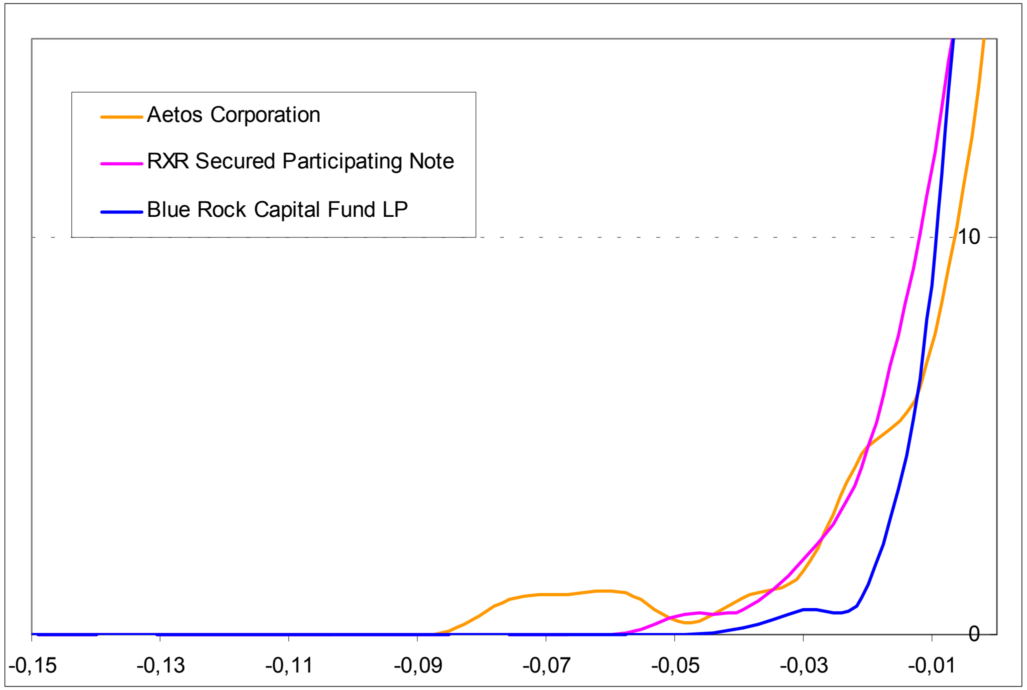


- **Analysis of 3 asset distributions and resulting allocations**
  - **Aetos C. has a fat tail : penalized by Granger VaR and even more by GLS VaR**
  - **RXR S. has a thin tail : favored by Granger VaR and GLS VaR**
  - **Blue R. has a thin extreme tail which quickly thickens : penalized by Granger VaR**

Alternative Risk Measures

Alternative risk measures

- **Recent works about risk measures properties** **(P. Artzner, F.Delbaen, J-M. Eber and D. Heath [1999], D. Tasche [2002], C. Acerbi [2002], H. Föllmer and A. Schied [2002])**
  - **widens the risk measure choice range**
- **Some choice criteria**
  - **coherence properties**
  - **numerical tractability**
- **Properties of optimal portfolios analysis**
  - **comparison of different optimal portfolios**

Expected shortfall

- Definition: mean of "losses " beyond the Value at Risk
- Properties
  - Coherent measure of risk
  - Spectral representation

$$ES_\alpha(X) = \frac{1}{\alpha}\int_0^\alpha VaR_u(X)du$$

➔ optimal portfolio may be very sensitive to extreme events if α is very low

- Algorithm
  - Linear optimization algorithms (see R.T Rockafellar & S. Uryasev [2000])

    ➔ may be based on the simplex optimization program
  - Quick computation

Downside risk

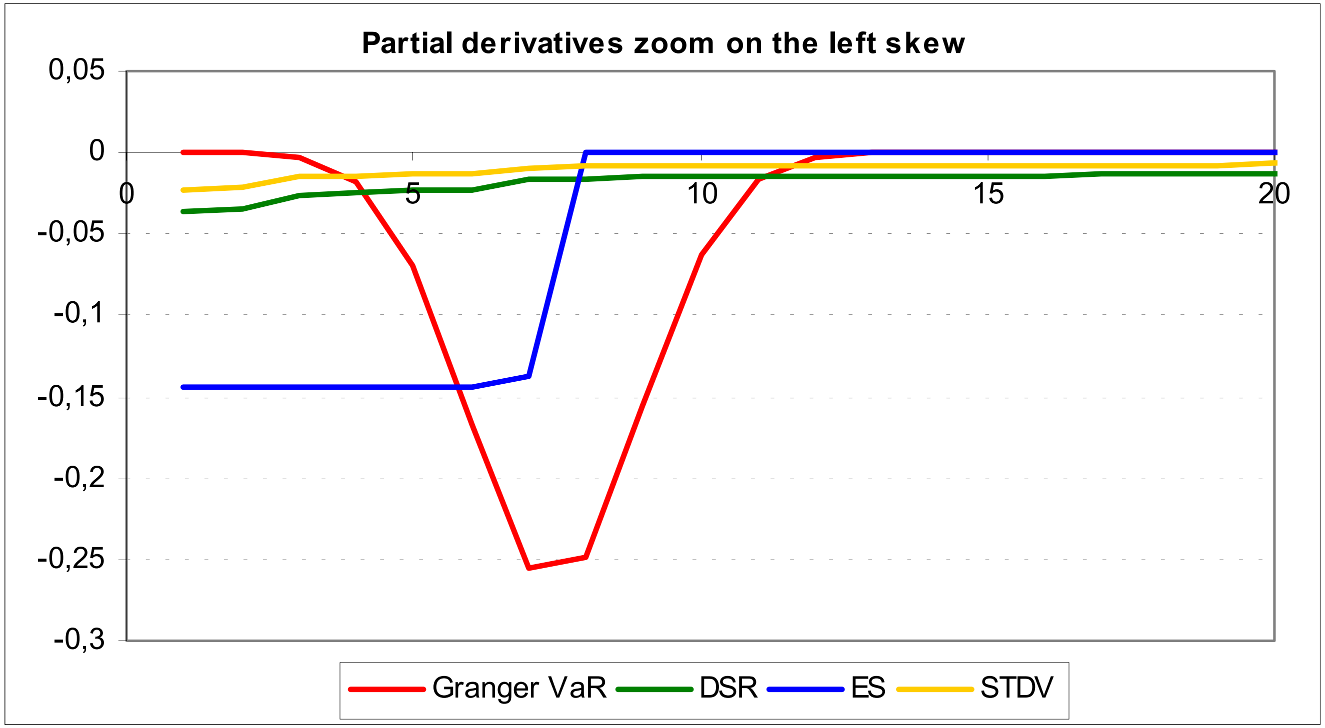
- **Definitions**
  - Let $x_1, x_2, \ldots x_n$ be the values of a portfolio (historical or simulated)
  - The downside risk is defined as follows

$$SV(X) = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left[\left(\overline{x} - x_i\right)^+\right]^2} - \overline{x}$$

- **Properties**
  - Coherent measure of risk (see T. Fischer [2001])
  - No spectral representation
    - ➔ fails to be comonotonic additive
  - Could be a good candidate to take into account the investors positive return preference
- **Algorithm**
  - Derived from the variance optimization
    - ➔ the Athayde's recursive algorithm

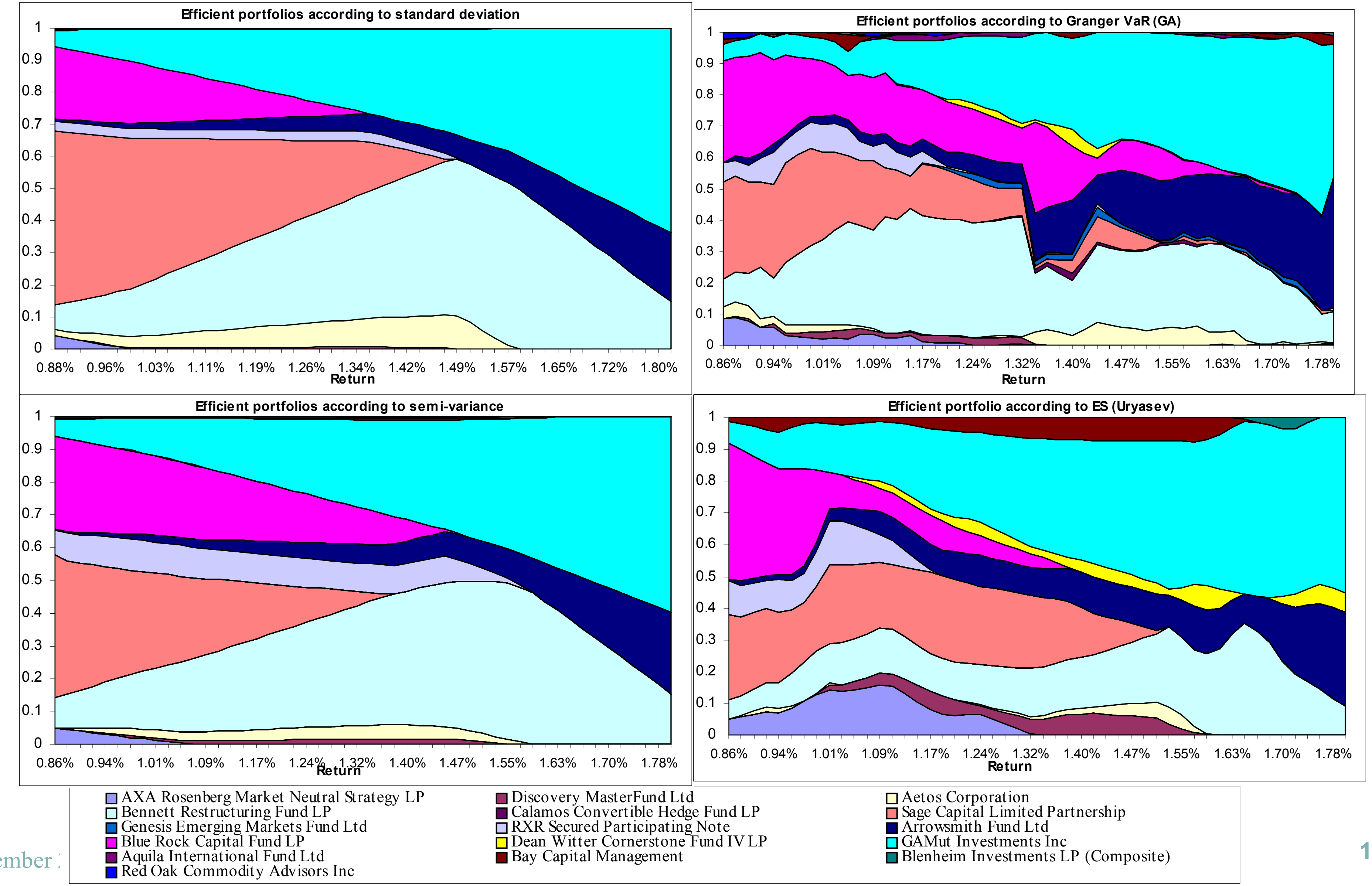
Contribution of rank statistics

- Decomposition of the risk measures as for the VaR case



- VaR and ES weights are concentrated on extreme rank statistics
- Variance and Downside risk weights exhibit a smoother weighting scheme

Optimal portfolios



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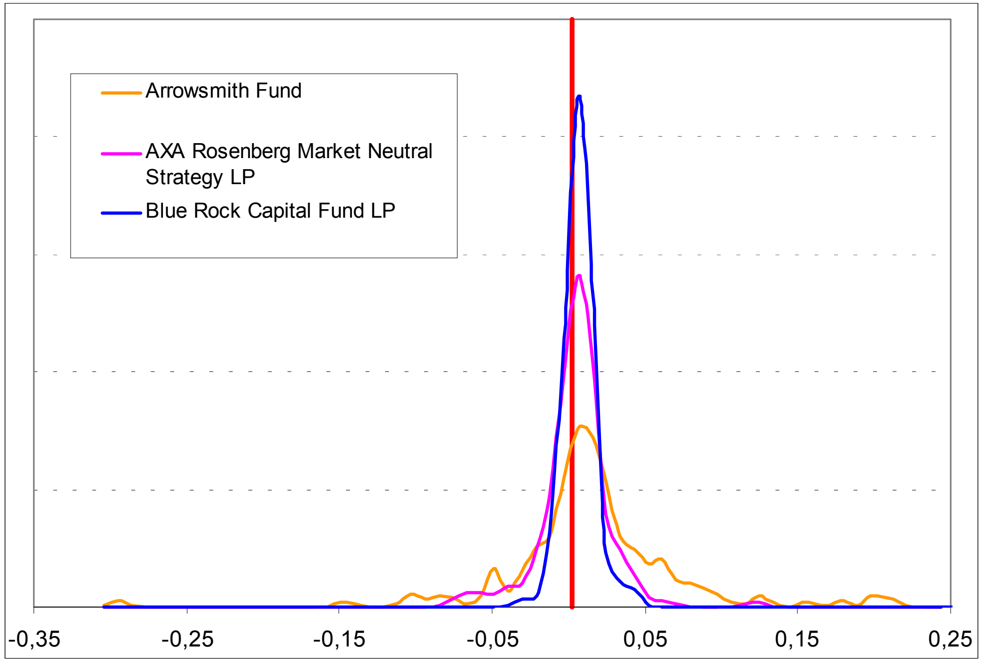
Aquila International Fund Ltd	0,0%	0,0%	0,0%
Bay Capital Management	1,6%	1,6%	0,4%
Blenheim Investments LP (Composite)	0,0%	0,0%	0,0%
Red Oak Commodity Advisors Inc	0,0%	0,0%	0,0%



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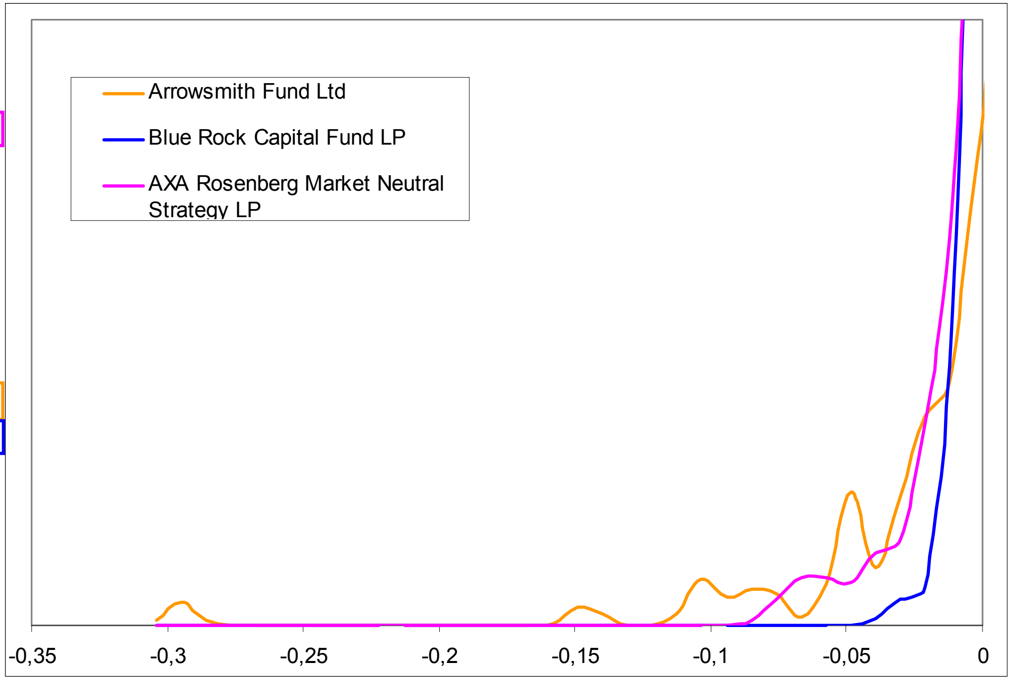

- **Analysis of 3 asset distributions and resulting allocations**
  - **Axa R. has very few points in the extreme tail : favored by Expected Shortfall**
  - **Arrowsmith. has an extreme low return : penalized by Downside Risk**
  - **Blue R. has a thin extreme tail which quickly thickens : penalized by VaR**