Trading book and credit risk: bending the binds

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based on a joint work with J-P. LAURENT and S. THOMAS.

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   - EU Corporate exposures: long only and long/short portfolios
   - Impact on 99.9% VaR
   - Drivers of risk (systematic vs idiosyncratic)
The RWA conundrum

- Basel framework: the Risk Weighted Assets (RWA)
  \[
  \text{Minimum Capital Requirement} = X\% \times RWA
  \] (1)

- Is a risk-based indicator a trustworthy one?

Source: Haldane's speech at FSA (9th April 2013) [1]
Credit risk in Basel 2.5 (IRC) and RWA variability

- **RWA for credit risk in the trading book**: Incremental Risk Charge (IRC)
  - No prescribed model (internal, often multi-factorial model for the default correlation).

Source: Haldane's speech at FSA (9th April 2013) [1]

- **Internal models implementations are in cause.**
RWA variability: Hypothetical Portfolio Exercises

*Dispersion of normalised IRC results for credit spread portfolios*

<table>
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<tr>
<th>Portfolio</th>
<th>Description</th>
</tr>
</thead>
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<td>P19</td>
<td>Sovereign CDS portfolio</td>
</tr>
<tr>
<td>P20</td>
<td>Sovereign bond/CDS portfolio</td>
</tr>
<tr>
<td>P21</td>
<td>Sector concentration portfolio</td>
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<tr>
<td>P22</td>
<td>Diversified index portfolio</td>
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<td>P23</td>
<td>Diversified index portfolio (higher concentration)</td>
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<td>P35</td>
<td>All-in portfolio comprising portfolios P19–P28</td>
</tr>
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</table>

Basel III FRTB: the Default Risk Charge (DRC)

- **RWA variability tackled**
  - Within the regulation philosophy, variability of RWA among financial institutions should mostly stem from discrepancies in activity, local jurisdictions or risk profiles.

- **Improving the RWA comparability among financial institutions**
  ⇒ Prescriptive constraints on the modelling choices for internal models

- **Basel III FRTB, RWA for credit risk:** Default Risk Charge (DRC)
  ⇒ PD,LGD, default correlation matrix
  ⇒ Based on a prescribed two-factor model for the default correlation.

- **Two papers in the literature addressing these questions**
  - **LAURENT, SESTIER and THOMAS (2015) [8]:** focuses on the correlation matrix estimation through a statistical approach
  - **WILKENS and PREDESCU (2016) [9]:** provides a full calibration methodology through an economic approach
Portfolio loss

- **One period portfolio loss**

\[
L = \sum_k EAD_k \times LGD_k \times \text{DefaultIndicator}_k
\]  

- Exposures (EAD) and Losses Given Default (LGD) assumed constant for simplicity.

⇒ Here, we focus on correlation modelling.

- **Trading book inventories**
  - Exposures may be long (sign +) or short (sign -).
  - CDS or bond exposures.

- **Latent variable model**
  - Default occurs if a latent variable, \( X_k \), lies below a threshold:

\[
\text{DefaultIndicator}_k = 1\{X_k \leq \text{threshold}_k\}
\]
Prescribed two-factor model

**Prescribed two-factor model**

"The Committee has decided to develop a more prescriptive DRC charge in the models-based framework. Banks using the internal model approach to calculate a default risk charge must use a two-factor default simulation model, which the Committee believes will reduce variation in market risk-weighted assets but be sufficiently risk sensitive as compared to multifactor models."

BCBS (2013) [5]

**Factor models**

\[ X_k = \beta_k Z + \sqrt{1 - \beta_k' \beta_k} \epsilon_k \]  

- \( Z \sim N(0, \mathbf{I}_J) \): systematic factor.
- \( \epsilon_k \sim N(0, 1) \): specific risk.
- \( \beta \in \mathbb{R}^{K,J} \): factor loadings.
- threshold \( k = \Phi^{-1}(p_k) \) with \( p_k \) the default probability of the obligor \( k \) and \( \Phi \) the Gaussian cdf.


**Not prescriptive: latent (endogeneous) or observable (exogeneous) factors**
Prescribed calibration data

- **Prescribed calibration data**

  "Default correlations must be based on credit spreads or on listed equity prices".  
  BCBS (2015) [13]

  "correlations [should] be calibrated over a one-year stress period [...] using [...] annual co-movements [...] which took place within the last ten years".  
  BCBS (2016) [7]

- Let’s consider \( X \in \mathbb{R}^{K \times T} \) the historical sample of centered returns (equity prices or CDS spreads), along two specifications:

  - **Sample** covariance matrix: \( \Sigma_{Sample} = T^{-1} X X^t \)
  - **Shrinked** covariance matrix: \( \Sigma_{Shrinkage} = \alpha \Sigma_{FactorModel} + (1 - \alpha) \Sigma_{Sample} \)

  \( \Rightarrow \) The initial correlation matrix is:  
  \( C_0 = (\text{diag}(\Sigma))^{-1/2} \Sigma (\text{diag}(\Sigma))^{-1/2} \)
Calibration approach

- No guidance by the BCBS on how to obtain a (J=2)-factor structure
  - Economic approach
    - Exogeneous variables only
    - System-wise
    - Need for an equity return model
  - Statistical approach
    - Exogeneous or endogeneous variables
    - Portfolio-wise
    - No need for an equity return model

- Nearest correlation matrix with a two-factor structure

\[
\begin{aligned}
\arg\min_{\beta} f_{obj}(\beta) &= \|C(\beta) - C_0\|_F \\
\text{subject to } \beta &\in \Omega = \{\beta \in \mathbb{R}^{K \times 2} | \beta_k' \beta_k \leq 1, k = 1, \ldots, K\}
\end{aligned}
\]

⇒ Constraint ensures that \(C(\beta) = \beta \beta^t + \text{diag}(I_d - \beta \beta^t)\) is positive semi-definite.

- PCA-based method and Spectral projected gradient method
  ANDERSEN et al. (2003) [14], BIRGIN et. al (2000, 2001) [15, 16]
## Unconstrained correlation matrix and $J$-factor model

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Data for estimating $C_0$</th>
<th>Period</th>
<th>Estimation method for $C_0$</th>
<th>Calibration method for the $J$-factor models</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Equity - P1</td>
<td>Equity returns</td>
<td>1</td>
<td>Sample correlation</td>
<td>PCA and SPG algorithms</td>
</tr>
<tr>
<td>(2) Equity - P1 Shrinked</td>
<td>Equity returns</td>
<td>1</td>
<td>Shrinkage ($\alpha_{shrink} = 0.32$)</td>
<td>PCA and SPG algorithms</td>
</tr>
<tr>
<td>(3) Equity - P1 Exogenous Factors</td>
<td>Equity returns</td>
<td>1</td>
<td>Sample correlation</td>
<td>Linear Regression</td>
</tr>
<tr>
<td>(4) Equity - P2</td>
<td>Equity returns</td>
<td>2</td>
<td>Sample correlation</td>
<td>PCA and SPG algorithms</td>
</tr>
<tr>
<td>(5) Equity - P2 Shrinked</td>
<td>Equity returns</td>
<td>2</td>
<td>Shrinkage ($\alpha_{shrink} = 0.43$)</td>
<td>PCA and SPG algorithms</td>
</tr>
<tr>
<td>(6) Equity - P2 Exogenous Factors</td>
<td>Equity returns</td>
<td>2</td>
<td>Sample correlation</td>
<td>Linear Regression</td>
</tr>
<tr>
<td>(7) IRBA</td>
<td>-</td>
<td>-</td>
<td>IRBA formula</td>
<td>PCA and SPG algorithms</td>
</tr>
<tr>
<td>(8) KMV - P2</td>
<td>-</td>
<td>2</td>
<td>GCorr methodology</td>
<td>PCA and SPG algorithms</td>
</tr>
<tr>
<td>(9) CDS - P2</td>
<td>CDS spreads</td>
<td>2</td>
<td>Sample correlation</td>
<td>PCA and SPG algorithms</td>
</tr>
</tbody>
</table>

Period 1: from 07/01/2008 to 07/01/2009. Period 2: from 09/01/2013 to 09/01/2014.
Correlation matrices - Distributions

1. Equity - P1
2. Equity - P1 - Shrinked
3. Equity - P1 - Exogenous Factors
4. Equity - P2
5. Equity - P2 - Shrinked
6. Equity - P2 - Exogenous Factors
7. IRBA
8. KMV - P2
9. CDS - P2

Frequency (%)

Pairwise correlation (%)

Unconstrained model  |  1-factor model  |  2-factor model  |  J-factor model
Specific-systematic decomposition of the loss

\[ L(Z, \varepsilon) = \sum_k EAD_k \times LGD_k \times 1 \left\{ \beta_k Z + \sqrt{1 - \beta_k^2} \leq \Phi^{-1}(p_k) \right\} \]

- **Hoeffding decomposition of the default losses**
  - Van der Vaart (2000) [17], Rosen & Saunders (2010) [12], Hoeffding (1948) [18].

\[
\begin{align*}
L(Z, \varepsilon) & = \mathbb{E}[L] \quad \text{\{ } \phi_0(L) : \text{Expected Loss} \} \\
& + \mathbb{E}[L|Z] - \mathbb{E}[L] \quad \text{\{ } \phi_1(L; Z) : \text{Systematic Loss} \} \\
& + \mathbb{E}[L|\varepsilon] - \mathbb{E}[L] \quad \text{\{ } \phi_2(L; \varepsilon) : \text{Specific Loss} \} \\
& + L(Z, \varepsilon) - \mathbb{E}[L|Z] - \mathbb{E}[L|\varepsilon] + \mathbb{E}[L] \quad \text{\{ } \phi_{1,2}(L; Z, \varepsilon) : \text{Interaction Loss} \}
\end{align*}
\]

- \( \phi_1(L; Z) \) corresponds (up to the expected loss term) to the heterogeneous Large Pool Approximation.
Portfolio risk and contributions

- **Portfolio risk**
  - Value-at-Risk: \( \text{VaR}_\alpha[L] = \inf \{l \in \mathbb{R} | P(L \leq l) \geq \alpha \} \)
  - Full allocation property: \( \text{VaR}_\alpha[L = L_1 + L_2] = E[L_1 | L = \text{VaR}_\alpha[L]] + E[L_2 | L = \text{VaR}_\alpha[L]] \)

- **Systematic-specific contribution of the portfolio risk**

\[
\text{VaR}_\alpha[L] = E[\phi_0 | L = \text{VaR}_\alpha[L]] \\
+ E[\phi_1(L; Z) | L = \text{VaR}_\alpha[L]] \\
+ E[\phi_2(L; \varepsilon) | L = \text{VaR}_\alpha[L]] \\
+ E[\phi_{1,2}(L; Z, \varepsilon) | L = \text{VaR}_\alpha[L]]
\]

- \( C_\emptyset : \text{Expected Loss Contribution} \)
- \( C_1(L; Z) : \text{Systematic Contribution} \)
- \( C_2(L; \varepsilon) : \text{Specific Contribution} \)
- \( C_{1,2}(L; Z, \varepsilon) : \text{Interaction Contribution} \)
Portfolios - Itraxx Europe - Corporates

- A diversification portfolio and a hedge portfolio are built.
- This parallels the distinction between the banking book (long positions, e.g. loans) and the banking book (long/short positions, e.g. in bonds, CDSs).

<table>
<thead>
<tr>
<th>Composition</th>
<th>Long only portfolio</th>
<th>Long/short portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long 125 names</td>
<td></td>
<td>Long 27 financial names</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Short 27 non-financial names</td>
</tr>
<tr>
<td>Exposures</td>
<td>Equaly weighted</td>
<td>Equaly weighted</td>
</tr>
<tr>
<td>Total exposure</td>
<td>= 1</td>
<td>Total exposure</td>
</tr>
<tr>
<td></td>
<td></td>
<td>= 0</td>
</tr>
</tbody>
</table>
1-year Default Probabilities

- 1-year Default Probabilities: Bloomberg Issuer Default Risk Methodology
Impacts on the risk - Long portfolio

Configurations: (1) Equity - P1; (2) Equity - P1 - Shrinked; (3) Equity - P1 - Exogenous Factors; (4) Equity - P2; (5) Equity - P2 - Shrinked; (6) Equity - P2 - Exogenous Factors; (7) IRBA; (8) KMV - P2; (9) CDS - P2. $J^*$-factor model is only active for “(1) Equity – P1” and “(4) Equity – P2” configurations.
Impacts on the risk - Long-short portfolio

Configurations: (1) Equity - P1; (2) Equity - P1 - Shrinked; (3) Equity - P1 - Exogenous Factors; (4) Equity - P2; (5) Equity - P2 - Shrinked; (6) Equity - P2 - Exogenous Factors; (7) IRBA; (8) KMV - P2; (9) CDS - P2. $J^*$-factor model is only active for “(1) Equity – P1” and “(4) Equity – P2” configurations.
Systematic contribution to the risk - Long portfolio

Configurations: (1) Equity - P1; (2) Equity - P1 - Shrinked; (3) Equity - P1 - Exogenous Factors; (4) Equity - P2; (5) Equity - P2 - Shrinked; (6) Equity - P2 - Exogenous Factors; (7) IRBA; (8) KMV - P2; (9) CDS - P2. $J^*$-factor model is only active for “(1) Equity – P1” and “(4) Equity – P2” configurations.
Systematic contribution to the risk - Long-short portfolio

Configurations: (1) Equity - P1; (2) Equity - P1 - Shrinked; (3) Equity - P1 - Exogenous Factors; (4) Equity - P2; (5) Equity - P2 - Shrinked; (6) Equity - P2 - Exogenous Factors; (7) IRBA; (8) KMV - P2; (9) CDS - P2. $J^*$-factor model is only active for “(1) Equity – P1” and “(4) Equity – P2” configurations.
Conclusions - RWA variability and comparability

• The RWA variability stemming from correlation modelling remains high.
  - It is a challenge regarding model comparability.
  - Two factor constraint is more active in stressed periods (2008)
  - The prescriptions might prove quite useful when dealing with a large number of assets: unconstrained correlation matrix (with small eigenvalues) would ease the building of opportunistic portfolios.

• Other main sources of variability
  - The high confidence level of the regulatory risk measure;
  - Disparities among correlation matrices (type of data and/or the calibration period).
⇒ Small changes in exposures or other parameters may lead to significant changes in the credit VaR, jeopardizing the comparability of RWA.

• The use of Large Pool Approximation is questionable: poor contribution to the VaR
  ⇒ Bending the binds does not seem fundamental enough yet . . .
  ⇒ Need for more research on impacts on regulatory risk of estimation and calibration methods of the correlation matrix . . .
Bibliography I


