Trading book and credit risk : bending the binds

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Phast SOLUTIONS

Contents

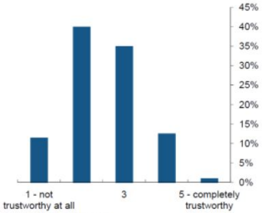
The RWA conundrum

- **Basel framework : the Risk Weighted Assets (RWA)**

$$\text{Minimum Capital Requirement} = X\% \times \text{RWA} \qquad (1)$$

- **Is a risk-based indicator a trustworthy one?**

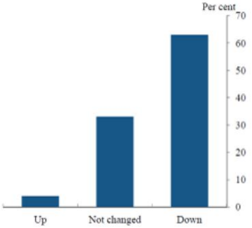
Chart 9: Survey responses to "How much do you trust risk weights?"(a)



Source: Barclays Capital.

(a) Based on survey responses of over 130 investors carried out in H1 2012, of perceptions over the previous year.

Chart 10: Survey responses to "Has your confidence in risk-weighted assets gone up or down?"(a)



Source: Barclays Capital.

(a) Based on survey responses of over 130 investors carried out in H1 2012, of perceptions over the previous year.

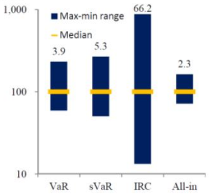
Source: Haldane's speech at FSA (9th April 2013) [1]

Credit risk in Basel 2.5 (IRC) and RWA variability

- **RWA for credit risk in the trading book**: Incremental Risk Charge (IRC)
  BCBS - Basel 2.5 (2009) [2]

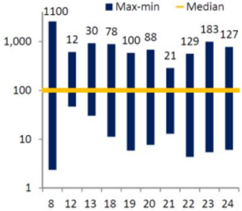
⇒ No prescribed model (internal, often multi-factorial model for the default correlation).

Chart 5: Risk weight variability in the trading book[(a)(b)(c)(d)]



Source: BCBS
(a) From the BCBS hypothetical portfolio exercise for the trading book.
(b) Sample consists of 15 banks.
(c) Values have been normalised by the median. For each model, the ranges represent the simple average of the normalised minima and maxima for all portfolios the model was applied to. For the all-in portfolio, the supervisory multiplier was held constant.
(d) Numbers on bars indicate maximum - minimum ratios.

Chart 6: Risk weight variability in the IRC model[(a)]



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(a) From the BCBS hypothetical portfolio exercise for the trading book.
(b) Sample consists of 15 banks.
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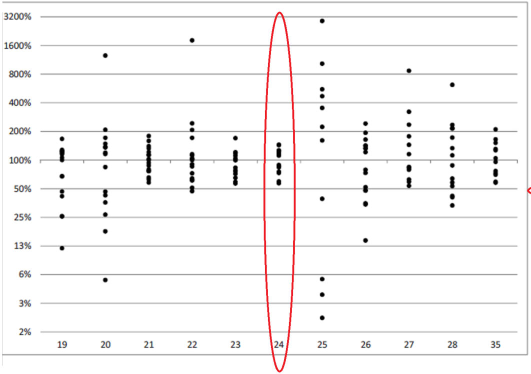
Source: Haldane's speech at FSA (9th April 2013) [1]

- **Internal models implementations are in cause.**

RWA variability : Hypothetical Portfolio Exercices

Dispersion of normalised IRC results for credit spread portfolios



Portfolio	Description
P19	Sovereign CDS portfolio
P20	Sovereign bond/CDS portfolio
P21	Sector concentration portfolio
P22	Diversified index portfolio
P23	Diversified index portfolio (higher concentration)
P24	Diversified corporate portfolio
P25	Index basis trade on iTraxx 5-year Europe index
P26	CDS bond basis on 5 financials
P27	Short index put on iTraxx Europe Crossover
P28	Quanto CDS on Spain with delta hedge
P35	All-in portfolio comprising portfolios P19–P28

Note: Normalisation is defined as dividing it by its median;
The vertical axis in each panel is a base 2 log scale.

Source: BCBS - Regulatory Consistency Assessment Program, 2nd report on RWA in the trading book (2013) [3]

Basel III FRTB: the Default Risk Charge (DRC)

- **RWA variability tackled**
  - Within the regulation philosophy, variability of RWA among financial institutions should mostly stem from discrepancies in activity, local jurisdictions or risk profiles.
- **Improving the RWA comparability among financial institutions**
  - ⇒ Prescriptive constraints on the modelling choices for internal models
- **Basel III FRTB, RWA for credit risk**: Default Risk Charge (DRC)
  BCBS - Fundamental Review of the Trading Book (2012, 2013, 2015, 2016) [4, 5, 6, 7]
  - ⇒ PD,LGD, default correlation matrix
  - ⇒ Based on a prescribed two-factor model for the default correlation.
- **Two papers in the literature addressing these questions**
  - LAURENT, SESTIER and THOMAS (2015) [8]: focuses on the correlation matrix estimation through a statistical approach
  - WILKENS and PREDESCU (2016) [9]: provides a full calibration methodology through an economic approach

Portfolio loss

- **One period portfolio loss**

$$L = \sum_k EAD_k \times LGD_k \times \text{DefaultIndicator}_k \quad (2)$$

- Exposures (EAD) and Losses Given Default (LGD) assumed constant for simplicity.

⇒ Here, we focus on correlation modelling.

- **Trading book inventories**
  - Exposures may be long (sign +) or short (sign -).
  - CDS or bond exposures.

- **Latent variable model**
  - Default occurs if a latent variable, X_k, lies below a threshold:

$$\text{DefaultIndicator}_k = 1_{\{X_k \leq \text{threshold}_k\}} \quad (3)$$

Prescribed two-factor model

- **Prescribed two-factor model**

  "The Committee has decided to develop a more prescriptive DRC charge in the models-based framework. Banks using the internal model approach to calculate a default risk charge must use a **two-factor default simulation model**, *which the Committee believes will* **reduce variation** *in* **market risk-weighted assets** *but be* **sufficiently risk sensitive** *as compared to multifactor models."*

  BCBS (2013) [5]

- **Factor models**

$$X_k = \beta_k Z + \sqrt{1 - \beta_k' \beta_k} \epsilon_k \tag{4}$$

  - $Z \sim N(\mathbf{0}, \mathbf{Id_J})$: systematic factor.
  - $\epsilon_k \sim N(0, 1)$: specific risk.
  - $\beta \in \mathbb{R}^{K,J}$: factor loadings.

- threshold$_k$ $= \Phi^{-1}(p_k)$ with p_k the default probability of the obligor k and Φ the Gaussian cdf.

  MERTON (1974) [10], BCBS (IRB) (2004) [11], ROSEN & SAUNDERS (2010) [12].

- **Not prescriptive: latent (endogeneous) or observable (exogeneous) factors**

Prescribed calibration data

- **Prescribed calibration data**

  "Default correlations must be based on credit spreads or on listed equity prices".
  BCBS (2015) [13]

  "correlations [should] be calibrated over a one-year stress period [...] using [...] annual co-movements [...] which took place within the last ten years".
  BCBS (2016) [7]

- Let's consider $X \in \mathbb{R}^{K \times T}$ the historical sample of centered returns (equity prices or CDS spreads), along two specifications:

$$\begin{aligned} \text{Sample covariance matrix :} \quad \Sigma_{Sample} &= T^{-1} X X^t \\ \text{Shrinked covariance matrix :} \quad \Sigma_{Shrinkage} &= \alpha \Sigma_{FactorModel} + (1-\alpha) \Sigma_{Sample} \end{aligned}$$

$\Rightarrow$ The initial correlation matrix is: $C_0 = (diag(\Sigma))^{-1/2} \Sigma (diag(\Sigma))^{-1/2}$

Calibration approach

- **No guidance by the BCBS on how to obtain a (J=2)-factor structure**

- Economic approach
  - Exogeneous variables only
  - System-wise
  - Need for an equity return model
- **Statistical approach**
  - Exogeneous or endogeneous variables
  - Portfolio-wise
  - No need for an equity return model

- **Nearest correlation matrix with a two-factor structure**

$$\begin{cases} \arg\min_\beta f_{obj}(\beta) & = \|C(\beta) - C_0\|_F \\ \text{subject to } \beta \in \Omega & = \{\beta \in \mathbb{R}^{K\times 2} | \beta_k' \beta_k \leq 1, k = 1, \ldots, K\} \end{cases}$$

⇒ Constraint ensures that $C(\beta) = \beta\beta^t + diag(Id - \beta\beta^t)$ is positive semi-definite.

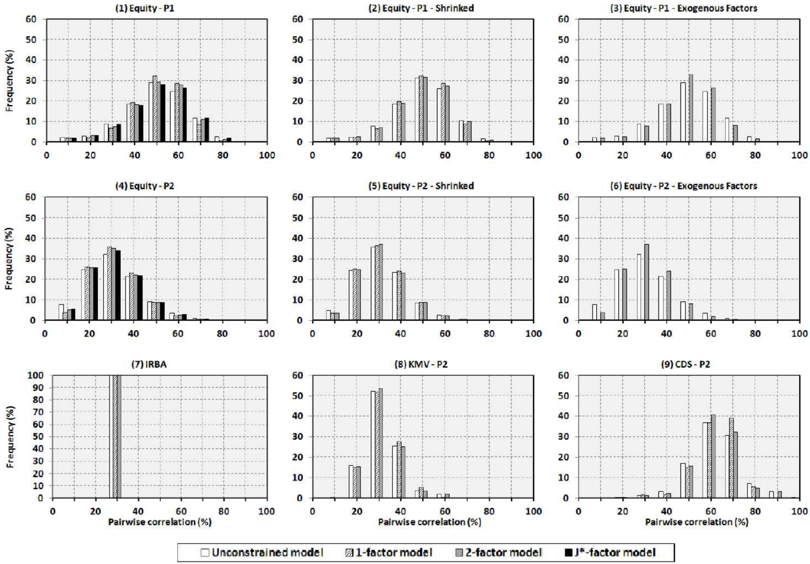
- PCA-based method and Spectral projected gradient method
  ANDERSEN et al. (2003) [14], BIRGIN et. al (2000, 2001) [15, 16]

Unconstrained correlation matrix and J-factor model

	Configuration	Data for estimating C_0	Period	Estimation method for C_0	Calibration method for the J-factor models
(1)	**Equity - P1**	Equity returns	1	Sample correlation	PCA and SPG algorithms
(2)	**Equity - P1 Shrinked**	Equity returns	1	Shrinkage ($\alpha_{shrink} = 0.32$)	PCA and SPG algorithms
(3)	**Equity - P1 Exogenous Factors**	Equity returns	1	Sample correlation	Linear Regression
(4)	**Equity - P2**	Equity returns	2	Sample correlation	PCA and SPG algorithms
(5)	**Equity - P2 Shrinked**	Equity returns	2	Shrinkage ($\alpha_{shrink} = 0.43$)	PCA and SPG algorithms
(6)	**Equity - P2 Exogenous Factors**	Equity returns	2	Sample correlation	Linear Regression
(7)	**IRBA**	-	-	IRBA formula	PCA and SPG algorithms
(8)	**KMV - P2**	-	2	GCorr methodology	PCA and SPG algorithms
(9)	**CDS - P2**	CDS spreads	2	Sample correlation	PCA and SPG algorithms

Period 1: from 07/01/2008 to 07/01/2009. Period 2: from 09/01/2013 to 09/01/2014.

Correlation matrices - Distributions



Specific-systematic decomposition of the loss

$$L(Z,\varepsilon) = \sum_k EAD_k \times LGD_k \times 1_{\{\beta_k Z + \sqrt{1-\beta_k' \beta_k}\epsilon_k \leq \Phi^{-1}(p_k)\}}$$

- **Hoeffding decomposition of the default losses**
  VAN DER VAART (2000) [17], ROSEN & SAUNDERS (2010) [12], HOEFFDING (1948) [18].

$$\begin{aligned} L(Z,\epsilon) &= \mathbb{E}[L] && \} \ \phi_\emptyset(L) : \text{ Expected Loss} \\ &+ \mathbb{E}[L|Z] - \mathbb{E}[L] && \} \ \phi_1(L;Z) : \text{ Systematic Loss} \\ &+ \mathbb{E}[L|\varepsilon] - \mathbb{E}[L] && \} \ \phi_2(L;\varepsilon) : \text{ Specific Loss} \\ &+ L(Z,\epsilon) - \mathbb{E}[L|Z] - \mathbb{E}[L|\varepsilon] + \mathbb{E}[L] && \} \ \phi_{1,2}(L;Z,\varepsilon) : \text{ Interaction Loss} \end{aligned}$$

- $\phi_1(L;Z)$ corresponds (up to the expected loss term) to the heterogeneous Large Pool Approximation.

Portfolio risk and contributions

- **Portfolio risk**

- Value-at-Risk: $VaR_\alpha[L] = \inf\{l \in \mathbb{R} | \mathbb{P}(L \leq l) \geq \alpha\}$
- Full allocation property: $VaR_\alpha[L = L_1 + L_2] = \mathbb{E}[L_1 | L = VaR_\alpha[L]] + \mathbb{E}[L_2 | L = VaR_\alpha[L]]$

- **Systematic-specific contribution of the portfolio risk**

$$
\begin{aligned}
VaR_\alpha[L] &= \mathbb{E}[\phi_\emptyset | L = VaR_\alpha[L]] && \} \; C_\emptyset : \text{Expected Loss Contribution} \\
&+ \mathbb{E}[\phi_1(L; Z) | L = VaR_\alpha[L]] && \} \; C_1(L; Z) : \text{Systematic Contribution} \\
&+ \mathbb{E}[\phi_2(L; \varepsilon) | L = VaR_\alpha[L]] && \} \; C_2(L; \varepsilon) : \text{Specific Contribution} \\
&+ \mathbb{E}[\phi_{1,2}(L; Z, \varepsilon) | L = VaR_\alpha[L]] && \} \; C_{1,2}(L; Z, \varepsilon) : \text{Interaction Contribution}
\end{aligned}
$$

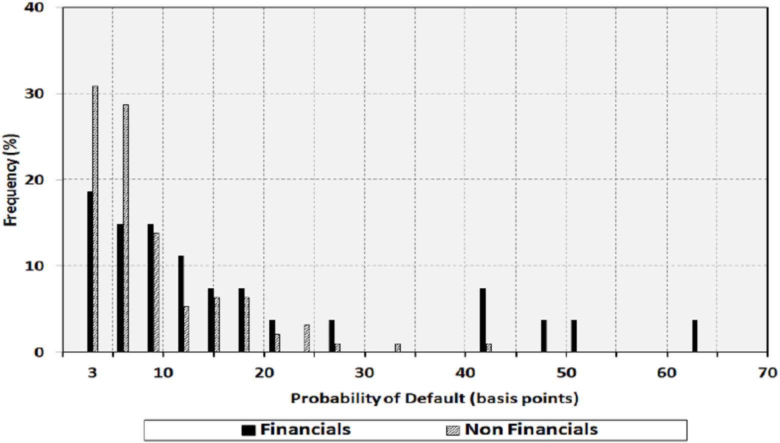
Portfolios - Itraxx Europe - Corporates

- A diversification portfolio and a hedge portfolio are built.
- This parallels the distinction between the banking book (long positions, e.g. loans) and the banking book (long/short positions, e.g. in bonds , CDSs).

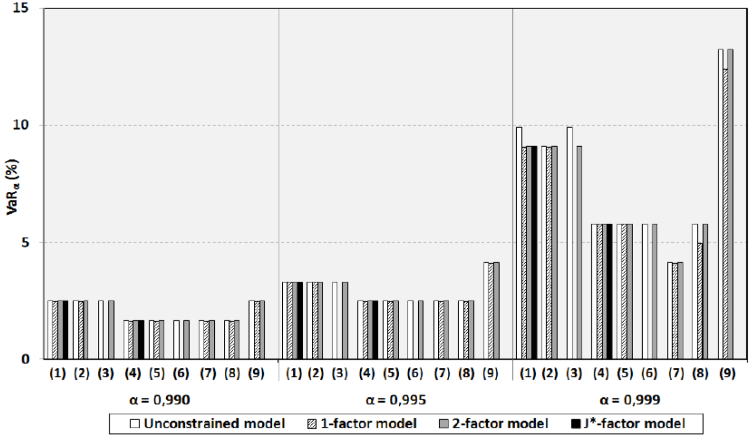
	Long only portfolio	Long/short portfolio
Composition	Long 125 names	Long 27 financial names Short 27 non-financial names
Exposures	Equaly weighted Total exposure = 1	Equaly weighted Total exposure = 0

1-year Default Probabilities

- **1-year Default Probabilities**: Bloomberg Issuer Default Risk Methodology

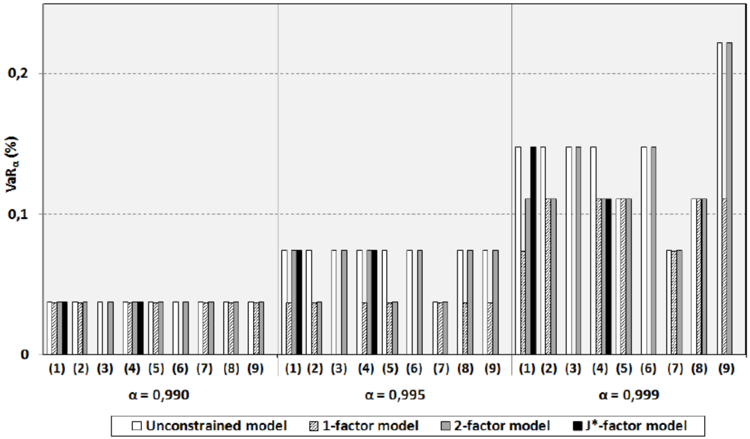


Impacts on the risk - Long portfolio



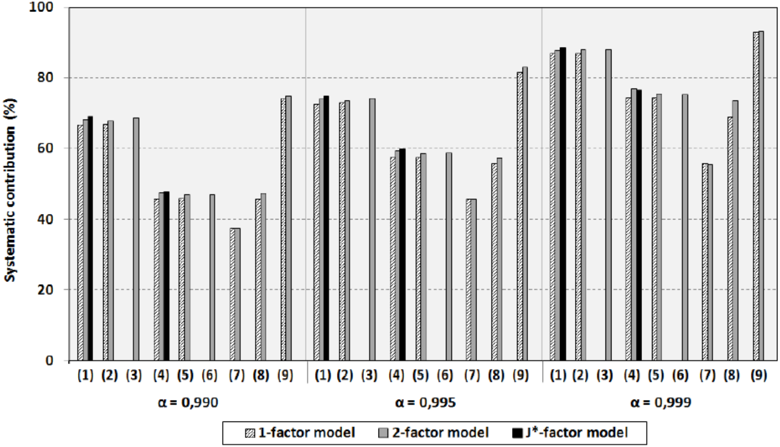
Configurations: (1) Equity - P1; (2) Equity - P1 - Shrinked; (3) Equity - P1 - Exogenous Factors; (4) Equity - P2; (5) Equity - P2 - Shrinked; (6) Equity - P2 - Exogenous Factors; (7) IRBA; (8) KMV - P2; (9) CDS - P2. J^*-factor model is only active for "(1) Equity – P1" and "(4) Equity – P2" configurations.

Impacts on the risk - Long-short portfolio



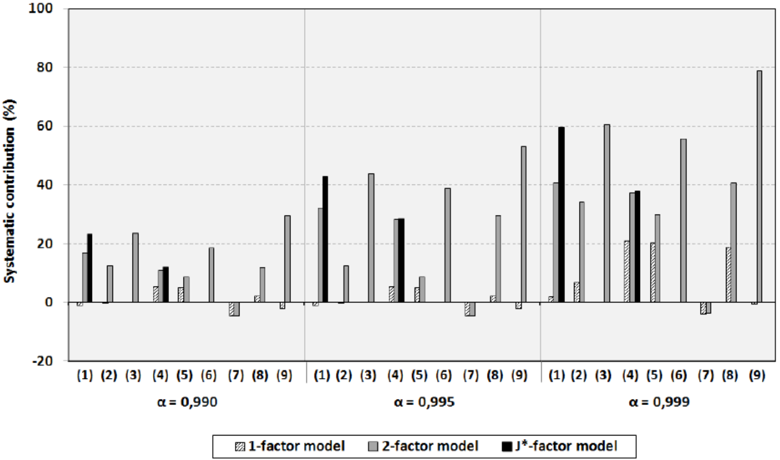
Configurations: (1) Equity - P1; (2) Equity - P1 - Shrinked; (3) Equity - P1 - Exogenous Factors; (4) Equity - P2; (5) Equity - P2 - Shrinked; (6) Equity - P2 - Exogenous Factors; (7) IRBA; (8) KMV - P2; (9) CDS - P2. J^*-factor model is only active for "(1) Equity – P1" and "(4) Equity – P2" configurations.

Systematic contribution to the risk - Long portfolio



Configurations: (1) Equity - P1; (2) Equity - P1 - Shrinked; (3) Equity - P1 - Exogenous Factors; (4) Equity - P2; (5) Equity - P2 - Shrinked; (6) Equity - P2 - Exogenous Factors; (7) IRBA; (8) KMV - P2; (9) CDS - P2. J^*-factor model is only active for "(1) Equity – P1" and "(4) Equity – P2" configurations.

Systematic contribution to the risk - Long-short portfolio



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Conclusions - RWA variability and comparability

- **The RWA variability stemming from correlation modelling remains high.**
  - It is a challenge regarding model comparability.
  - Two factor constraint is more active in stressed periods (2008)
  - The prescriptions might prove quite useful when dealing with a large number of assets: unconstrained correlation matrix (with small eigenvalues) would ease the building of opportunistic portfolios.
- **Other main sources of variability**
  - The high confidence level of the regulatory risk measure;
  - Disparities among correlation matrices (type of data and/or the calibration period).

$\Rightarrow$ Small changes in exposures or other parameters may lead to significant changes in the credit VaR, jeopardizing the comparability of RWA.

- **The use of Large Pool Approximation is questionable: poor contribution to the VaR**

$\Rightarrow$ **Bending the binds does not seem fundamental enough yet . . .**

$\Rightarrow$ **Need for more research on impacts on regulatory risk of estimation and calibration methods of the correlation matrix . . .**

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